合肥學院
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Metaheuristics for Smart Manufacturing

6. Simulated Annealing

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Outline

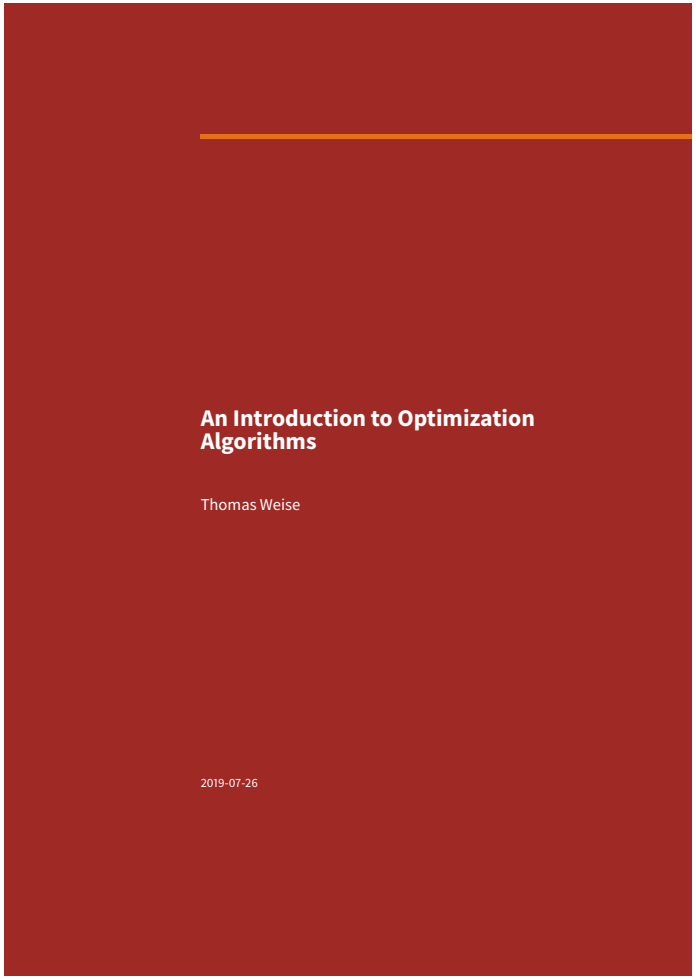
The slides are available at http://iao.hfuu.edu.cn/155, the book at http://thomasweise.github.io/aitoa, and the source code at http://www.github.com/thomasWeise/aitoa-code

course book

course material

An Introduction to Optimization Algorithms

The contents of this course are available as free electronic book *"An Introduction to Optimization Algorithms"* [1] at http://thomasweise.github.io/aitoa in pdf, html, azw3, and epub format, created with our bookbuildeR tool chain.



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- A hill climbing algorithm is likely to get stuck at local optima, which may vary in quality.
- We found that we can utilize this variance of the result quality by restarting the optimization process when it could not improve any more.
- Such a restart is costly, as it forces the local search to start completely from scratch (while we, of course, remember the best-ever solution in a variable hidden from the algorithm).

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- We will subsequently spend time to re-discover them in the hope that this will happen in a way that allows us to eventually reach the global optimum itself.
- But maybe there is a less-costly way?

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  3. The probability also decreases with the number of performed search steps.

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- An initial, brittle crystal structure is transformed to a much better configuration by stepping over good and bad states

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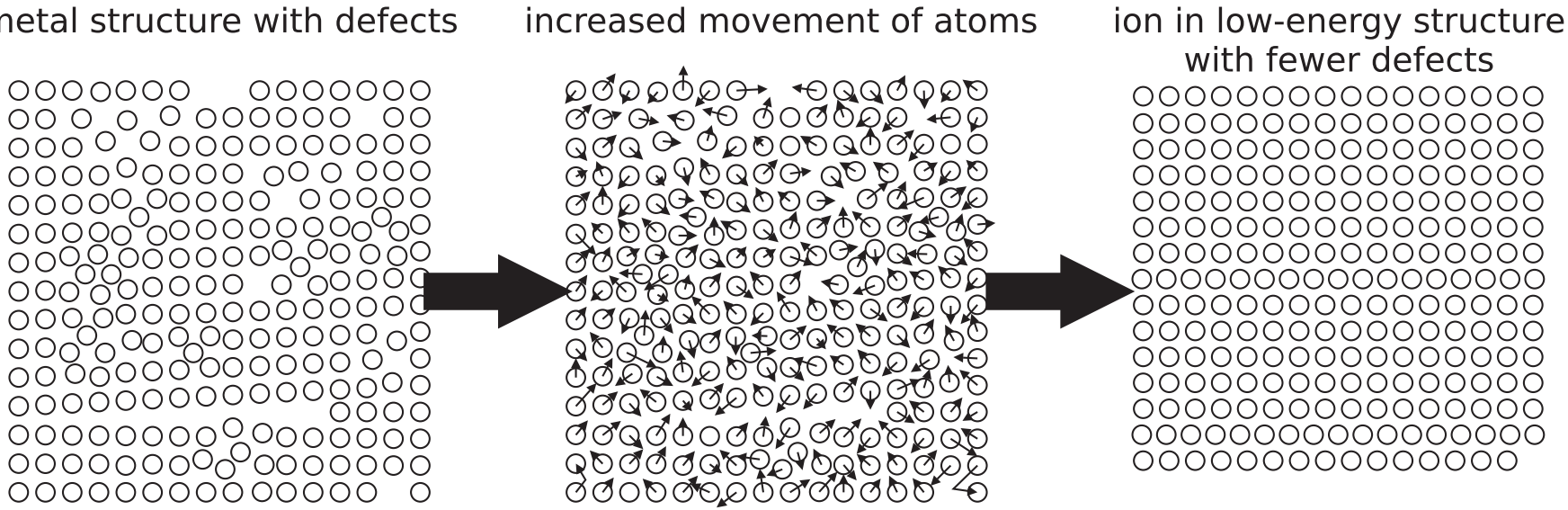

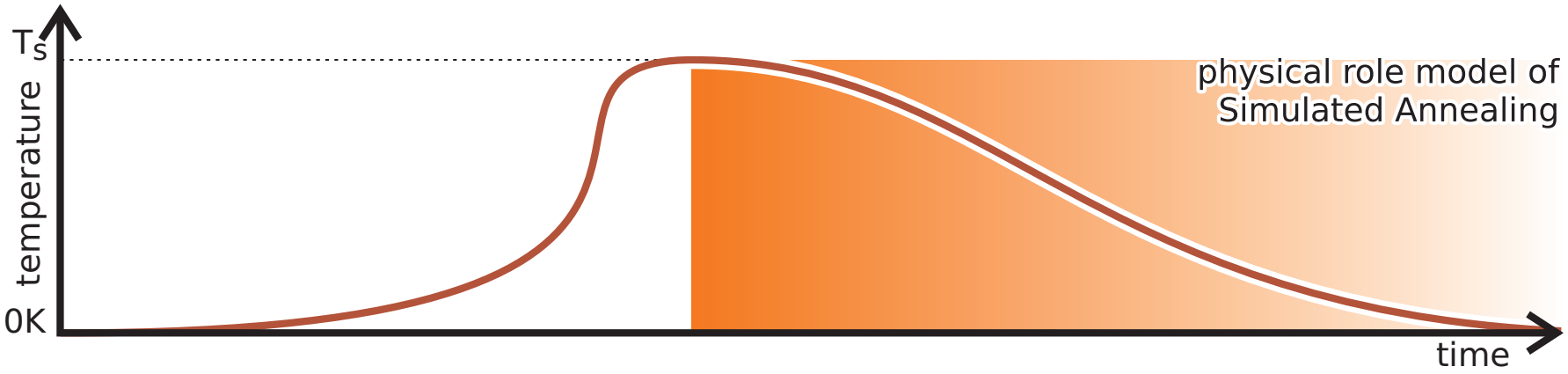

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- With each such state, there is an energy E associated: If many of the particles move around quickly or are in an unfavourable position, the energy E is high and if they form a nice crystal and don't move, the energy is low
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$$k_b = 1.380650524 * 10^{-27} J/K \text{ is the Boltzmann constant} \quad (1)$$

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- A system at temperature T has the probability $e^{-\frac{E}{k_B * T}}$ to be in a macro state with energy E.
- In other words: The higher the temperature, the higher the chance to be in a high-energy state

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- $P(\Delta E)$ is the probability that the new state s_2 will be accepted

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  - If the new solution is worse ($\Delta E > 0$), the acceptance probability then gets smaller the larger ΔE is and gets smaller the smaller the so-called "temperature" $T \geq 0$ is.

Annealing

- The temperature $T > 0$ and the objective value difference $\Delta E > 0$ enter the equation in an exponential term $e^{-\frac{\Delta E}{T}}$ and it holds that $e^{-a} < e^{-b} \forall a > b$ and $e^{-a} \in [0, 1] \forall a > 0$.

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- Then, the search progresses wildly any may accept even significantly worse solutions.
- As the process "cools" down, the search tends to accept fewer and fewer worse solutions and more likely such which are only a bit worse.
- Eventually, at temperature $T = 0$, the algorithm only accepts better solutions.
- T is actually a monotonously decreasing function $T(\tau)$ called the "temperature schedule" and it holds that $\lim_{\tau \to \infty} T(\tau) = 0$.

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  2. the logarithmic schedule, with start temperature T_s, it has a parameter $\epsilon \in (0, 1)$

$$T(\tau) = \frac{T_s}{\ln\left(\epsilon(\tau - 1) + e\right)} \tag{6}$$

Abstract Class for Temperature Schedules

Listing: Abstract Class for Temperature Schedules.

```
public abstract class TemperatureSchedule {

  public abstract double temperature(long tau);

}
```

Exponential Temperature Schedule

Listing: Exponential Temperature Schedule.

```
public class Exponential extends TemperatureSchedule {

  public double temperature(long tau) {
    return (this.startTemperature * Math.pow(
          (1d - this.epsilon), (tau - 1L)));
  }
}
```

Logarithmic Temperature Schedule

Listing: Logarithmic Temperature Schedule.

```
public class Logarithmic extends TemperatureSchedule {

public double temperature(long tau) {
    if (tau >= Long.MAX_VALUE) return 0d;
    return (this.startTemperature / Math.log(
              ((tau - 1L) * this.epsilon) + Math.E));
  }
}
```

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- We found from our previous experiments that a hill climber can do about 30'000'000 steps on `swv15` and 97'000'000 on `abz7` within these 3min.

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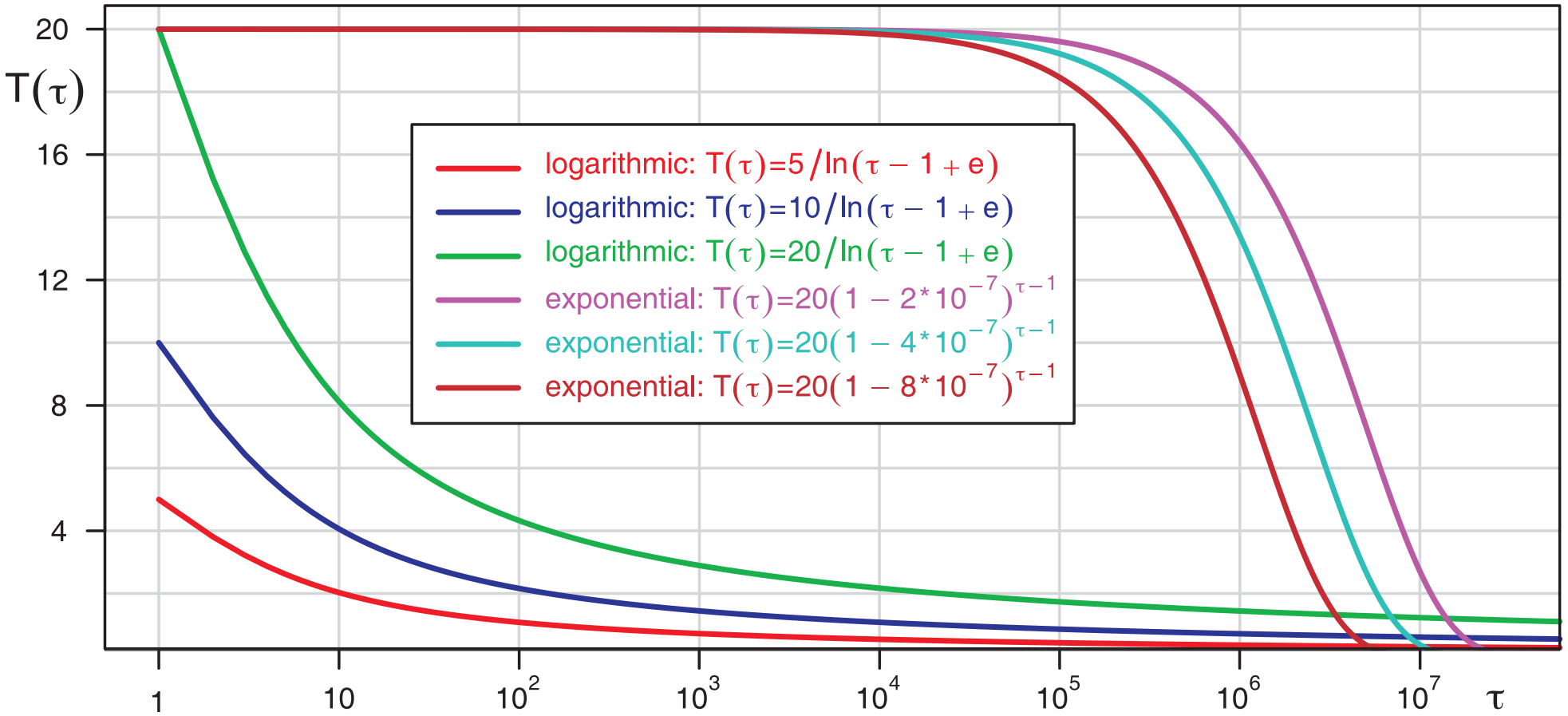
20
16
12
8
4
T(τ)
1
10
10^2
10^3
10^4
10^5
10^6
10^7
τ
logarithmic: T(τ)=5/ln(τ − 1 + e)
logarithmic: T(τ)=10/ln(τ − 1 + e)
logarithmic: T(τ)=20/ln(τ − 1 + e)
exponential: T(τ)=20(1 − 2*10^−7)^τ−1
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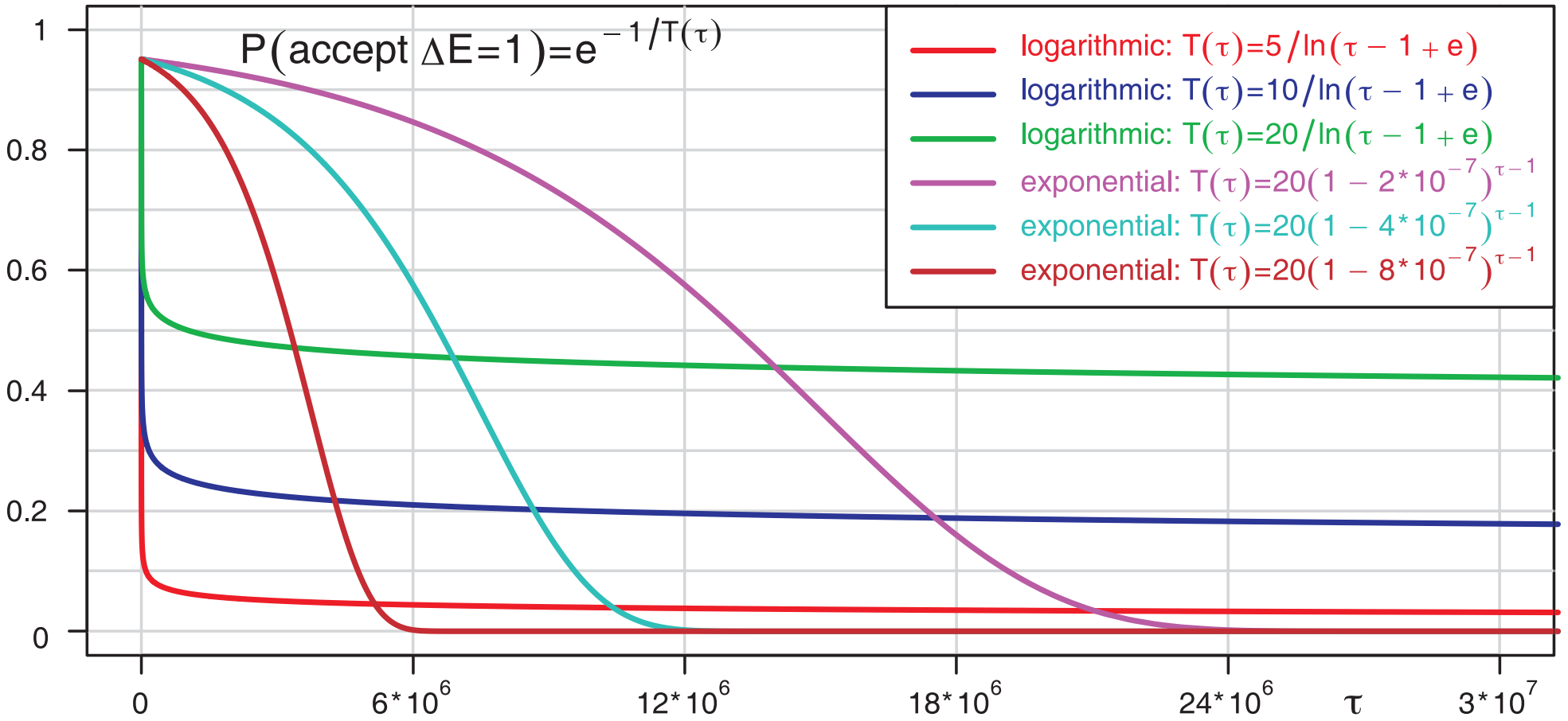
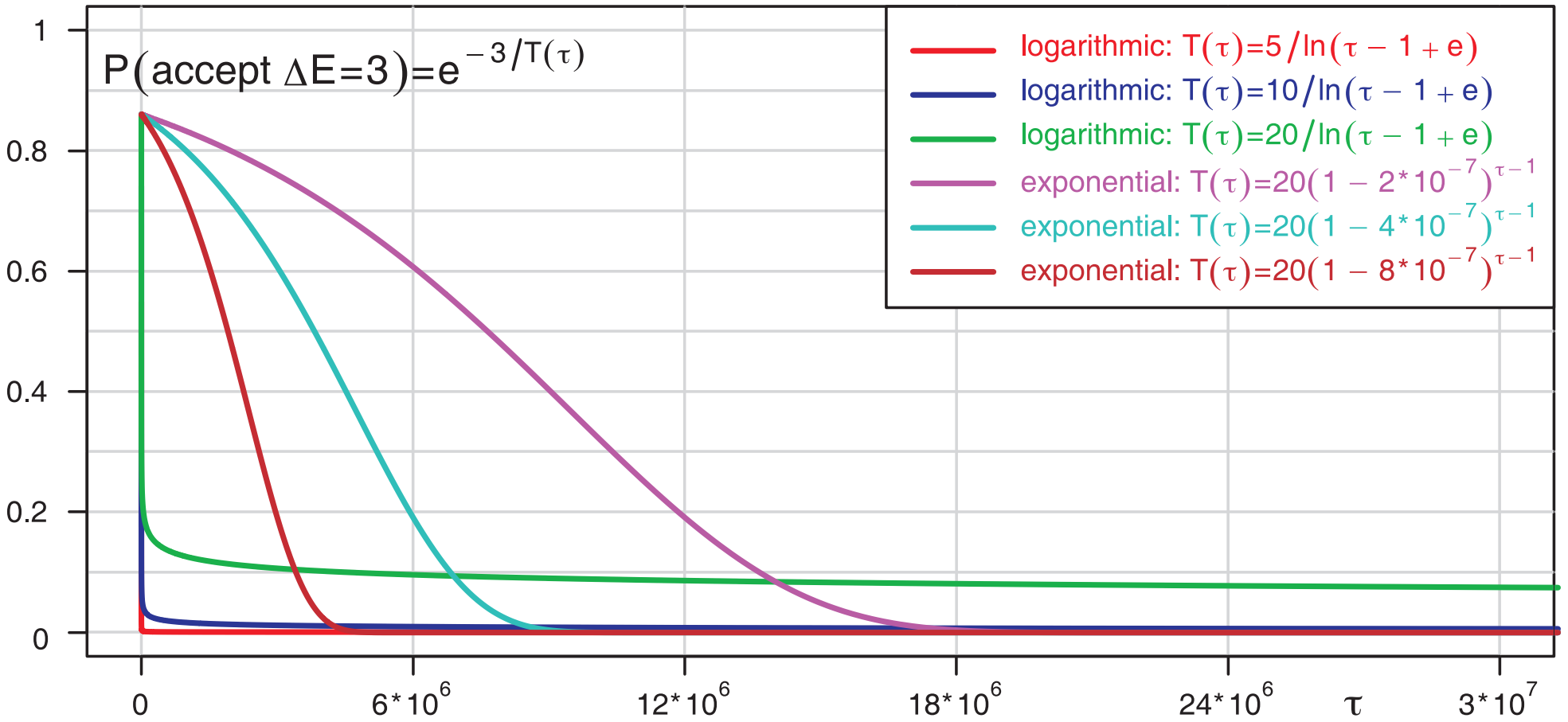

P(accept ΔE=1)=e^{−1/T(τ)}
logarithmic: T(τ)=5/ln(τ − 1 + e)
logarithmic: T(τ)=10/ln(τ − 1 + e)
logarithmic: T(τ)=20/ln(τ − 1 + e)
exponential: T(τ)=20(1 − 2*10^{−7})^{τ−1}
exponential: T(τ)=20(1 − 4*10^{−7})^{τ−1}
exponential: T(τ)=20(1 − 8*10^{−7})^{τ−1}
1
0.8
0.6
0.4
0.2
0
0
6*10^6
12*10^6
18*10^6
24*10^6
τ
3*10^7

How to use?


P(accept ΔE=3)=e^{−3/T(τ)}
logarithmic: T(τ)=5/ln(τ − 1 + e)
logarithmic: T(τ)=10/ln(τ − 1 + e)
logarithmic: T(τ)=20/ln(τ − 1 + e)
exponential: T(τ)=20(1 − 2*10^{−7})^{τ−1}
exponential: T(τ)=20(1 − 4*10^{−7})^{τ−1}
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0.8
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0
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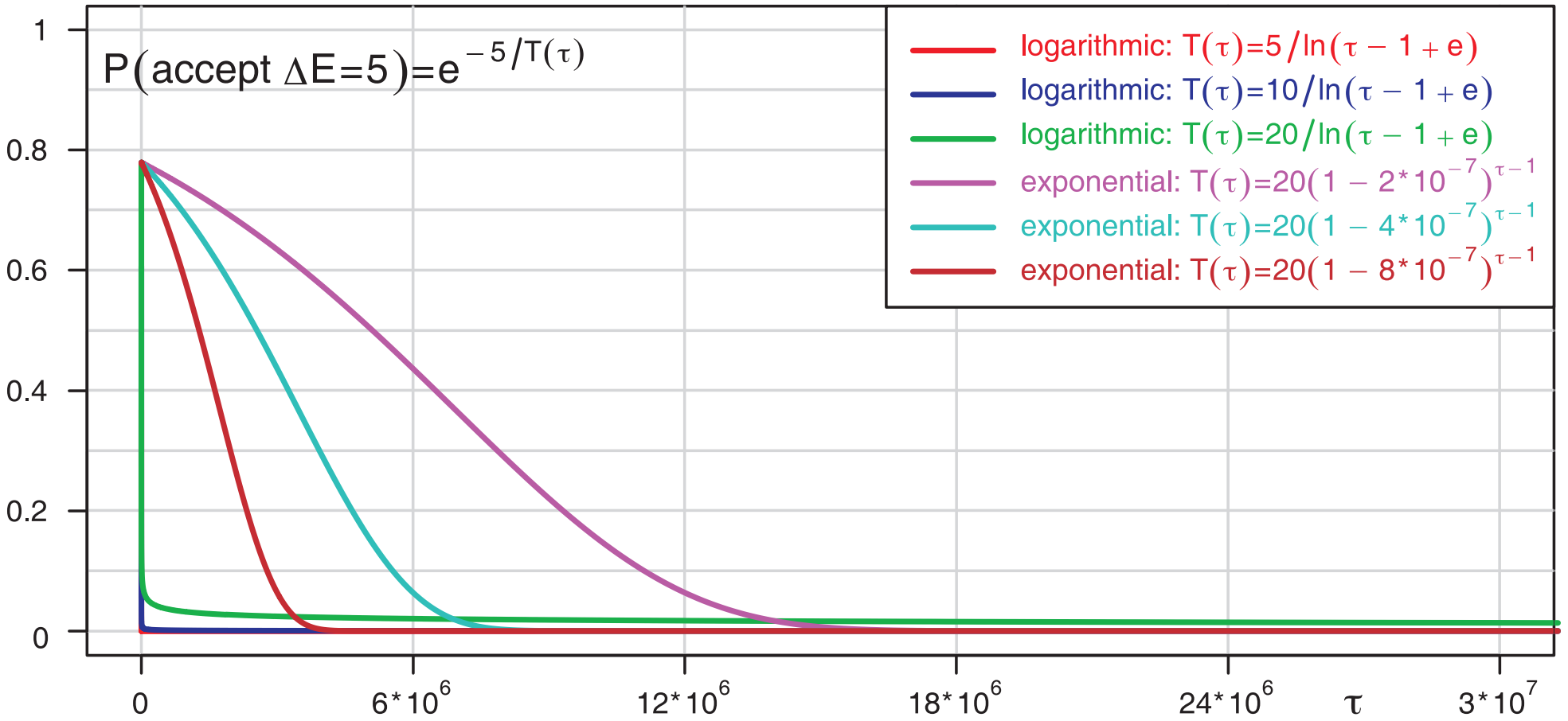
P(accept ΔE=5)=e^{−5/T(τ)}
logarithmic: T(τ)=5/ln(τ − 1 + e)
logarithmic: T(τ)=10/ln(τ − 1 + e)
logarithmic: T(τ)=20/ln(τ − 1 + e)
exponential: T(τ)=20(1 − 2*10^{−7})^{τ−1}
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  5. go back to 2 (until the time is up)

Section Outline

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  4. `sa_l_10_1swap`: Simulated Annealing with Logarithmic Schedule at $T_s = 10$ with $\epsilon = 1$

So what do we get?

		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	ea4096_1swap_5	680	712	712	10	20s	4'145'924
	sa_e_20_2e-7_1swap	663	673	672	5	92s	22'456'822
	sa_e_20_4e-7_1swap	**658**	674	675	5	55s	13'388'301
	sa_e_20_8e-7_1swap	663	675	675	6	36s	8'625'161
	sa_l_10_1swap	659	**672**	**671**	**4**	86s	21'271'077
la24	ea4096_1swap_5	945	976	975	15	4s	1'601'925
	sa_e_20_2e-7_1swap	938	**949**	**946**	**8**	27s	12'358'941
	sa_e_20_4e-7_1swap	**935**	949	**946**	9	16s	7'135'423
	sa_e_20_8e-7_1swap	**935**	951	950	8	9s	4'044'217
	sa_l_10_1swap	938	953	950	11	7s	3'210'824
swv15	ea4096_1swap_5	3413	3543	3539	66	169s	22'266'887
	sa_e_20_2e-7_1swap	2937	**2990**	**2988**	28	148s	21'949'073
	sa_e_20_4e-7_1swap	2941	2993	2993	**28**	128s	18'244'751
	sa_e_20_8e-7_1swap	**2936**	3000	3002	28	111s	16'029'528
	sa_l_10_1swap	2964	3021	3018	30	141s	21'252'052
yn4	ea4096_1swap_5	1034	1068	1068	18	37s	5'943'196
	sa_e_20_2e-7_1swap	973	**985**	**985**	**5**	113s	20'676'041
	sa_e_20_4e-7_1swap	**971**	987	986	7	68s	12'193'934
	sa_e_20_8e-7_1swap	972	988	988	7	58s	10'178'219
	sa_l_10_1swap	975	997	996	11	108s	19'850'143

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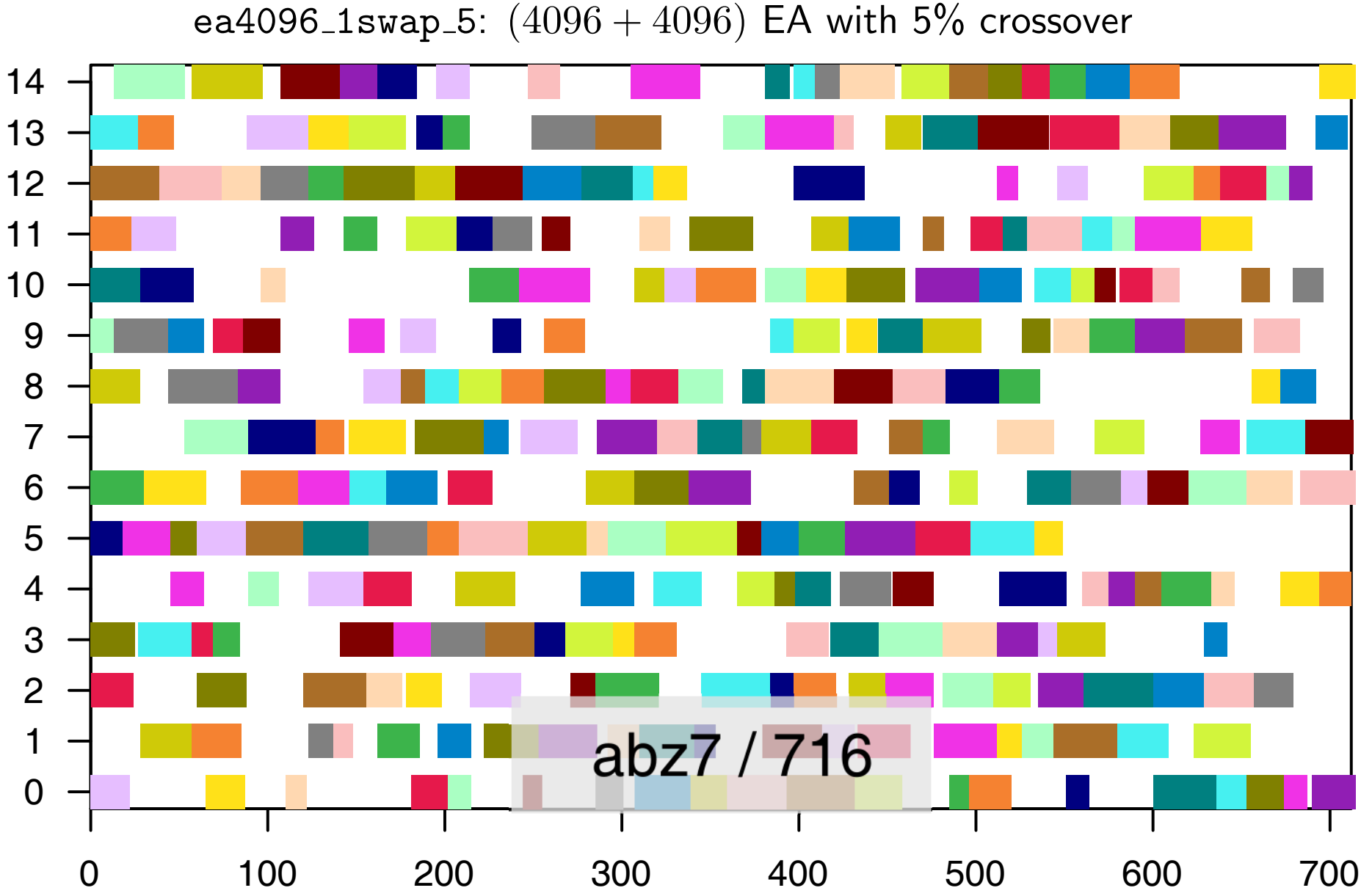

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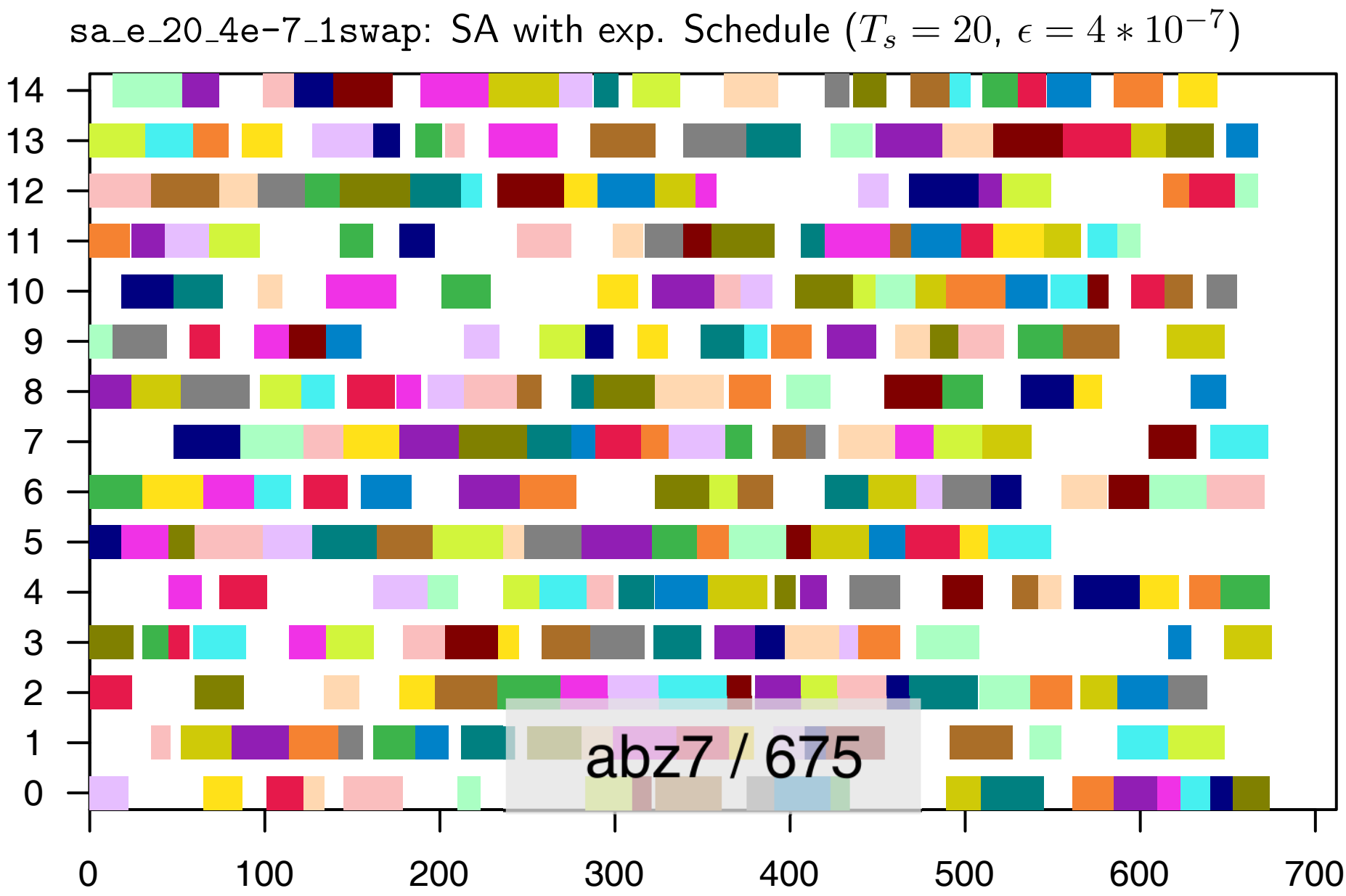

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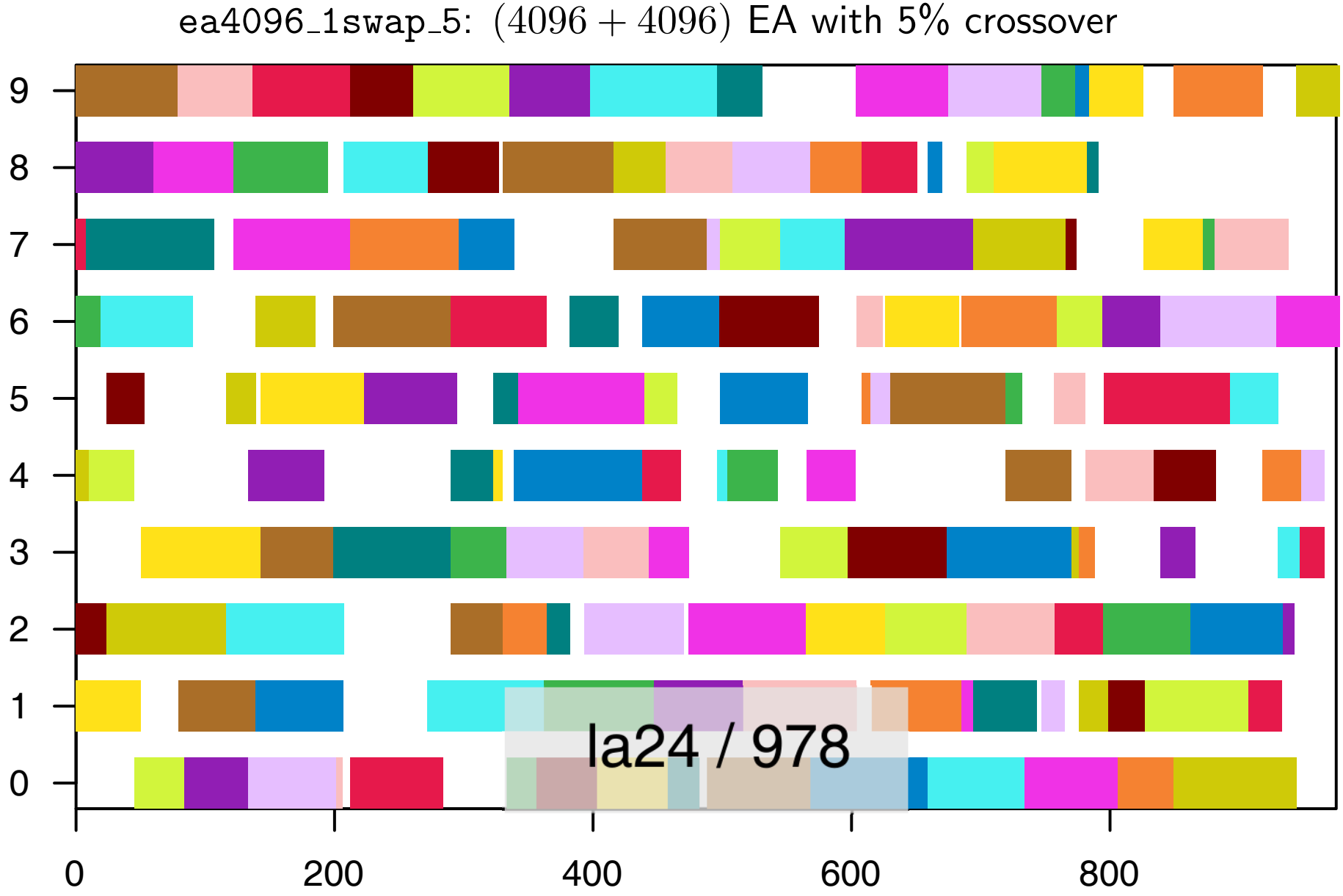

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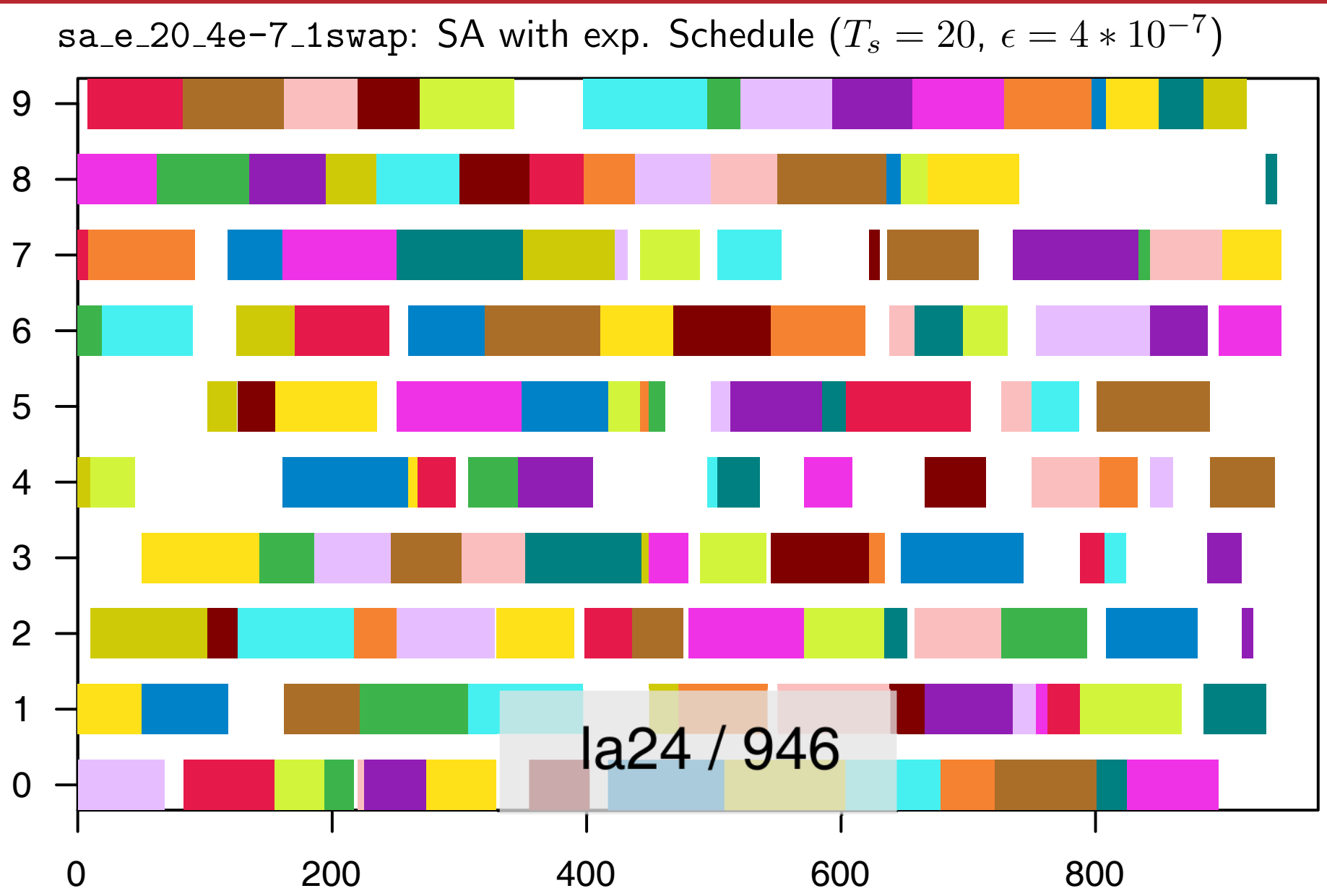

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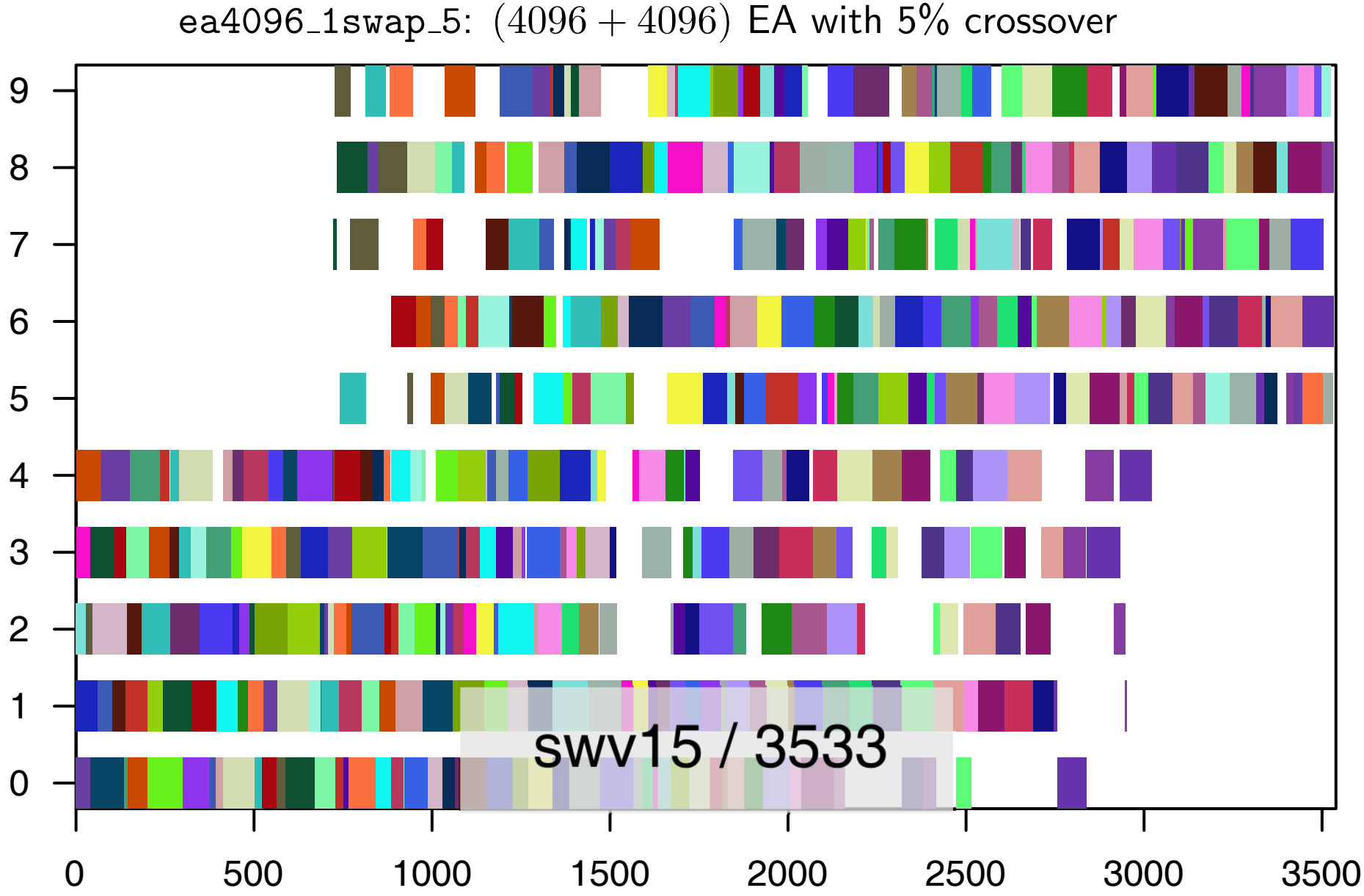

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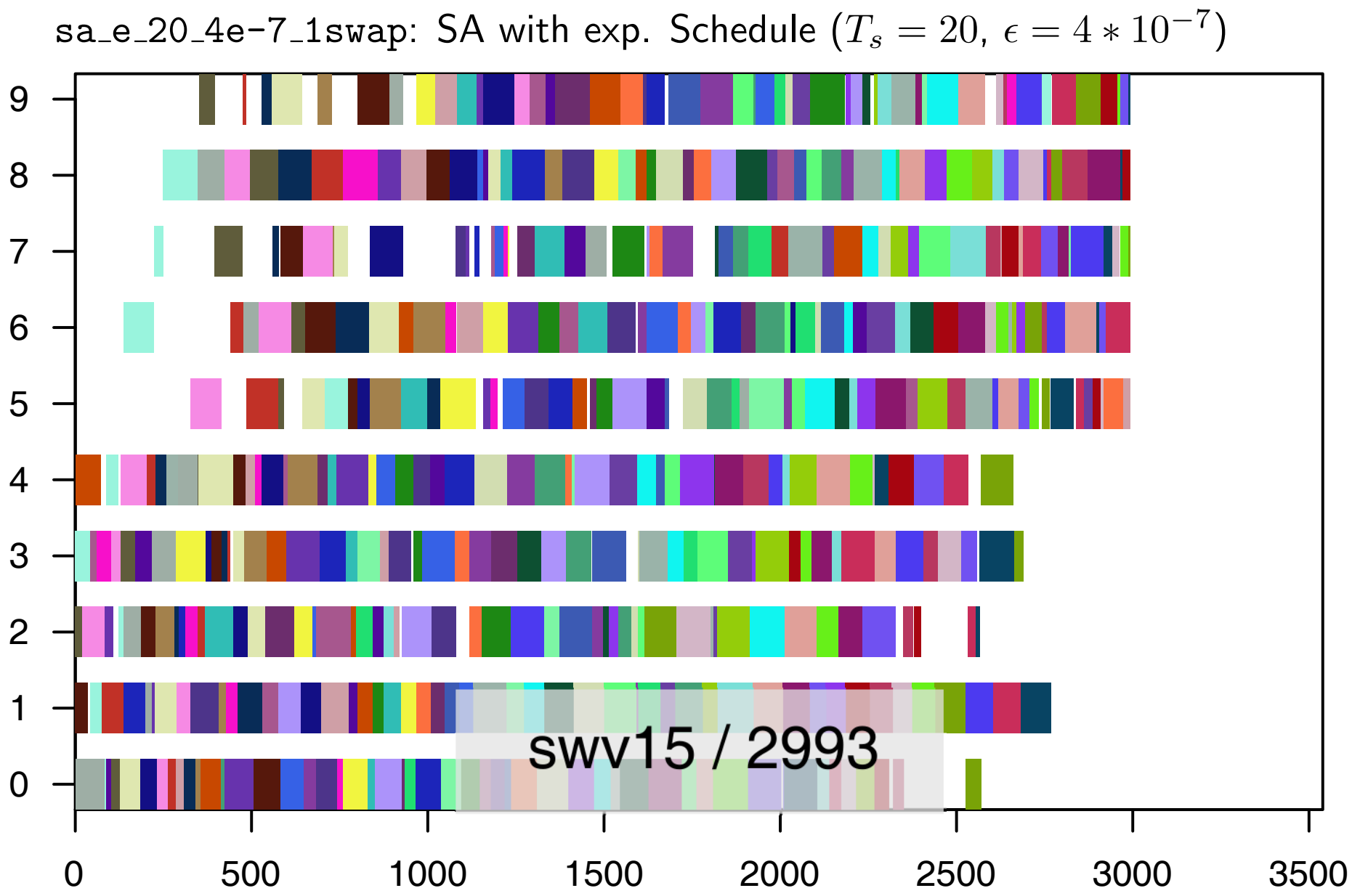

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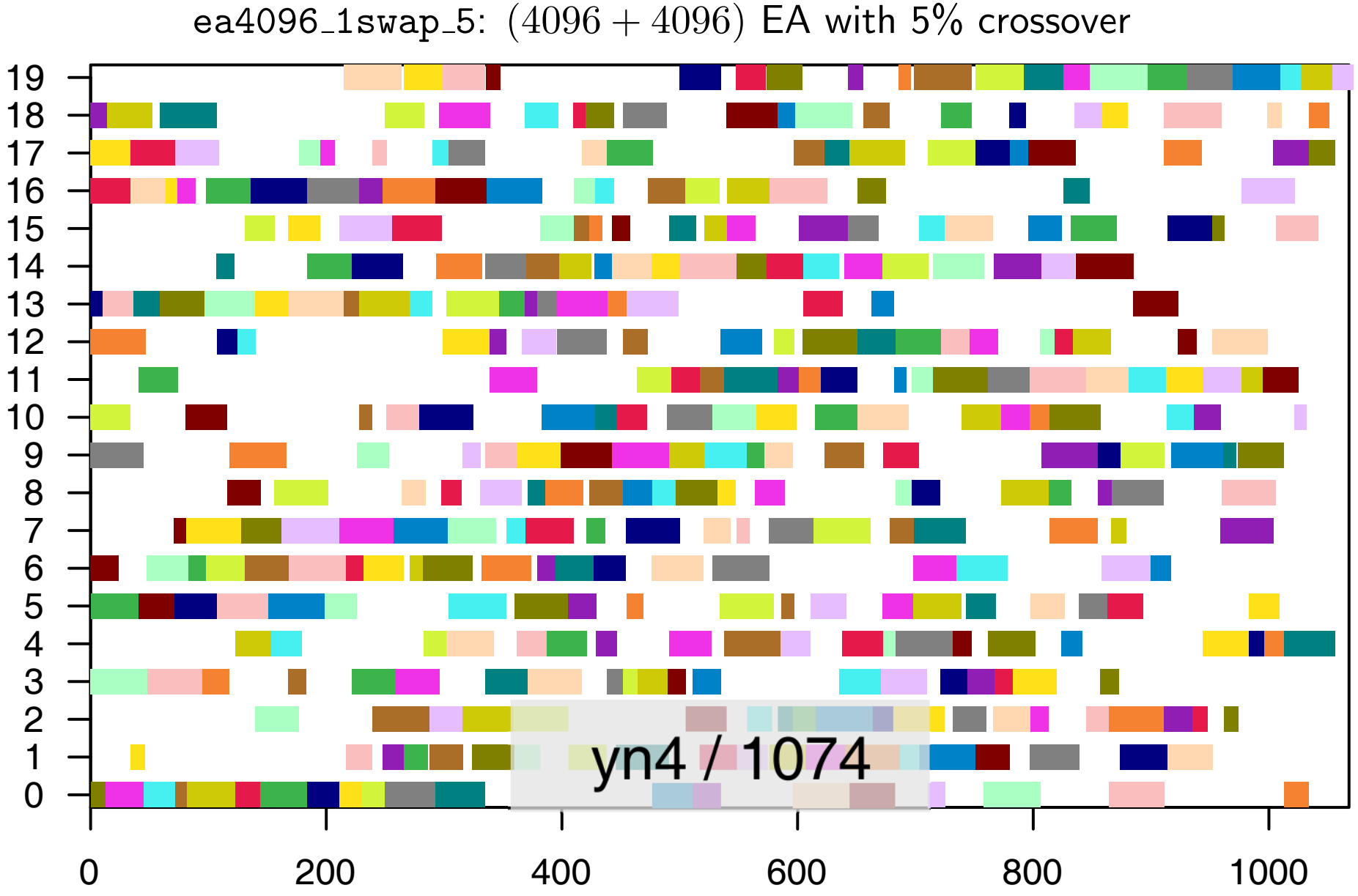

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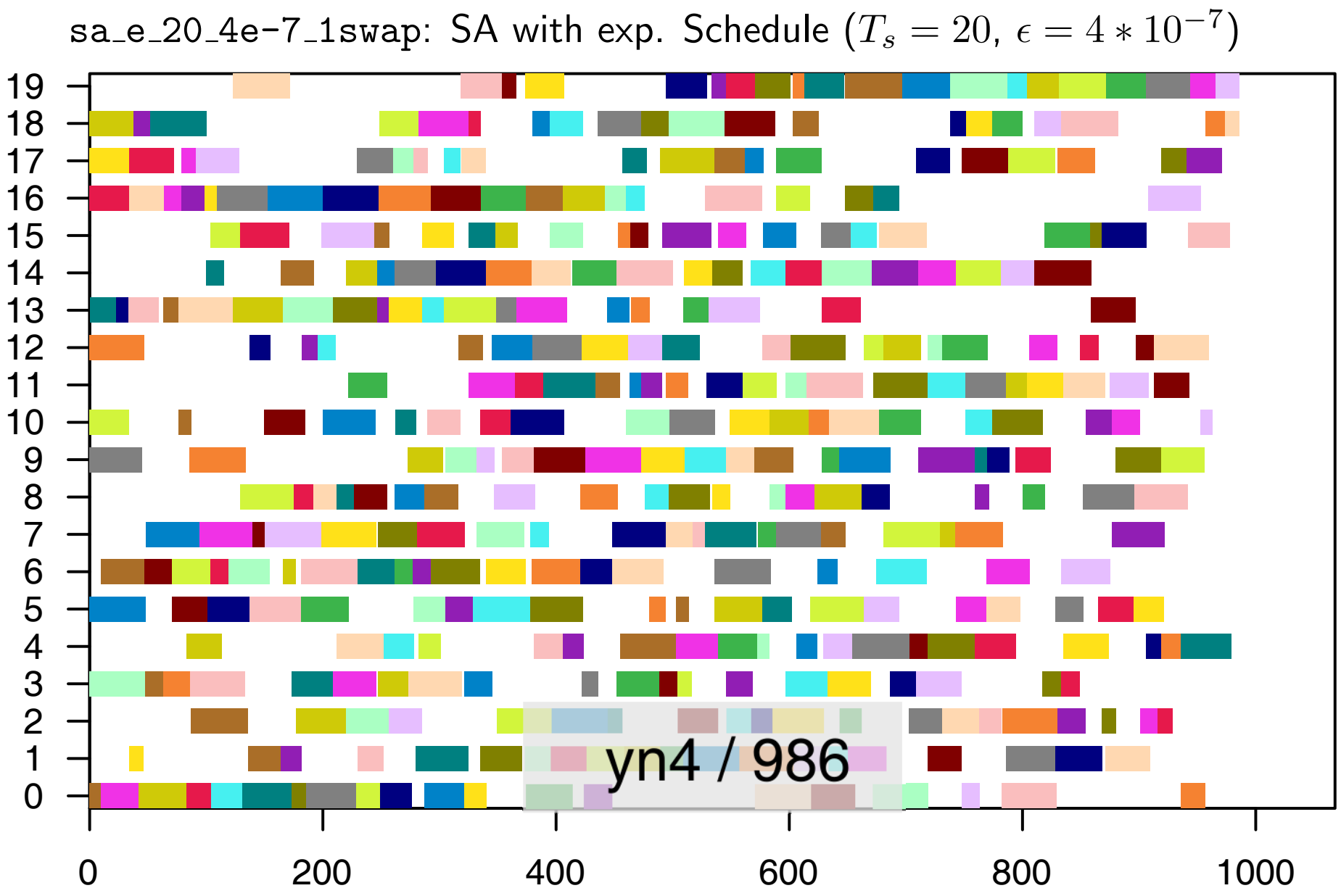

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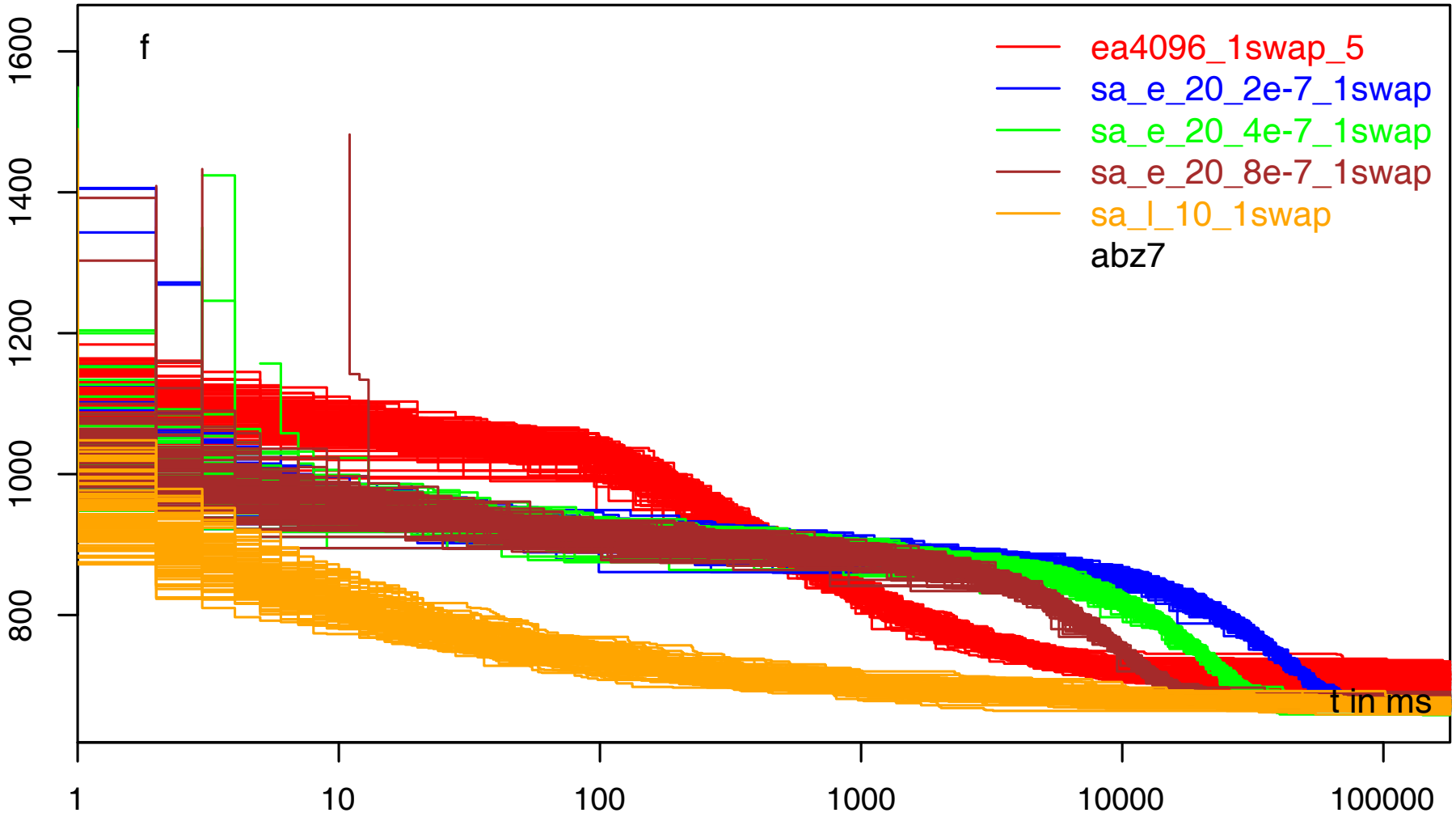

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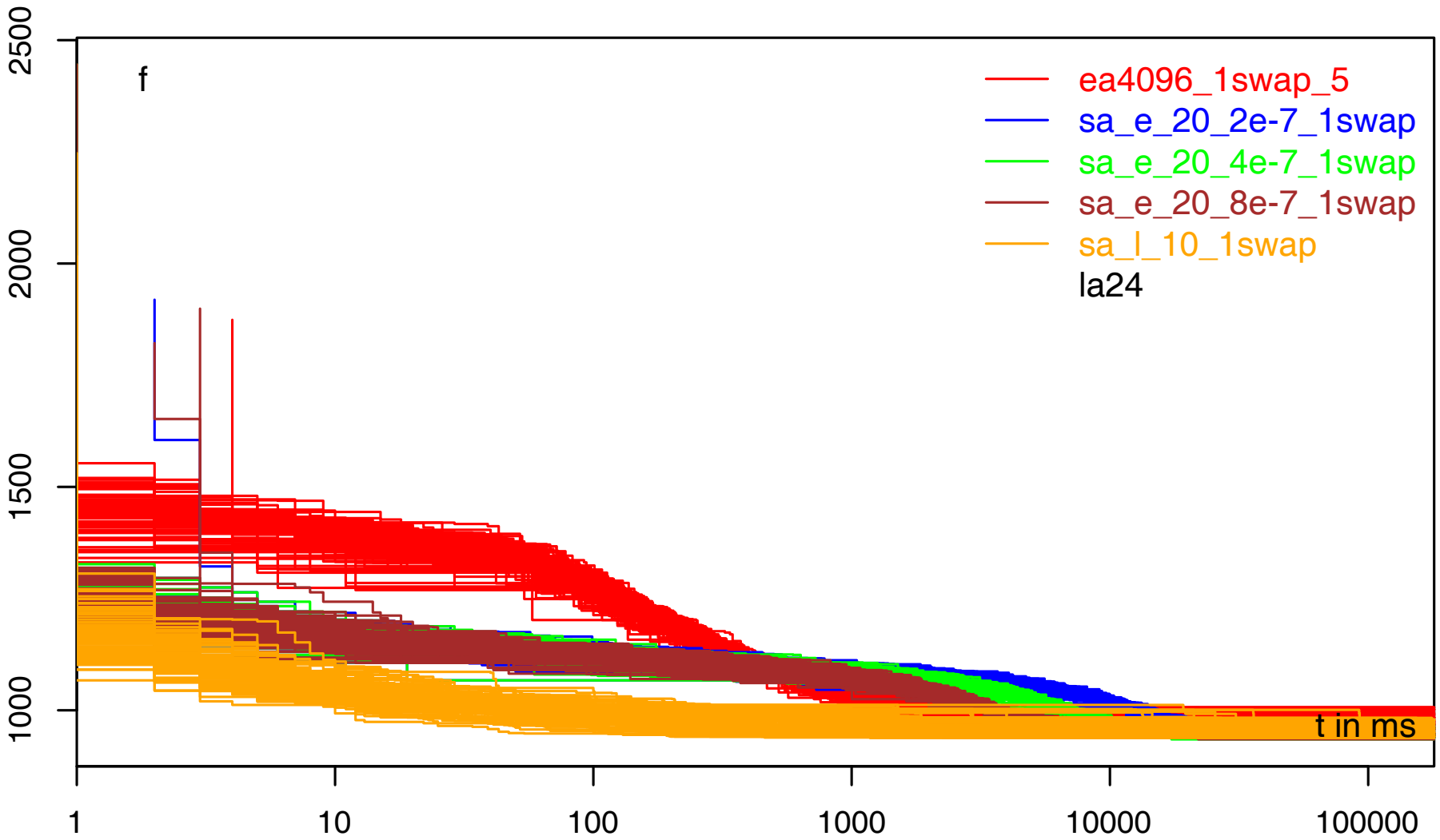

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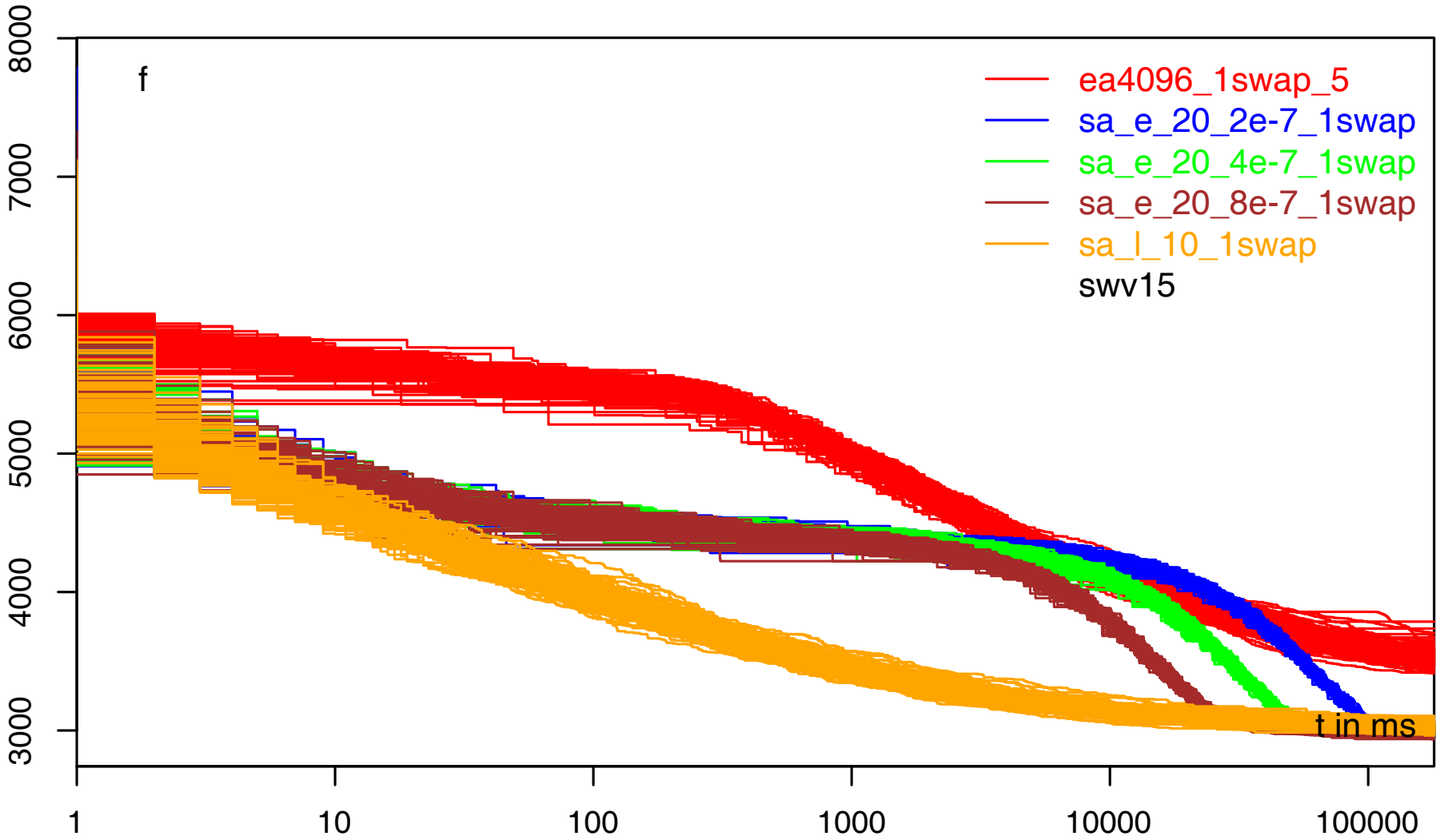

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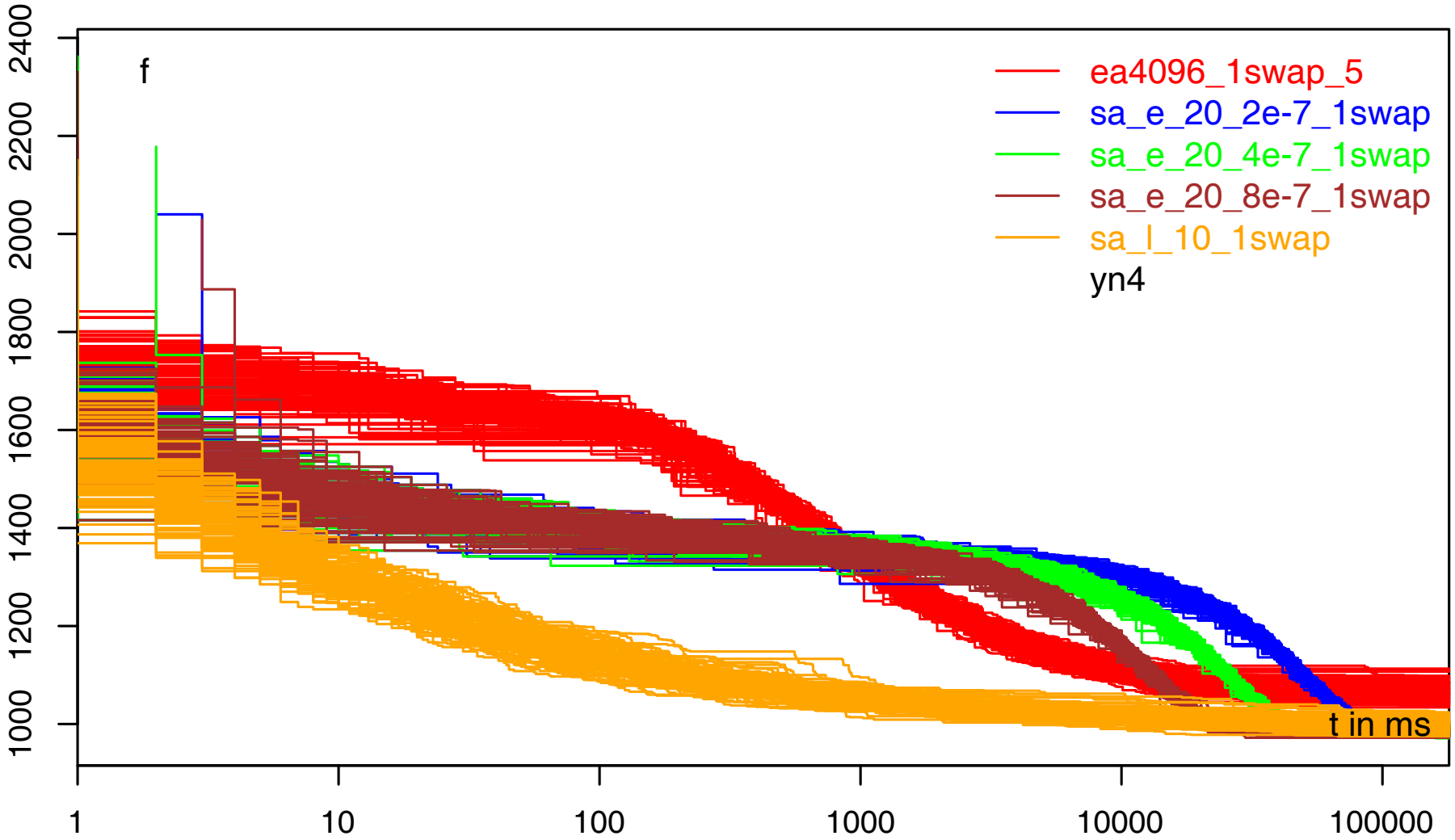

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  - Compared to the hill climber with restarts, it offers a "softer" way to escape from local optima which sacrifices less solution information.

谢谢

Thank you

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Caspar David Friedrich, “Der Wanderer über dem Nebelmeer”, 1818
http://en.wikipedia.org/wiki/Wanderer_above_the_Sea_of_Fog

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