Metaheuristics for Smart Manufacturing

5. Evolutionary Algorithms

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Outline

The slides are available at http://iao.hfuu.edu.cn/155, the book at http://thomasweise.github.io/aitoa, and the source code at http://www.github.com/thomasWeise/aitoa-code

course book

course material

An Introduction to Optimization Algorithms

The contents of this course are available as free electronic book *"An Introduction to Optimization Algorithms"* [1] at http://thomasweise.github.io/aitoa in pdf, html, azw3, and epub format, created with our bookbuildeR tool chain.

Section Outline

1. Introduction
2. Algorithm Concept: Population
3. Experiment and Analysis: Population
4. Algorithm Concept: Binary Operator
5. Experiment and Analysis: Binary Operator

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  2. they may still land again in a local optimum.
- Idea: Why not investigate multiple points in the search space at once and use the additional information in a clever way?

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- This has a couple of advantages:
  - we are less likely to get trapped in a local optimum.
  - we are more likely to find a better (local) optimum.
  - if we have different good points from the search space in our population, we can try to use this additional information...

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- Let us choose $\mu = \lambda$ and test the two values $\mu = \lambda = 2048$ and $\mu = \lambda = 4096$.

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		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	hcr_256+5%_1swap	723	742	743	**7**	21s	5'681'591
	ea2048_1swap	695	719	718	13	11s	2'581'614
	ea4096_1swap	**688**	**716**	**716**	12	19s	4'416'129
la24	hcr_256+5%_1swap	970	997	998	**9**	6s	3'470'368
	ea2048_1swap	945	983	983	16	2s	927'000
	ea4096_1swap	**941**	**980**	**978**	14	5s	1'897'387
swv15	hcr_256+5%_1swap	3701	3850	3857	**40**	60s	9'874'102
	ea2048_1swap	**3395**	3535	**3530**	78	128s	19'290'521
	ea4096_1swap	3397	**3533**	3533	54	171s	25'073'630
yn4	hcr_256+5%_1swap	1095	1129	1130	**14**	22s	4'676'669
	ea2048_1swap	1032	1082	1082	22	26s	4'792'622
	ea4096_1swap	**1020**	**1076**	**1074**	21	39s	6'907'692

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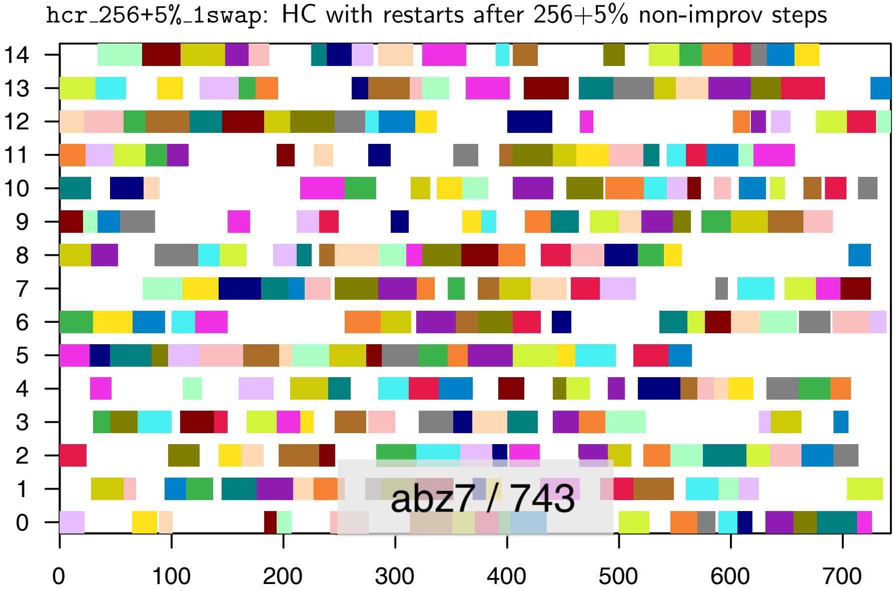

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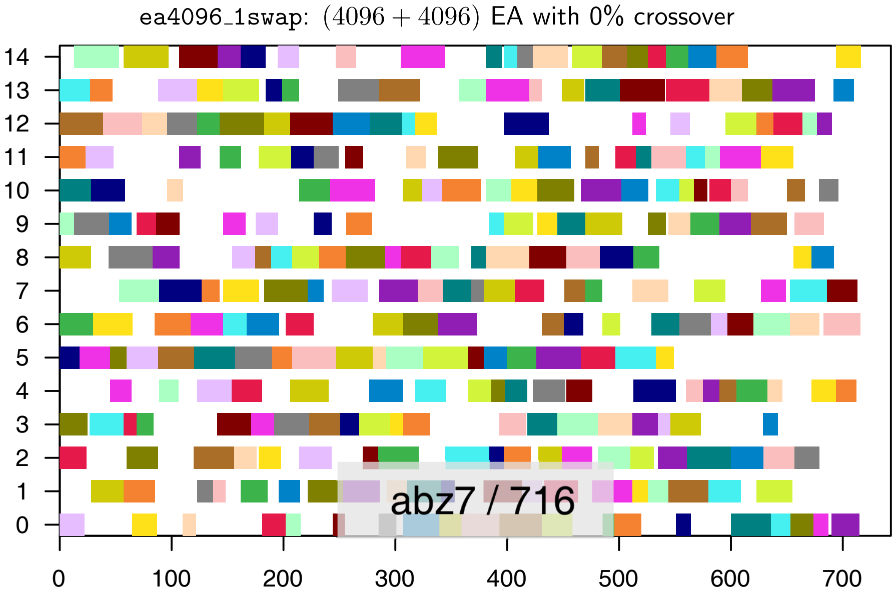

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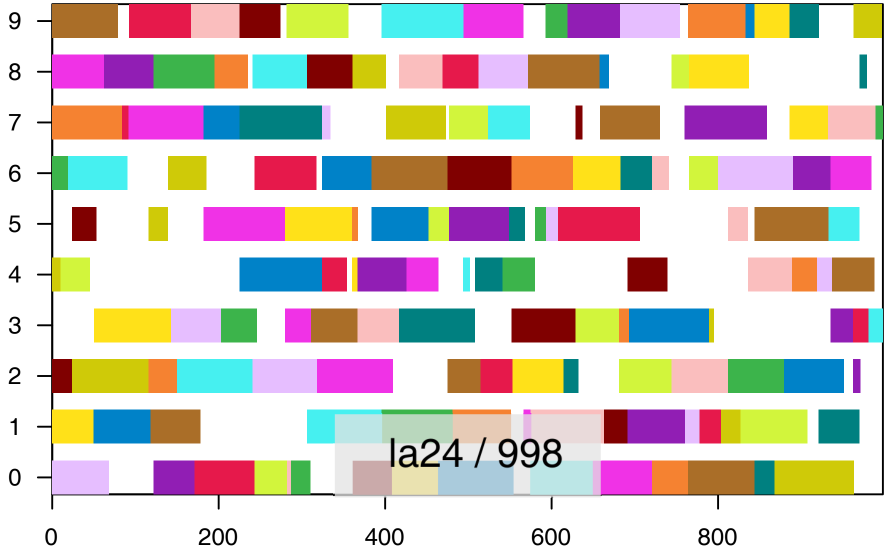
hcr_256+5%_1swap: HC with restarts after 256+5% non-improv steps



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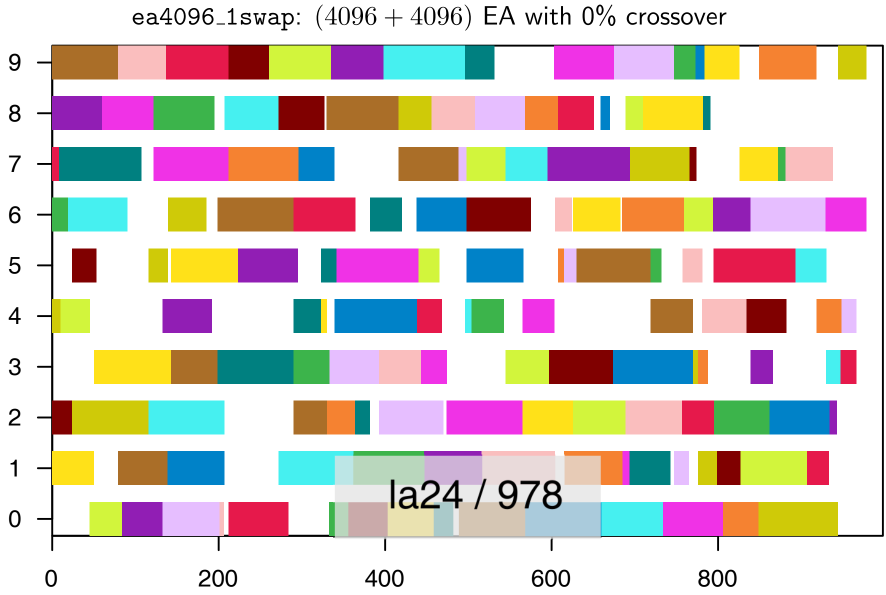

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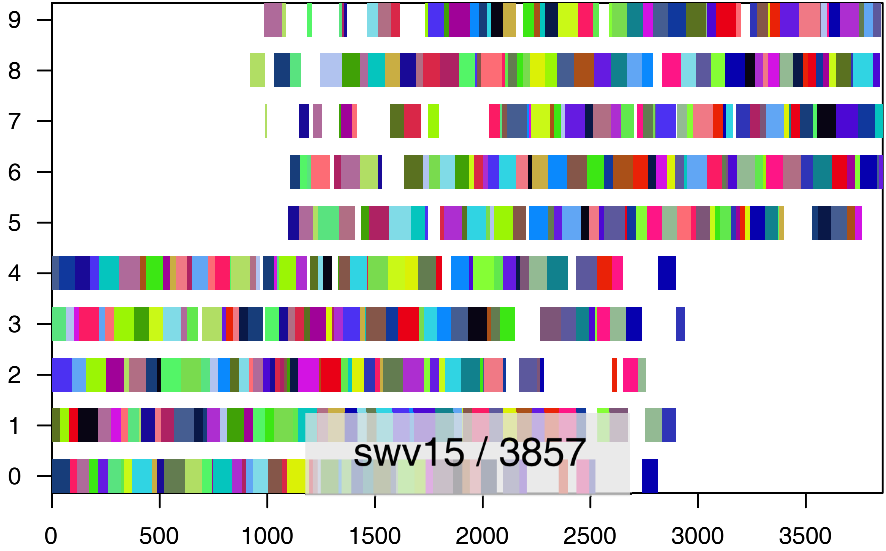

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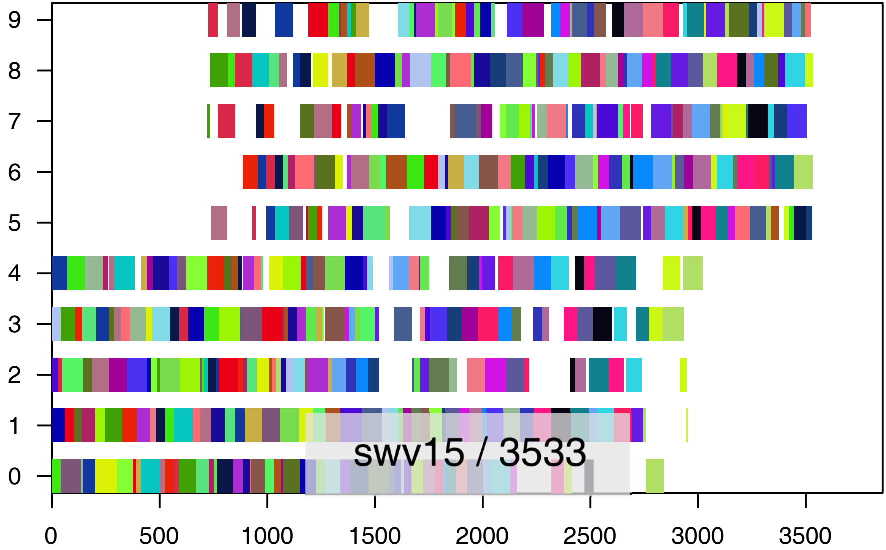
ea4096_1swap: $(4096 + 4096)$ EA with 0% crossover



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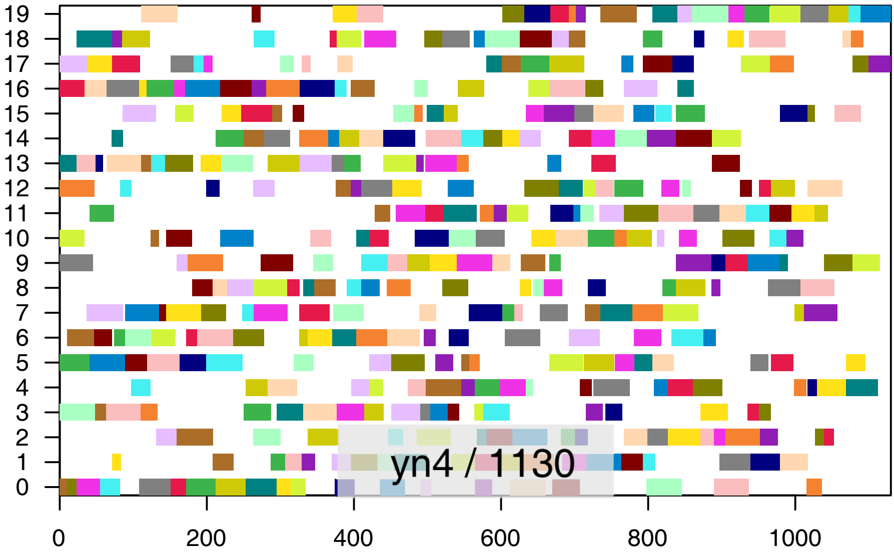

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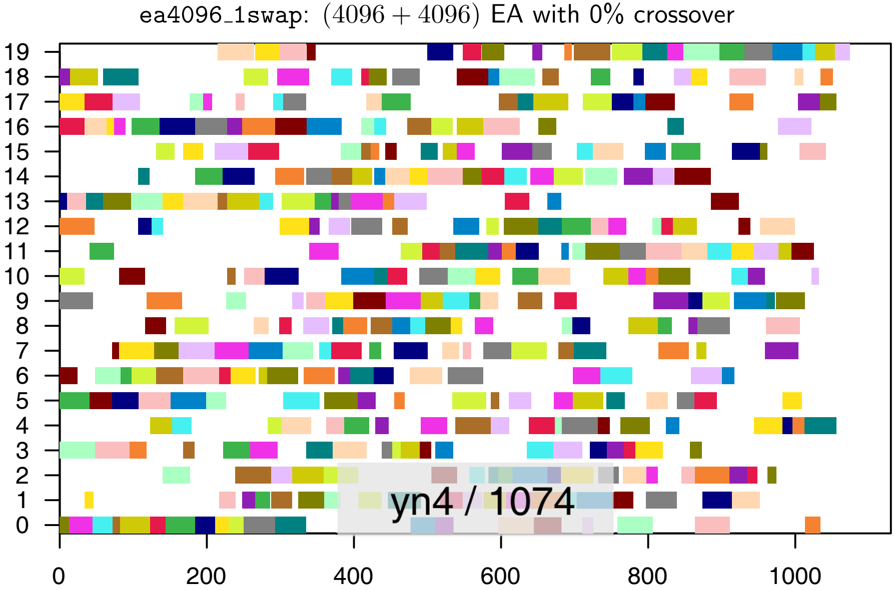

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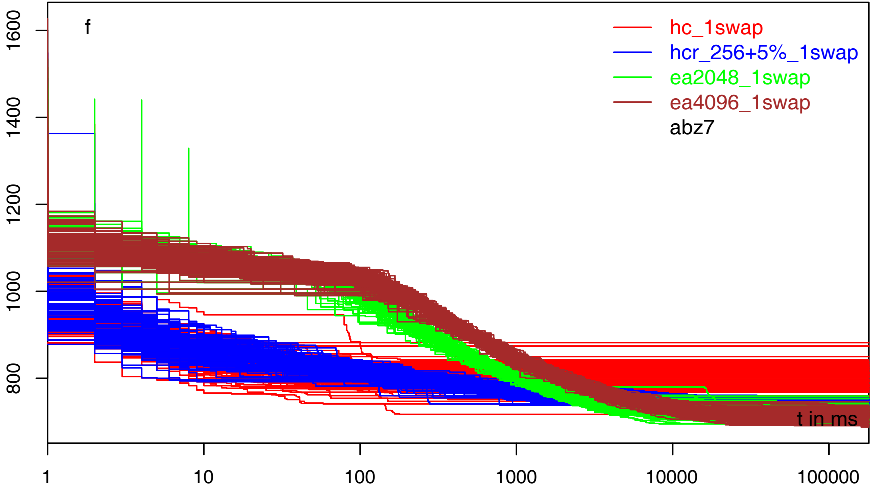

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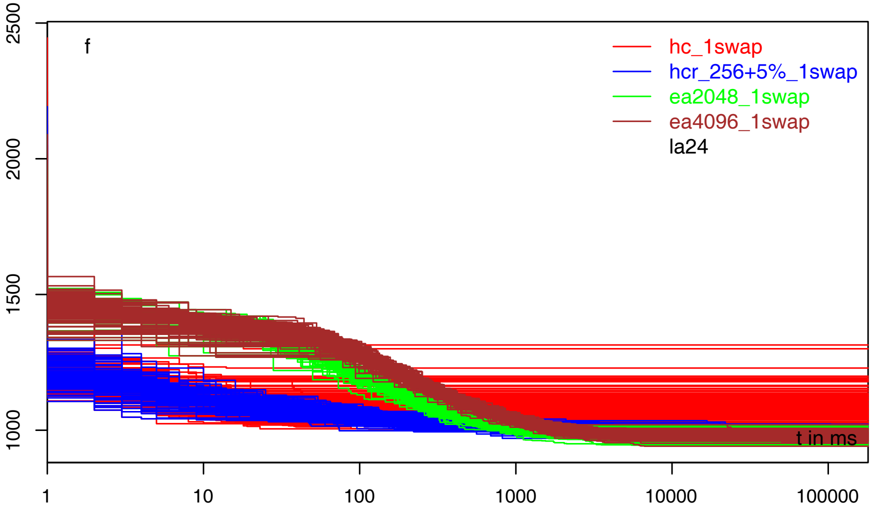

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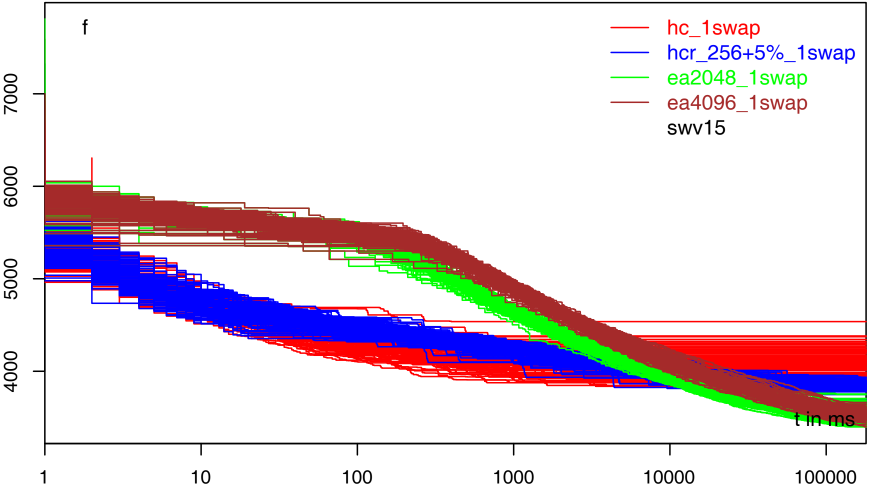

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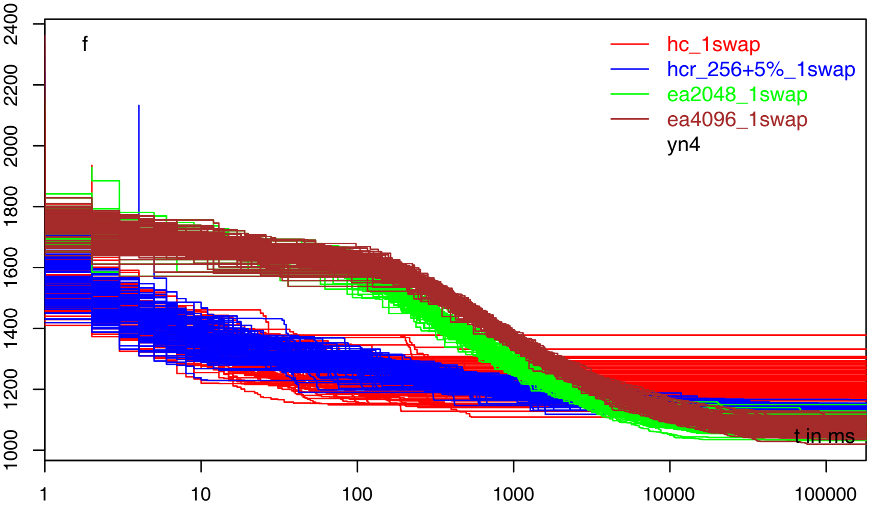

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- This is dilemma of "Exploration versus Exploitation" [2, 9–11].

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- This is the idea of the *crossover* or *recombination* operator in Evolutionary Algorithms. [2, 3, 7]

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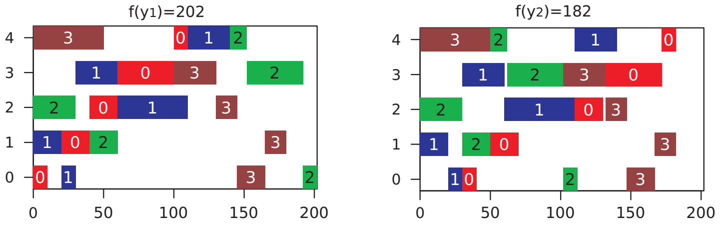
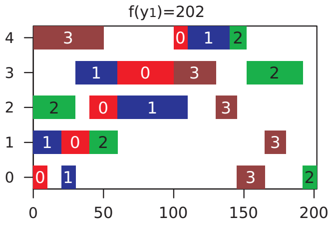
    5. Mark the first unmarked occurrence of J as “already used” in $x2$.

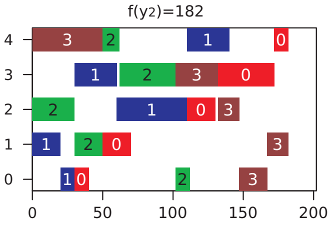
Example for Sequence Recombination



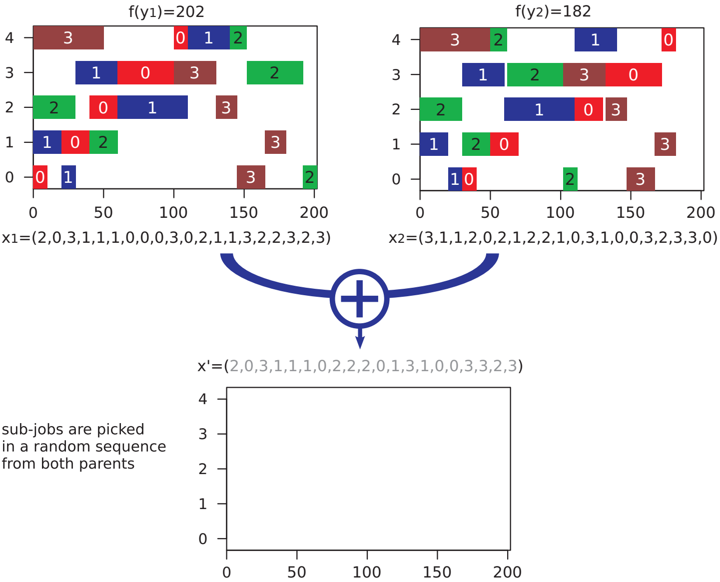
Example for Sequence Recombination



x1=(2,0,3,1,1,1,0,0,0,3,0,2,1,1,3,2,2,3,2,3)

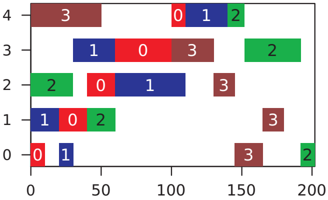


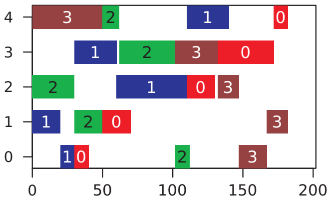
x2=(3,1,1,2,0,2,1,2,2,1,0,3,1,0,0,3,2,3,3,0)



sub-jobs are picked
in a random sequence
from both parents

Example for Sequence Recombination



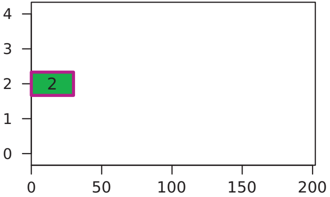


x1=(2,0,3,1,1,1,0,0,0,3,0,2,1,1,3,2,2,3,2,3) x2=(3,1,1,~~2~~,0,2,1,2,2,1,0,3,1,0,0,3,2,3,3,0)

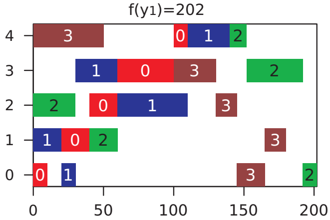
x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)

random sequence in which the sub-jobs are picked:

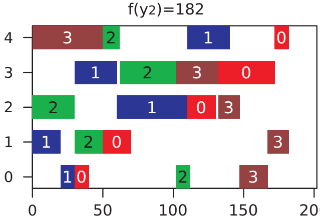
x1



Example for Sequence Recombination



x_1=(2,0,3,1,1,1,0,0,0,3,0,2,1,1,3,2,2,3,2,3)

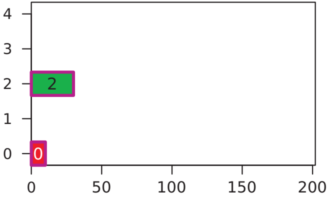


x_2=(3,1,1,~~2~~,~~0~~,2,1,2,2,1,0,3,1,0,0,3,2,3,3,0)

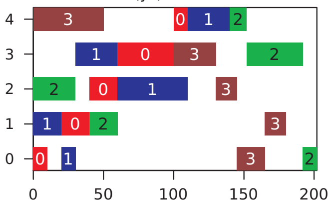
x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)

random sequence in which the sub-jobs are picked:

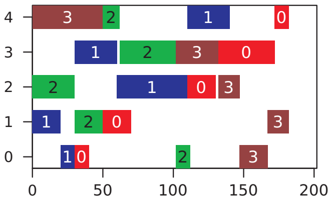
x_1, x_1



Example for Sequence Recombination



$x_1=(2,0,3,1,1,1,0,0,0,3,0,2,1,1,3,2,2,3,2,3)$

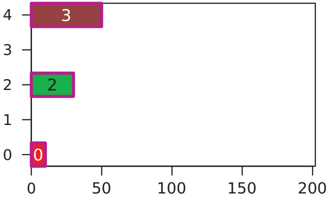


$x_2=(\cancel{3},1,1,\cancel{2},\cancel{0},2,1,2,2,1,0,3,1,0,0,3,2,3,3,0)$

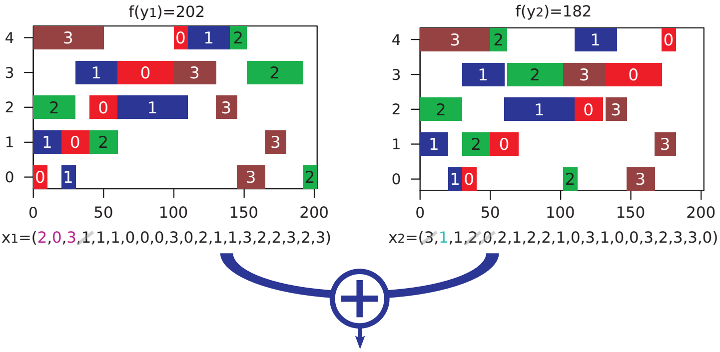
$x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)$

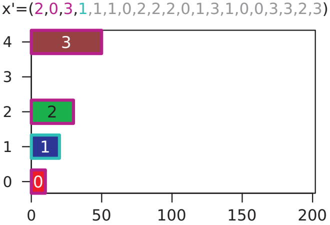
random sequence in which the sub-jobs are picked:

x_1, x_1, x_1



Example for Sequence Recombination

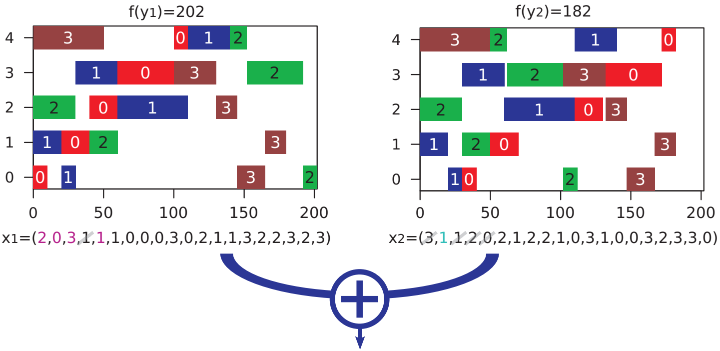




random sequence in which the sub-jobs are picked:

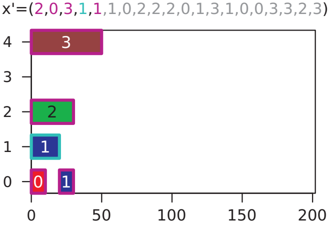
x1, x1, x1, x2

Example for Sequence Recombination



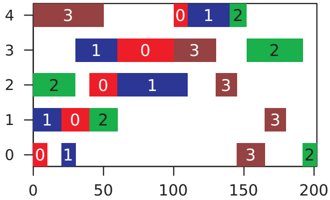
random sequence in which the sub-jobs are picked:

x1, x1, x1, x2, x1



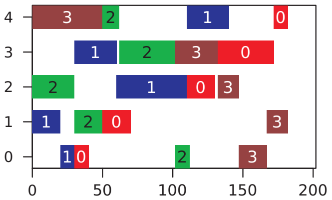
Example for Sequence Recombination

f(y_1)=202



x1=(2,0,3,~~1~~,1,1,0,0,0,3,0,2,1,1,3,2,2,3,2,3)

f(y_2)=182

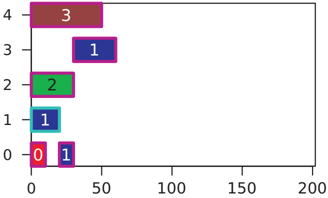


x2=(~~3~~,1,~~1~~,~~2~~,~~0~~,2,~~1~~,2,2,1,0,3,1,0,0,3,2,3,3,0)

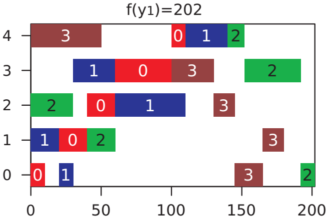
x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)

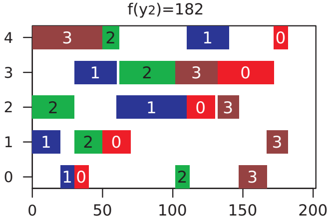
random sequence in which the sub-jobs are picked:

x1, x1, x1, x2, x1, x1



Example for Sequence Recombination

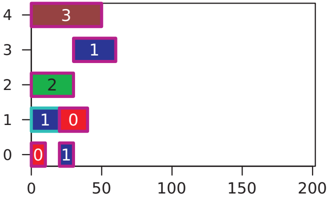




x1=(2,0,3,~~1~~,1,1,0,0,0,3,0,2,1,1,3,2,2,3,2,3)

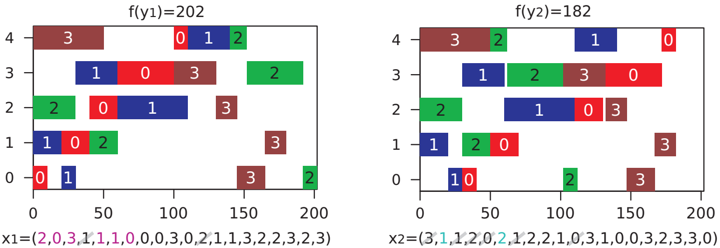
x2=(~~3~~,1,~~1~~,~~2~~,~~0~~,2,~~1~~,2,2,1,~~0~~,3,1,0,0,3,2,3,3,0)

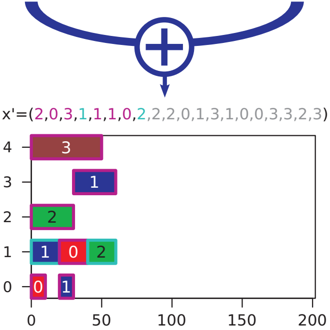
x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)



random sequence in which the sub-jobs are picked:

x1, x1, x1, x2, x1, x1, x1

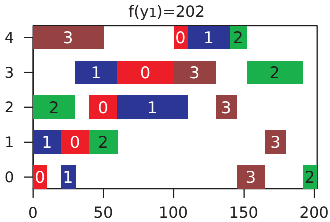


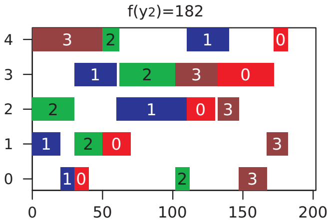


random sequence in which the sub-jobs are picked:

x1, x1, x1, x2, x1, x1, x1, x2

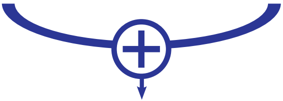
Example for Sequence Recombination



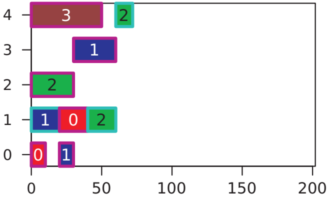


$x_1=(2,0,3,\cancel{1},1,1,0,0,0,3,0,\cancel{2},1,1,3,\cancel{2},2,3,2,3)$

$x_2=(\cancel{3},1,\cancel{1},\cancel{2},\cancel{0},2,\cancel{1},2,2,1,\cancel{0},3,1,0,0,3,2,3,3,0)$



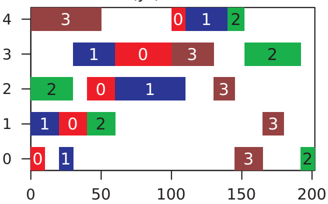
$x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)$



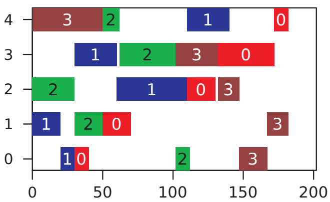
random sequence in which the sub-jobs are picked:

x_1, x_1, x_1, x_2, x_1, x_1, x_1, x_2, x_2

Example for Sequence Recombination



x1=(2,0,3,~~1~~,1,1,0,0,0,3,0,~~2~~,1,1,3,~~2~~,~~2~~,3,2,3)

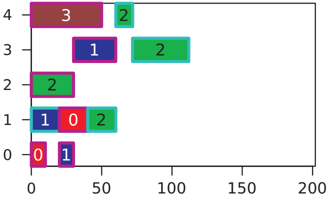


x2=(~~3~~,1,~~1~~,~~2~~,~~0~~,2,~~1~~,2,2,1,~~0~~,3,1,0,0,3,2,3,3,0)

x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)

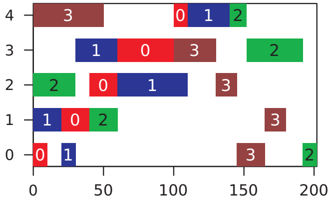
random sequence in which the sub-jobs are picked:

x1, x1, x1, x2, x1, x1, x1, x2, x2, x2

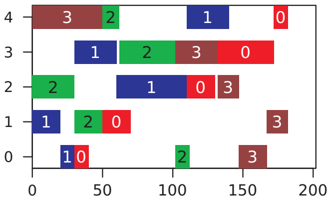


Example for Sequence Recombination

f(y1)=202



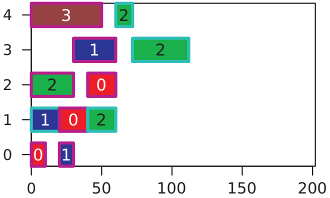
f(y2)=182



x1=(2,0,3,~~1~~,1,1,0,0,0,3,0,~~2~~,1,1,3,~~2~~,~~2~~,3,2,3)

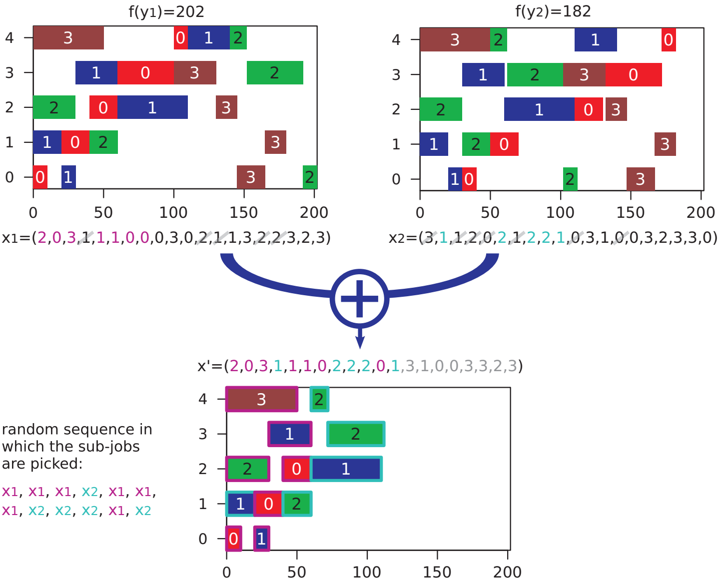
x2=(~~3~~,1,~~1~~,~~2~~,~~0~~,2,~~1~~,2,2,1,~~0~~,3,1,~~0~~,0,3,2,3,3,0)

x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)



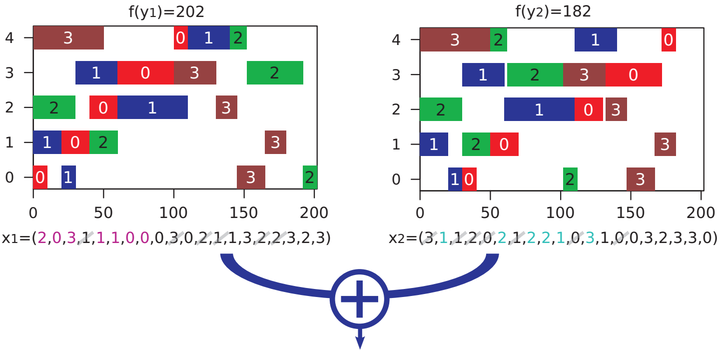
random sequence in which the sub-jobs are picked:

x1, x1, x1, x2, x1, x1, x1, x2, x2, x2, x1



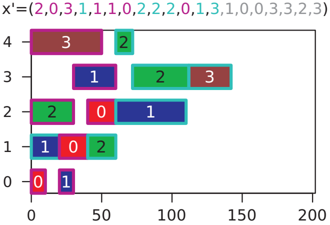
random sequence in which the sub-jobs are picked:

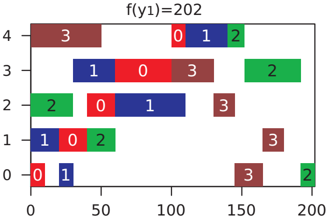
x_1, x_1, x_1, x_2, x_1, x_1, x_1, x_2, x_2, x_2, x_1, x_2

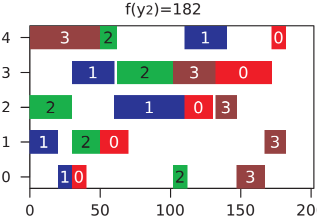


random sequence in which the sub-jobs are picked:

x1, x1, x1, x2, x1, x1, x1, x2, x2, x2, x1, x2, x2





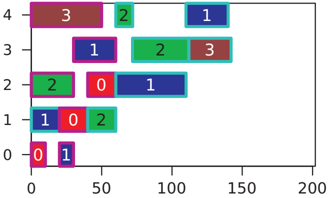


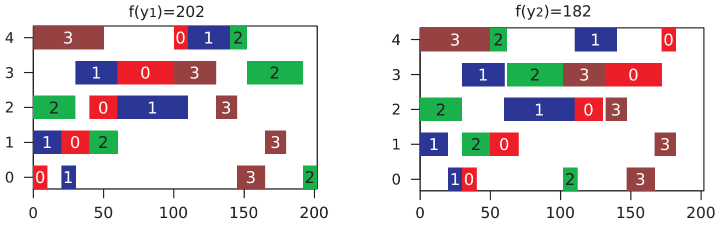
x1=(2,0,3,1,1,1,0,0,0,3,0,2,1,1,3,2,2,3,2,3) x2=(3,1,1,2,0,2,1,2,2,1,0,3,1,0,0,3,2,3,3,0)

x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)

random sequence in which the sub-jobs are picked:

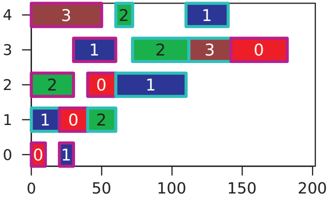
x1, x1, x1, x2, x1, x1, x1, x2, x2, x2, x1, x2, x2, x2





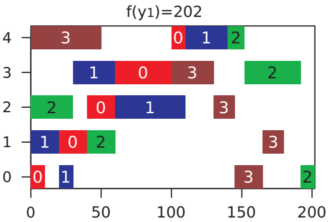
x_1=(2,0,3,1,1,1,0,0,0,3,0,2,1,1,3,2,2,3,2,3) x_2=(3,1,1,2,0,2,1,2,2,1,0,3,1,0,0,3,2,3,3,0)

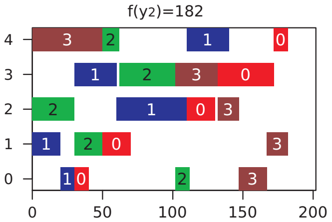
x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)



random sequence in which the sub-jobs are picked:

x_1, x_1, x_1, x_2, x_1, x_1, x_1, x_2, x_2, x_2, x_1, x_2, x_2, x_2, x_1

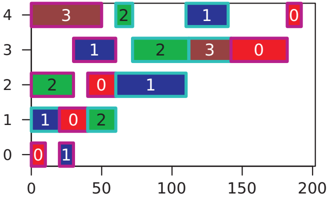




x1=(2,0,3,1,1,1,0,0,0,3,0,2,1,1,3,2,2,3,2,3)

x2=(3,1,1,2,0,2,1,2,2,1,0,3,1,0,0,3,2,3,3,0)

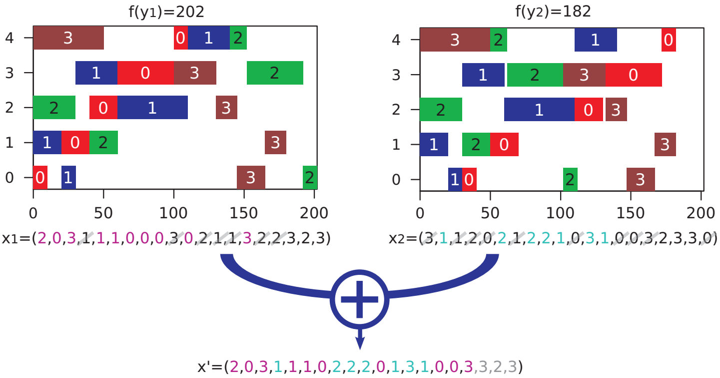
x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)

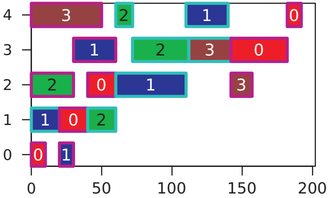


random sequence in which the sub-jobs are picked:

x1, x1, x1, x2, x1, x1, x1, x2, x2, x2, x1, x2, x2, x2, x1, x1

Example for Sequence Recombination

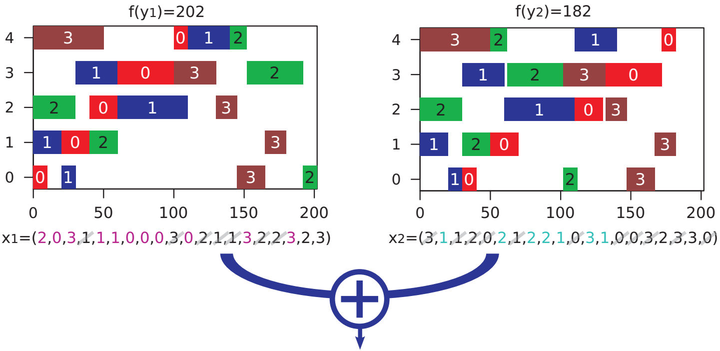




random sequence in which the sub-jobs are picked:

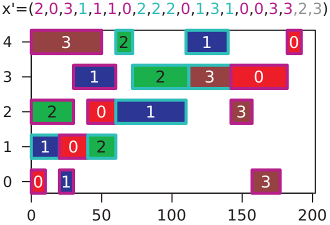
x1, x1, x1, x2, x1, x1, x1, x2, x2, x2, x1, x2, x2, x2, x1, x1, x1

Example for Sequence Recombination

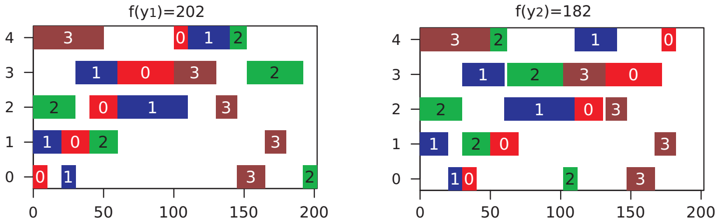


random sequence in which the sub-jobs are picked:

x1, x1, x1, x2, x1, x1, x1, x2, x2, x2, x1, x2, x2, x2, x1, x1, x1, x1



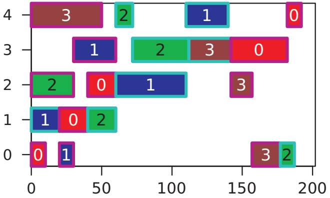
Example for Sequence Recombination



x_1=(2,0,3,~~1~~,1,1,0,0,0,~~3~~,0,~~2~~,~~1~~,~~1~~,3,~~2~~,~~2~~,3,~~2~~,3)

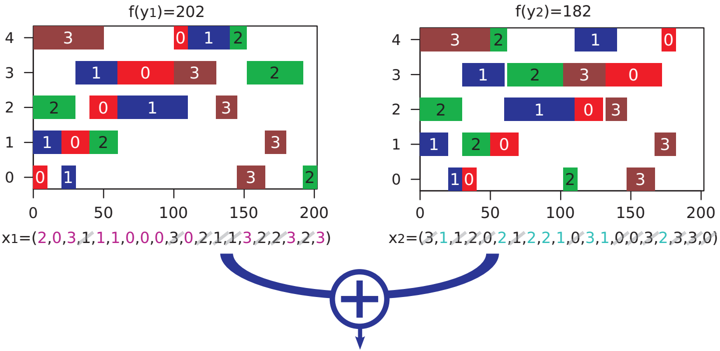
x_2=(~~3~~,1,~~1~~,~~2~~,~~0~~,2,~~1~~,2,2,1,~~0~~,3,1,~~0~~,~~0~~,~~3~~,2,~~3~~,3,~~0~~)

x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)



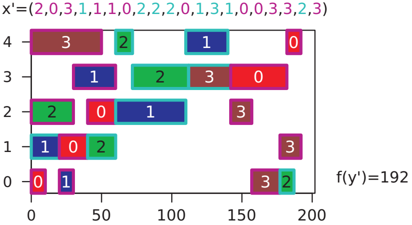
random sequence in which the sub-jobs are picked:

x_1, x_1, x_1, x_2, x_1, x_1, x_1, x_2, x_2, x_2, x_1, x_2, x_2, x_2, x_1, x_1, x_1, x_1, x_2

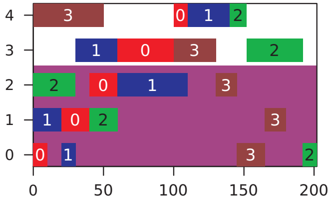


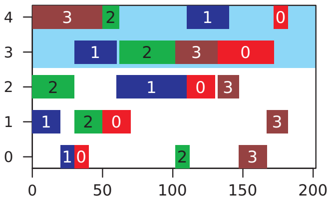
random sequence in which the sub-jobs are picked:

x1, x1, x1, x2, x1, x1, x1, x2, x2, x2, x1, x2, x2, x2, x1, x1, x1, x1, x2, x1



Example for Sequence Recombination





x1=(2,0,3,~~1~~,1,1,0,0,0,~~3~~,0,~~2~~,~~1~~,~~1~~,3,~~2~~,~~2~~,3,~~2~~,3)

x2=(~~3~~,1,~~1~~,~~2~~,~~0~~,2,~~1~~,2,2,1,~~0~~,3,1,~~0~~,~~0~~,~~3~~,2,~~3~~,~~3~~,~~0~~)

x'=(2,0,3,1,1,1,0,2,2,2,0,1,3,1,0,0,3,3,2,3)

random sequence in which the sub-jobs are picked:

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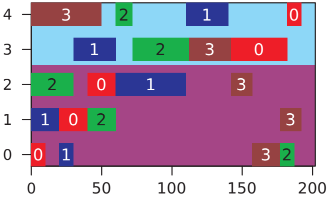


f(y')=192

The Binary Operator in Java

Listing: Recombination for our Representation

```java
public class JSSPBinaryOperatorSequence implements IBinarySearchOperator<int[]> {

  public void apply(int[] x0, int[] x1, int[] dest, Random random) {
    boolean[] done_x0 = this.m_done_x0;
    Arrays.fill(done_x0, false);
    boolean[] done_x1 = this.m_done_x1;
    Arrays.fill(done_x1, false);

    int length = done_x0.length;
    int desti = 0, x0i = 0, x1i = 0;

    for (;;) {
      int add = random.nextBoolean() ? x0[x0i] : x1[x1i];
      dest[desti++] = add;
      if (desti >= length) return;

      for (int i = x0i;; i++) {
        if ((x0[i] == add) && (!done_x0[i])) {
          done_x0[i] = true; break;
        }
      }
      while (done_x0[x0i]) x0i++;

      for (int i = x1i;; i++) {
        if ((x1[i] == add) && (!done_x1[i])) {
          done_x1[i] = true; break;
        }
      }
      while (done_x1[x1i]) x1i++;
    }
  }
}
```

Section Outline

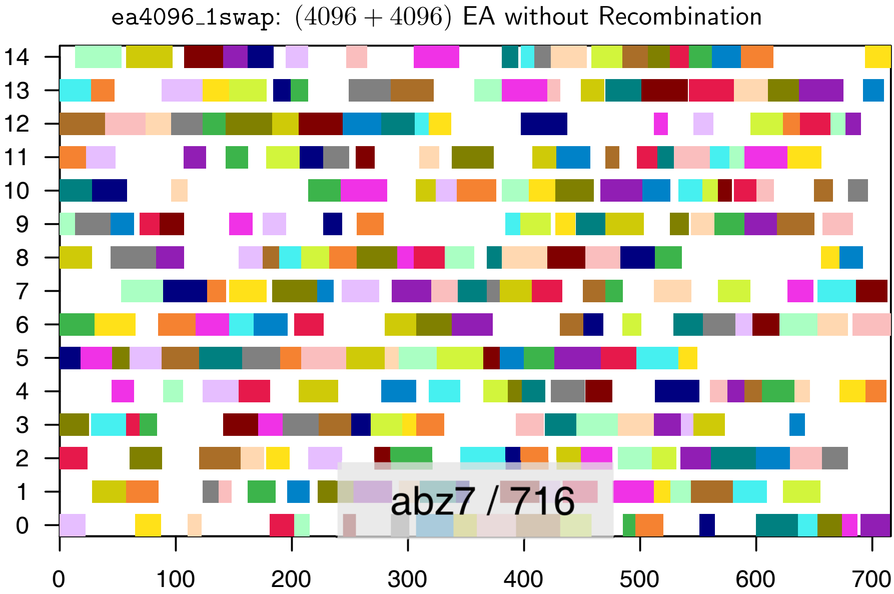
So what do we get?

- We now test the same EAs as before, but apply the binary operator at 5%

So what do we get?

		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	ea2048_1swap	695	719	718	13	11s	2'581'614
	ea4096_1swap	688	716	716	12	19s	4'416'129
	ea2048_1swap_5	689	713	**712**	11	12s	2'641'808
	ea4096_1swap_5	**680**	**712**	**712**	**10**	20s	4'145'924
la24	ea2048_1swap	945	983	983	16	2s	927'000
	ea4096_1swap	**941**	980	978	**14**	5s	1'897'387
	ea2048_1swap_5	948	980	982	15	2s	789'223
	ea4096_1swap_5	945	**976**	**975**	15	4s	1'601'925
swv15	ea2048_1swap	3395	3535	**3530**	78	128s	1'9290'521
	ea4096_1swap	3397	**3533**	3533	**54**	171s	25'073'630
	ea2048_1swap_5	**3390**	3545	3536	81	117s	15'999'092
	ea4096_1swap_5	3413	3543	3539	66	169s	22'266'887
yn4	ea2048_1swap	1032	1082	1082	22	26s	4'792'622
	ea4096_1swap	**1020**	1076	1074	21	39s	6'907'692
	ea2048_1swap_5	1027	1072	1072	19	19s	3'212'839
	ea4096_1swap_5	1034	**1068**	**1068**	**18**	37s	5'943'196

So what do we get?



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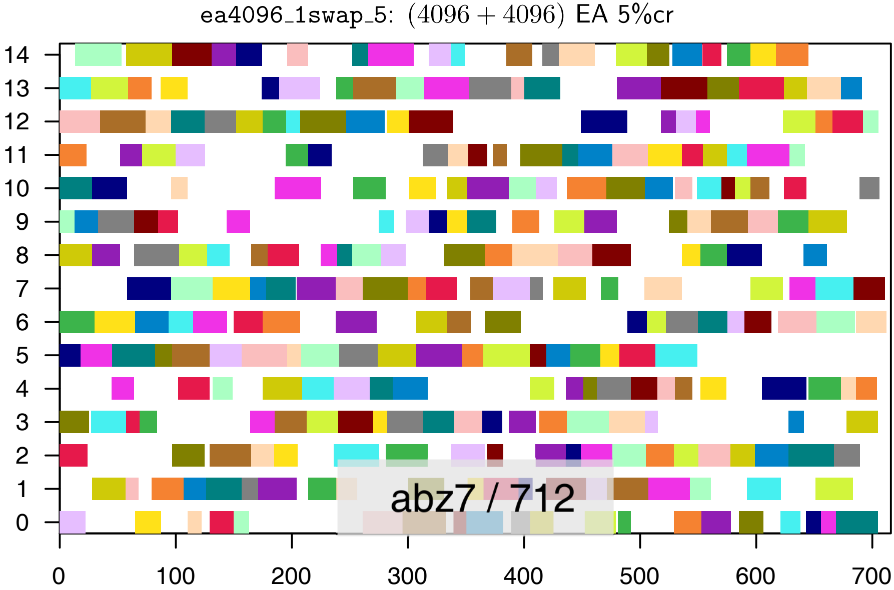

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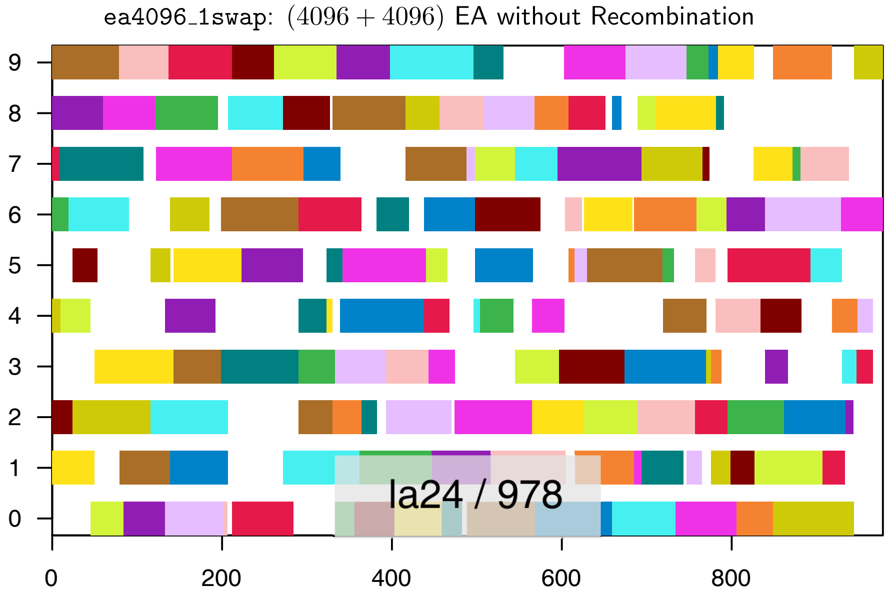

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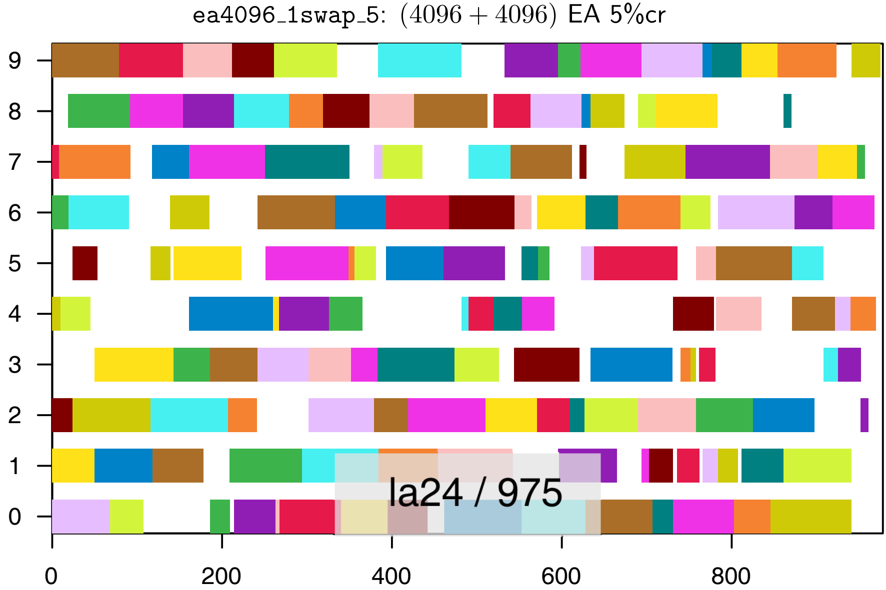
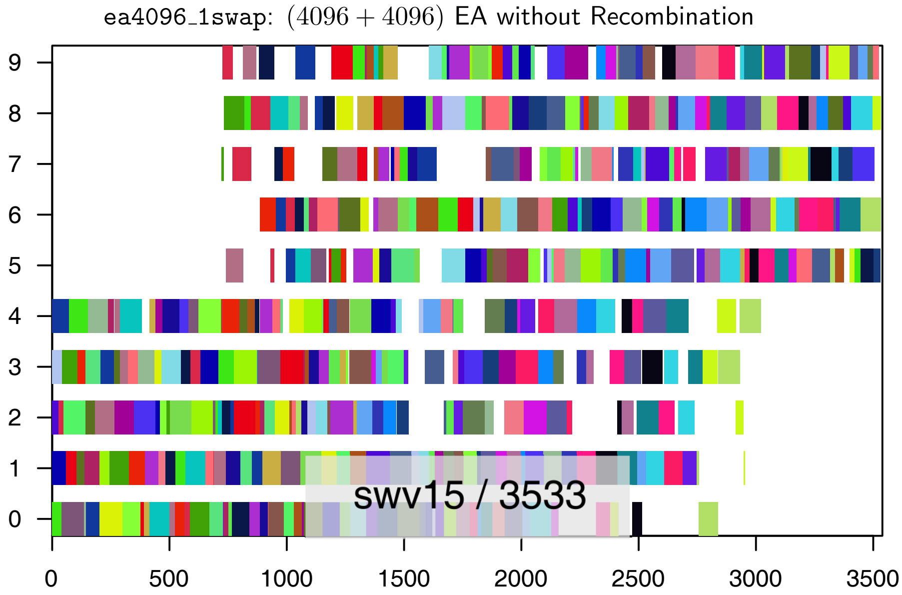
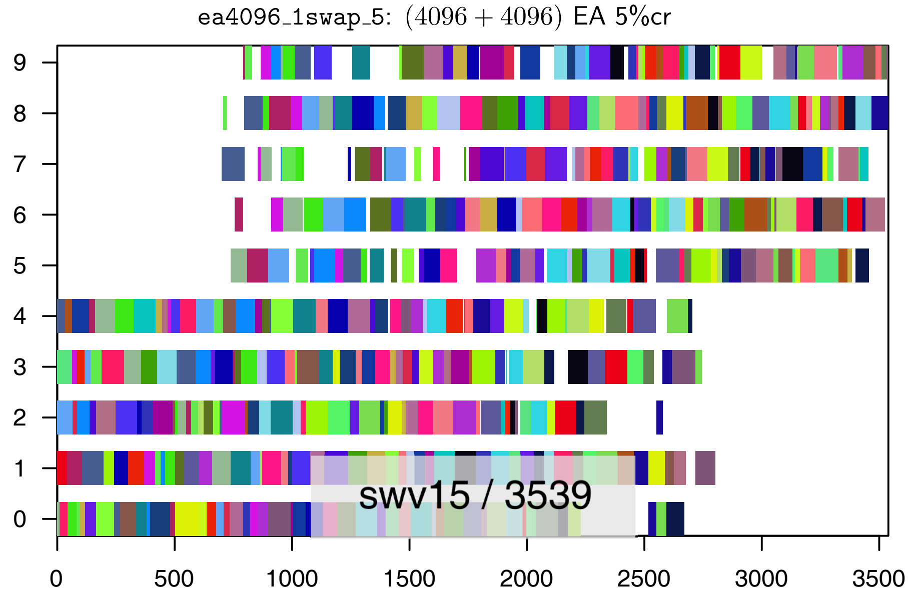
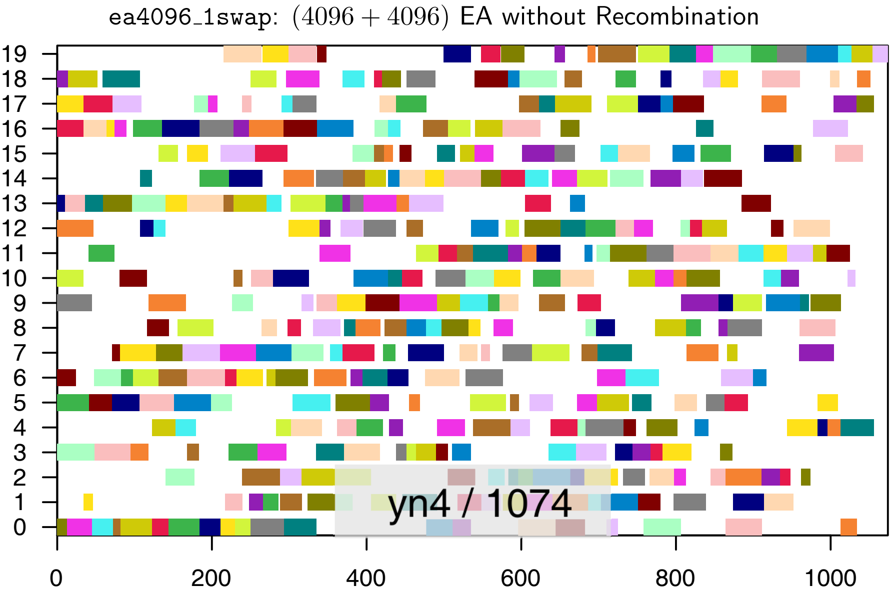



ea4096_1swap: $(4096+4096)$ EA without Recombination
9
8
7
6
5
4
3
2
1
0
0
500
1000
1500
2000
2500
3000
3500
swv15 / 3533

So what do we get?




ea4096_1swap: $(4096+4096)$ EA without Recombination
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0
200
400
600
800
1000
yn4 / 1074

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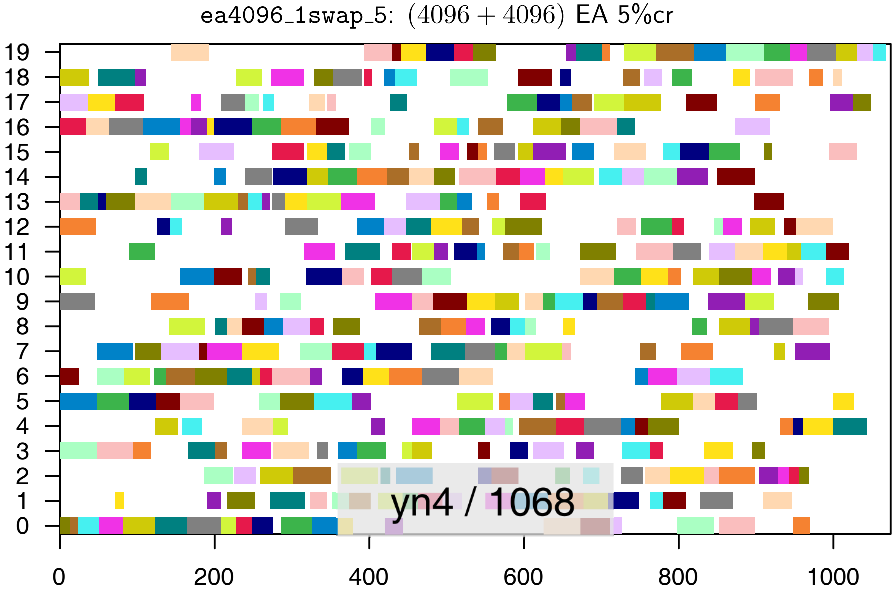

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What progress does the algorithm make over time?

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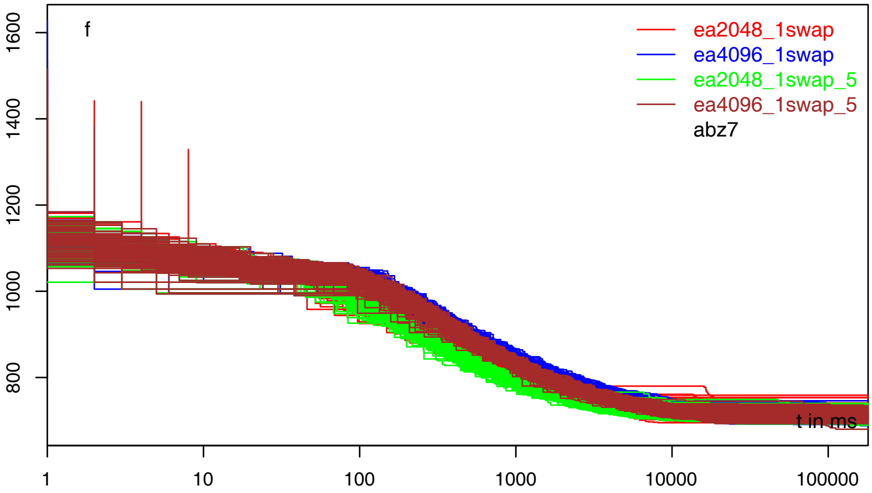

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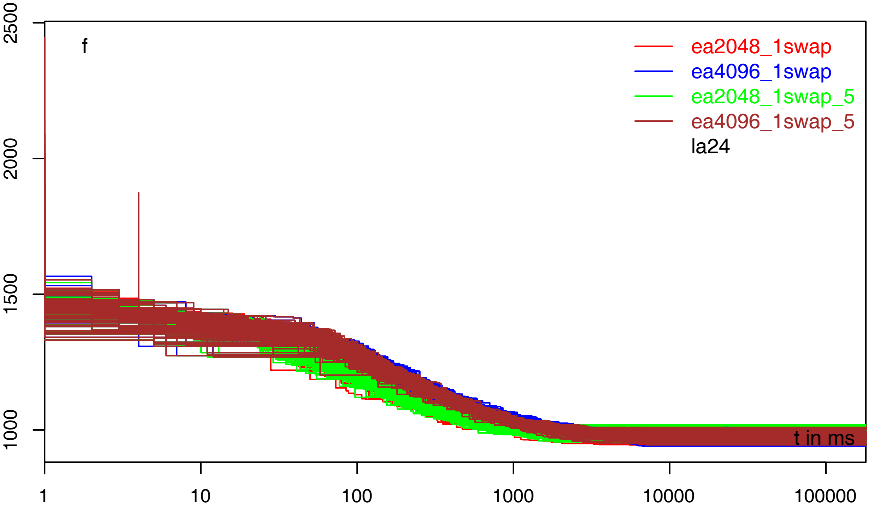

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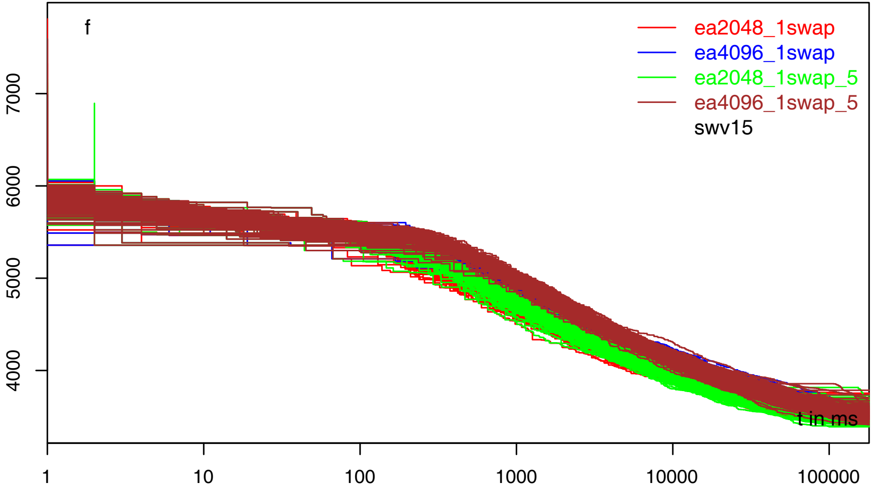

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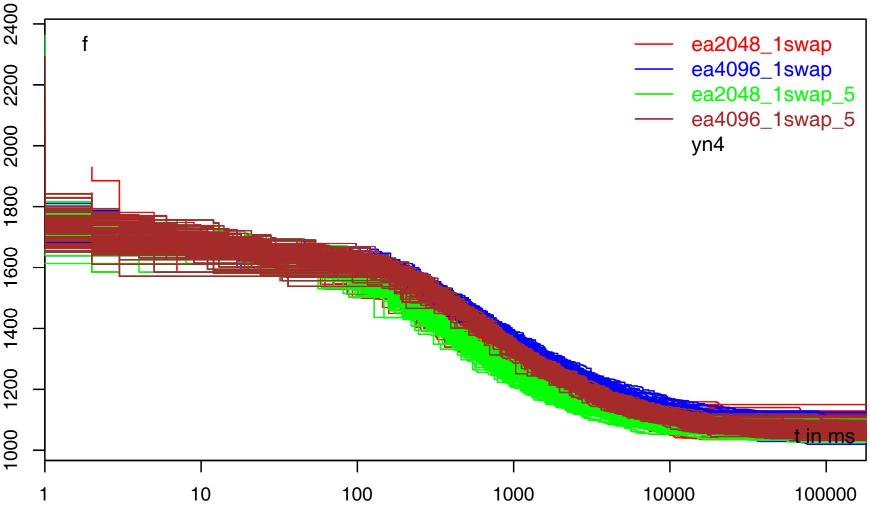

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- Anyway, overall, using the binary operator brings some gain.

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- ... but even small improvements might be worthwhile.

谢谢
Thank you

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Shushan District, Hefei, Anhui, China

Caspar David Friedrich, “Der Wanderer über dem Nebelmeer”, 1818
http://en.wikipedia.org/wiki/Wanderer_above_the_Sea_of_Fog

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