Metaheuristics for Smart Manufacturing

9. Why is optimization difficult?

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Outline

1. Premature Convergence
2. Ruggedness & Causality
3. Deceptiveness & Neutrality
4. Epistasis
5. Scalability
6. Summary

The slides are available at http://iao.hfuu.edu.cn/155, the book at http://thomasweise.github.io/aitoa, and the source code at http://www.github.com/thomasWeise/aitoa-code

course book

course material

An Introduction to Optimization Algorithms

The contents of this course are available as free electronic book *"An Introduction to Optimization Algorithms"* [1] at http://thomasweise.github.io/aitoa in pdf, html, azw3, and epub format, created with our bookbuildeR tool chain.

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  - There may be different aspects that render a problem difficult.

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- What features of problems or algorithms allow us to get good solutions?
- When/why can we not get good solutions?

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Convergence

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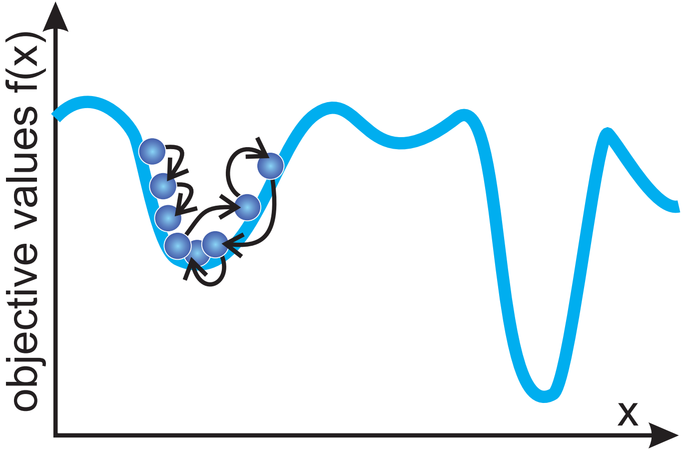
Definition (multi-modality)

A function/optimization problem is multi-modal if it has more than one minimum / maximum /optimum. [4–8]

Section Outline

- Premature Convergence = convergence to local optimum [2, 3]



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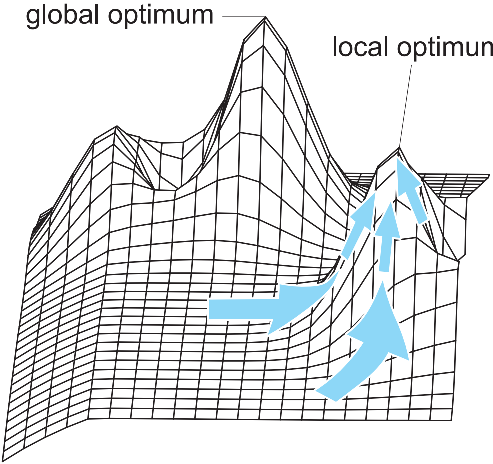

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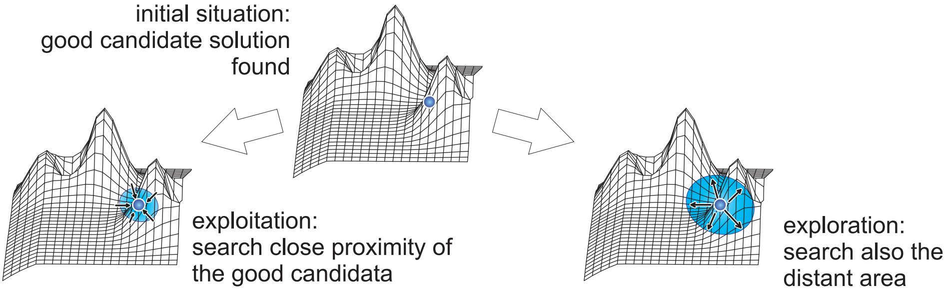
- This is called the Exploration versus Exploitation Dilemma [9–16]



Countermeasures

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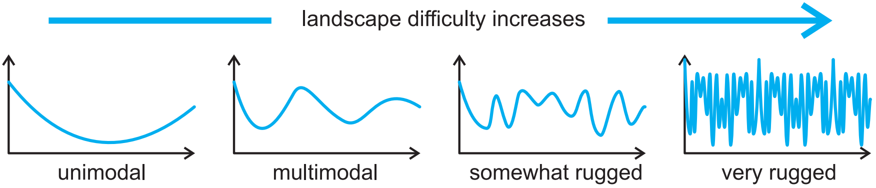
  4. Sharing, Niching, and Clearing in EAs [27–40]:
     - In an EA, give solutions that are very similar to each other a worse fitness

Ruggedness and Weak Causality



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- What happens if the causality in a problem is weak?
- The "memory" inside the search becomes useless, because a better solution may appear anywhere, not just next to a good solution (and the same holds for worse solutions the other way around...)

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        - Combinations of Evolutionary Algorithms with local search, or combinations of Evolutionary Algorithms with other concepts from Machine Learning

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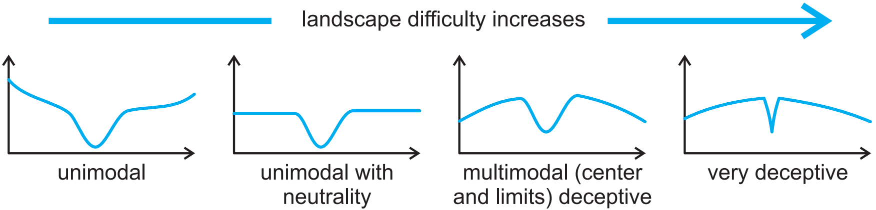
   - Update M, then repeat

Section Outline

Deceptiveness

- Gradient and information lead optimizer away from optimum [60, 68, 69]



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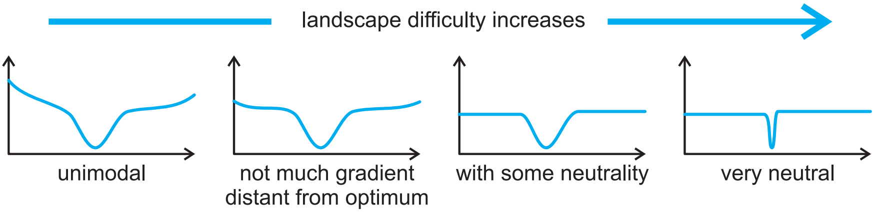
     - Frequency Fitness Assignment [78]

Neutrality

- Neutrality: Many candidate solutions have same objective values
- Little or no information gained from sampling the solution space



- Why??
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Epistasis: The other root of all evil...

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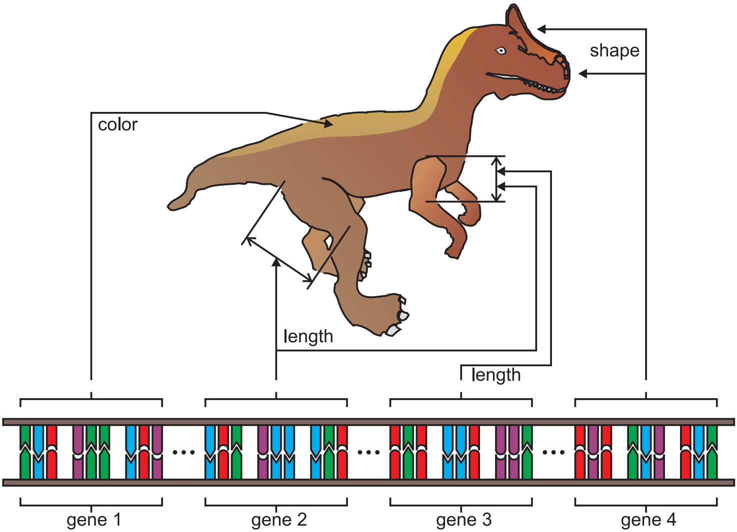
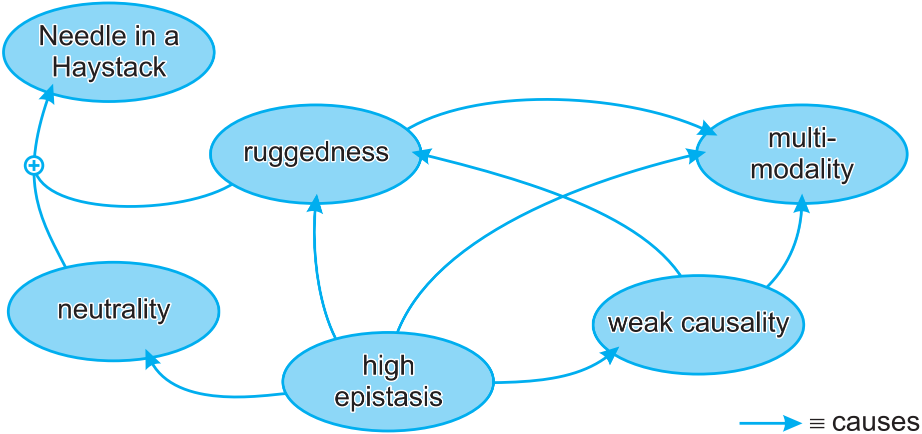
- non-epistatic (separable) problems can be solved efficiently by decomposition


shape
color
length
length
gene 1
gene 2
gene 3
gene 4

Influence of Epistasis



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   - For example: In binary operator, try to always pass such variables together to the offspring and to not separate them

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5. If epistasis is limited: cooperative-coevolution approach [103–105] (see later)

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- Example: search in $(1 \dots 10)^n$
- any algorithm (for non-trivial problems) takes longer for larger inputs...

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     - Distribution: Use multiple computers in a network [123, 124], a cluster, or a grid

- Countermeasures
  1. Parallelization and distribution
  2. Indirect Representation 1: Generative [108, 109]

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     - points in search space have few variables, search space is smaller, can be explored more easily

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     - They are mapped to larger, more complex candidate solutions by a simple functional mapping γ

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  2. Indirect Representation 1: Generative [108, 109]:
     - points in search space have few variables, search space is smaller, can be explored more easily
     - They are mapped to larger, more complex candidate solutions by a simple functional mapping γ
     - Utilizes/assumes symmetries in the candidate solutions

Countermeasures

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  3. Indirect Representation 2: Development [110, 111]

Countermeasures

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     - But: Mapping γ is more complex, a simulation which incorporates feedback from an environment or the objective function

Countermeasures

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  1. Parallelization and distribution
  2. Indirect Representation 1: Generative [108, 109]
  3. Indirect Representation 2: Development [110, 111]:
     - Similar to generative mapping, the search space is smaller
     - But: Mapping γ is more complex, a simulation which incorporates feedback from an environment or the objective function
     - Better behavior than generative mappings

Countermeasures

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  1. Parallelization and distribution
  2. Indirect Representation 1: Generative [108, 109]
  3. Indirect Representation 2: Development [110, 111]
  4. Exploiting Separability, e.g., with coevolution [103–105, 112, 113]

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     - Try to divide the problem into (almost) unrelated problems with smaller search spaces

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     - Try to divide the problem into (almost) unrelated problems with smaller search spaces
     - Solve them more or less separately, combine solutions to get overall solution, and repeat

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     - Try to divide the problem into (almost) unrelated problems with smaller search spaces
     - Solve them more or less separately, combine solutions to get overall solution, and repeat
     - Cooperative Coevolution [103, 105]: Use an EA that can find our how to divide the problem by itself and then applies the above

Countermeasures

- Countermeasures
  1. Parallelization and distribution
  2. Indirect Representation 1: Generative [108, 109]
  3. Indirect Representation 2: Development [110, 111]
  4. Exploiting Separability, e.g., with coevolution [103–105, 112, 113]
  5. Using multiple algorithms at once [114] or portfolios [115]

Section Outline

- Question: Can an optimization algorithm A be better than algorithm B?

No Free Lunch Theorem

- Question: Can an optimization algorithm A be better than algorithm B?
- Question: Can an optimization algorithm A be better than Random Sampling?

No Free Lunch Theorem

- Question: Can an optimization algorithm A be better than algorithm B?
- Question: Can an optimization algorithm A be better than Random Sampling?
- Wolpert and Macready [125] – No Free Lunch Theorem: Over all optimization problems ϕ over finite domains, the sum of the probabilities to reach a certain objective value y after m steps with algorithm A is the same as with algorithm B

$$\sum_{\forall\phi} P(y|\phi, m, A) = \sum_{\forall\phi} P(y|\phi, m, B) \tag{1}$$

No Free Lunch Theorem

- According to the No Free Lunch Theorem, the answers are:

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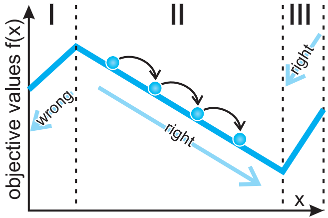
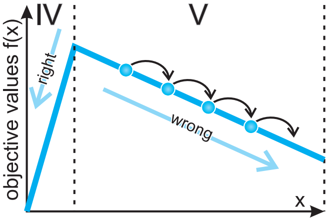
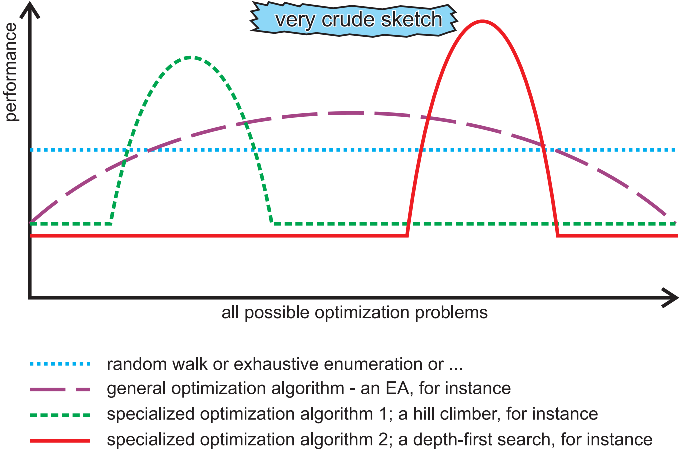
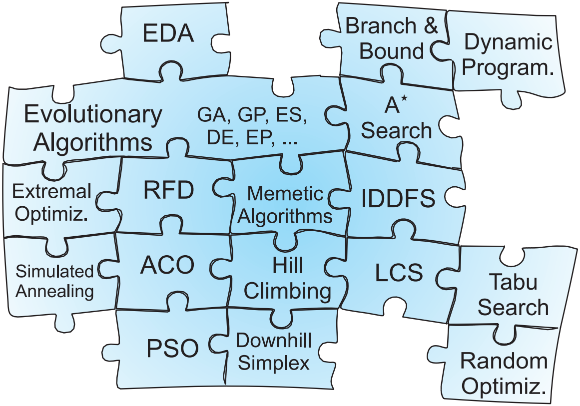




- Different algorithms are good for different problems



- Different algorithms are good for different problems and not all possible problems actually occur in practice [34, 126, 127]



Summary

- Optimization is difficult
- Metaheuristic optimizers may converge prematurely or non-uniformly
- Ruggedness is not good
- Deceptiveness is not good
- Neutrality is not good
- Epistasis is always bad – and often a representation issue!
- Large problem scales are not good
- No Free Lunch Theorem

谢谢

Thank you

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Caspar David Friedrich, “Der Wanderer über dem Nebelmeer”, 1818
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