合肥學院
HEFEI UNIVERSITY

Metaheuristics for Smart Manufacturing

4. Stochastic Hill Climbing

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Outline

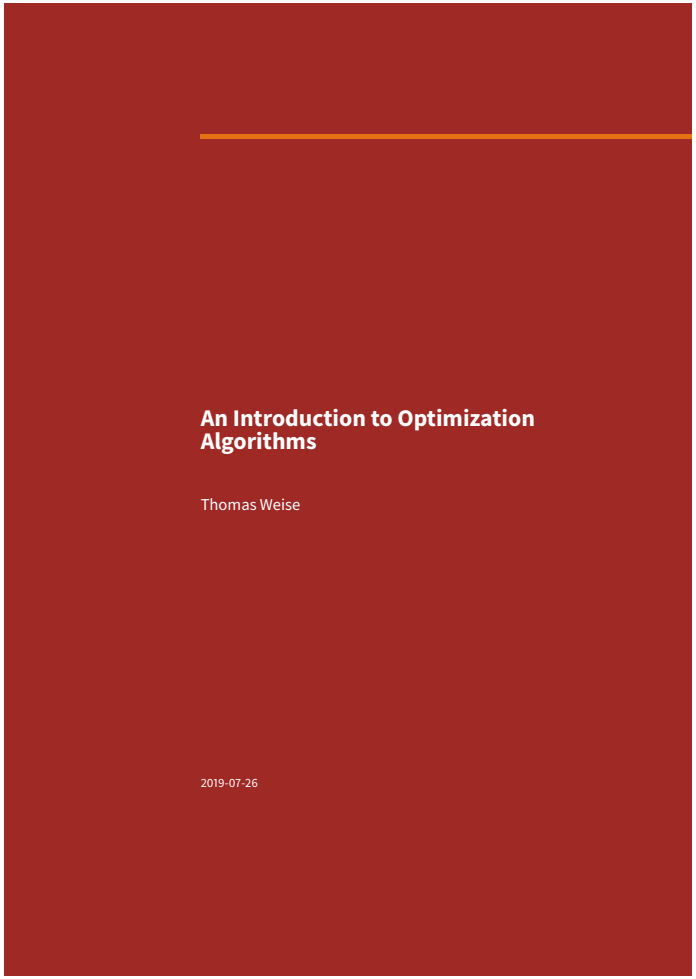
The slides are available at http://iao.hfuu.edu.cn/155, the book at http://thomasweise.github.io/aitoa, and the source code at http://www.github.com/thomasWeise/aitoa-code

course book

course material

An Introduction to Optimization Algorithms

The contents of this course are available as free electronic book *"An Introduction to Optimization Algorithms"* [1] at http://thomasweise.github.io/aitoa in pdf, html, azw3, and epub format, created with our bookbuildeR tool chain.



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  4. go back to 2 (until the time is up)

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- We hope to find one of them and then continue trying to do the same from there.

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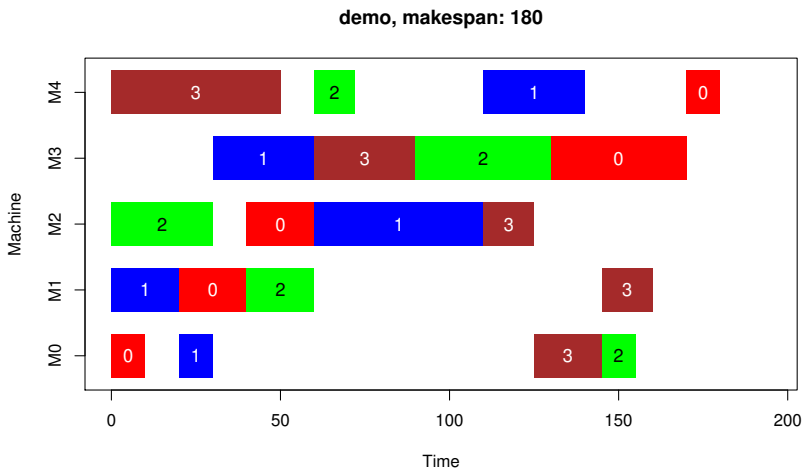

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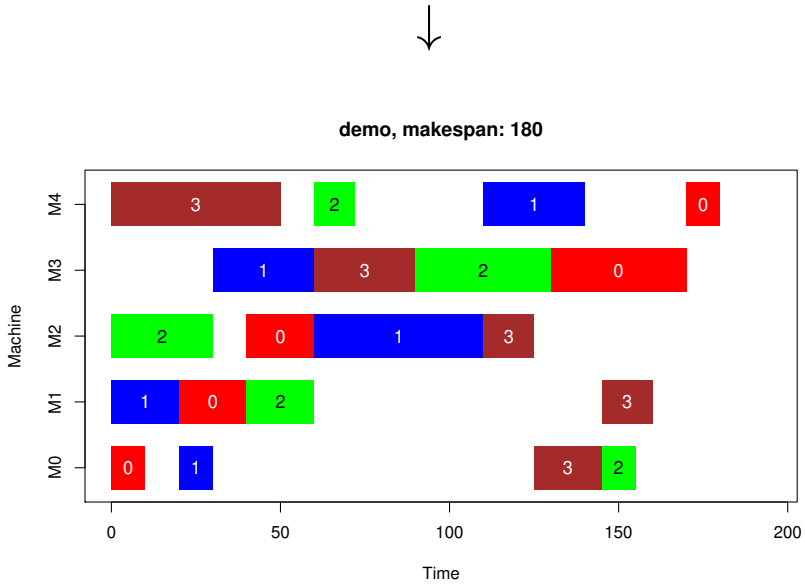

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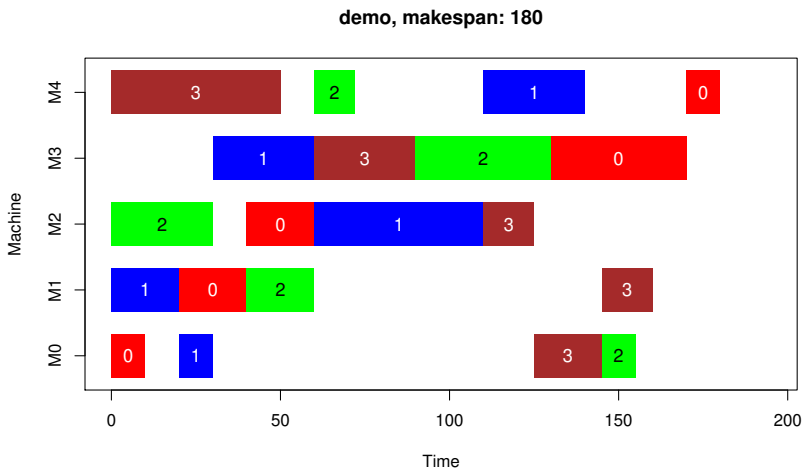

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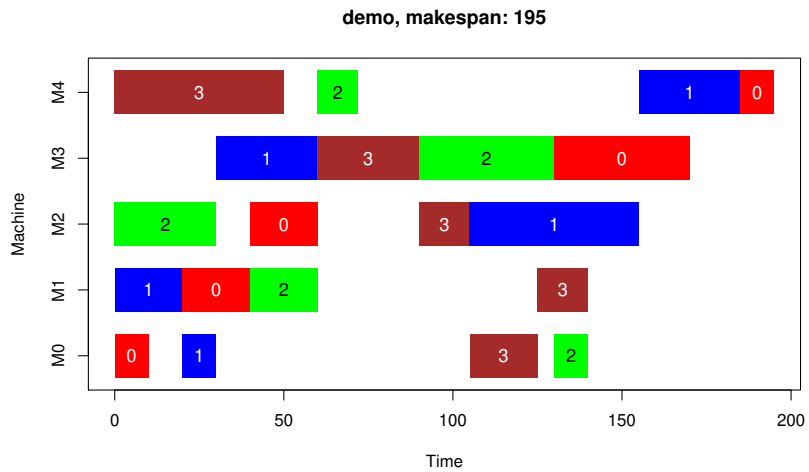

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  - To make sure that the result is different, we can first check if the job IDs are different and if not, pick two new indices i and j.
- Many modifications will lead to worse results, but some can be improvements.

Interface for Creating Modified Copy of Solution

Listing: An Java interface for generating a modified copy of a solution.

```
public interface IUnarySearchOperator<X> {

  public abstract void apply(X x, X dest, Random random);

}
```

Create Modified Copy of Existing Solution

Listing: Swap sub-jobs of two different jobs.

```
public class JSSPUnaryOperator1Swap
    implements IUnarySearchOperator<int[]> {

  public void apply(int[] x, int[] dest, Random random) {
    System.arraycopy(x, 0, dest, 0, x.length);

    int i = random.nextInt(dest.length);
    int job_i = dest[i];

    for (;;) {
      int j = random.nextInt(dest.length);
      int job_j = dest[j];
      if (job_i != job_j) {
        dest[i] = job_j;
        dest[j] = job_i;
        return;
      }
    }
  }
}
```

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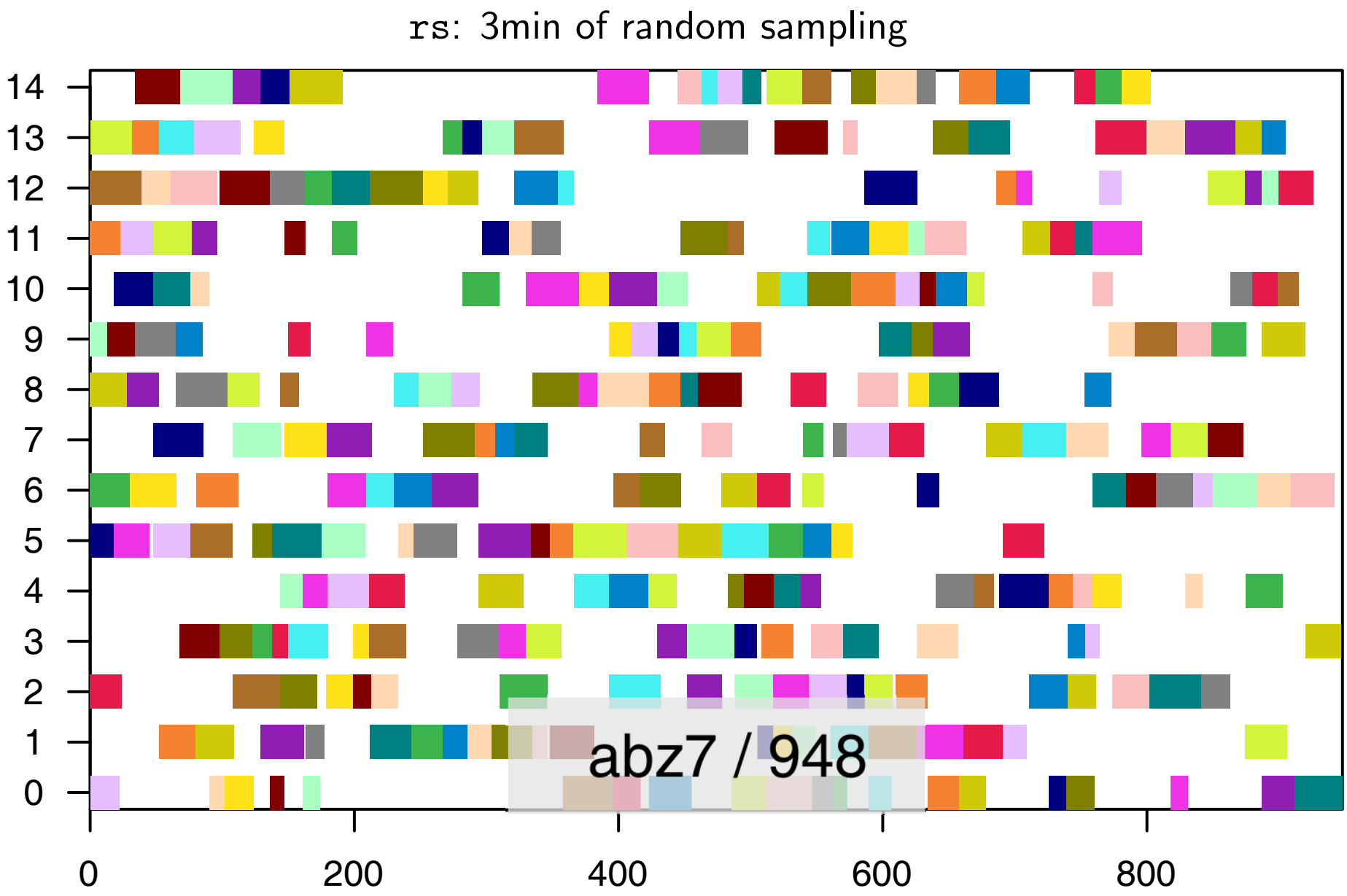

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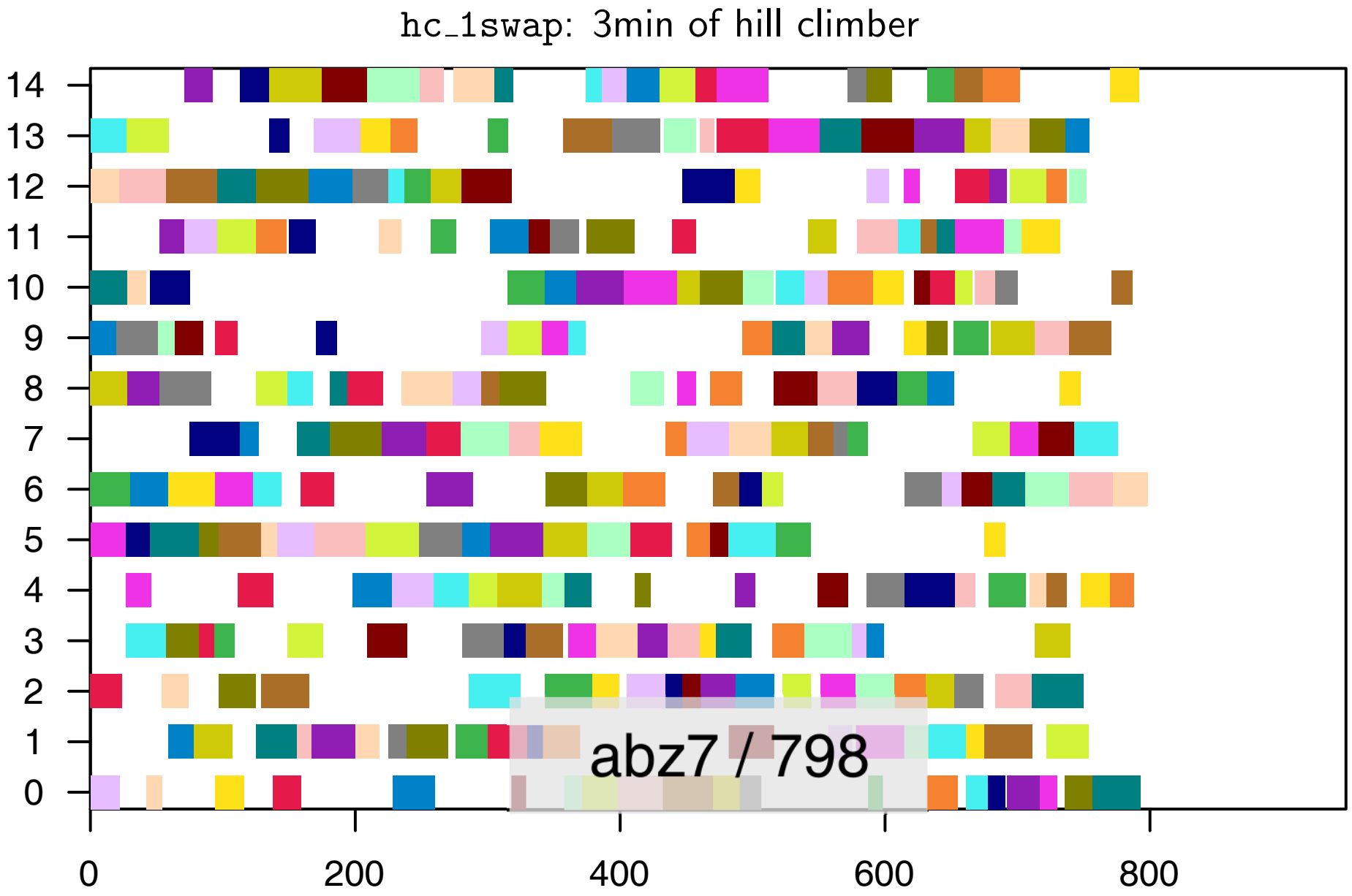

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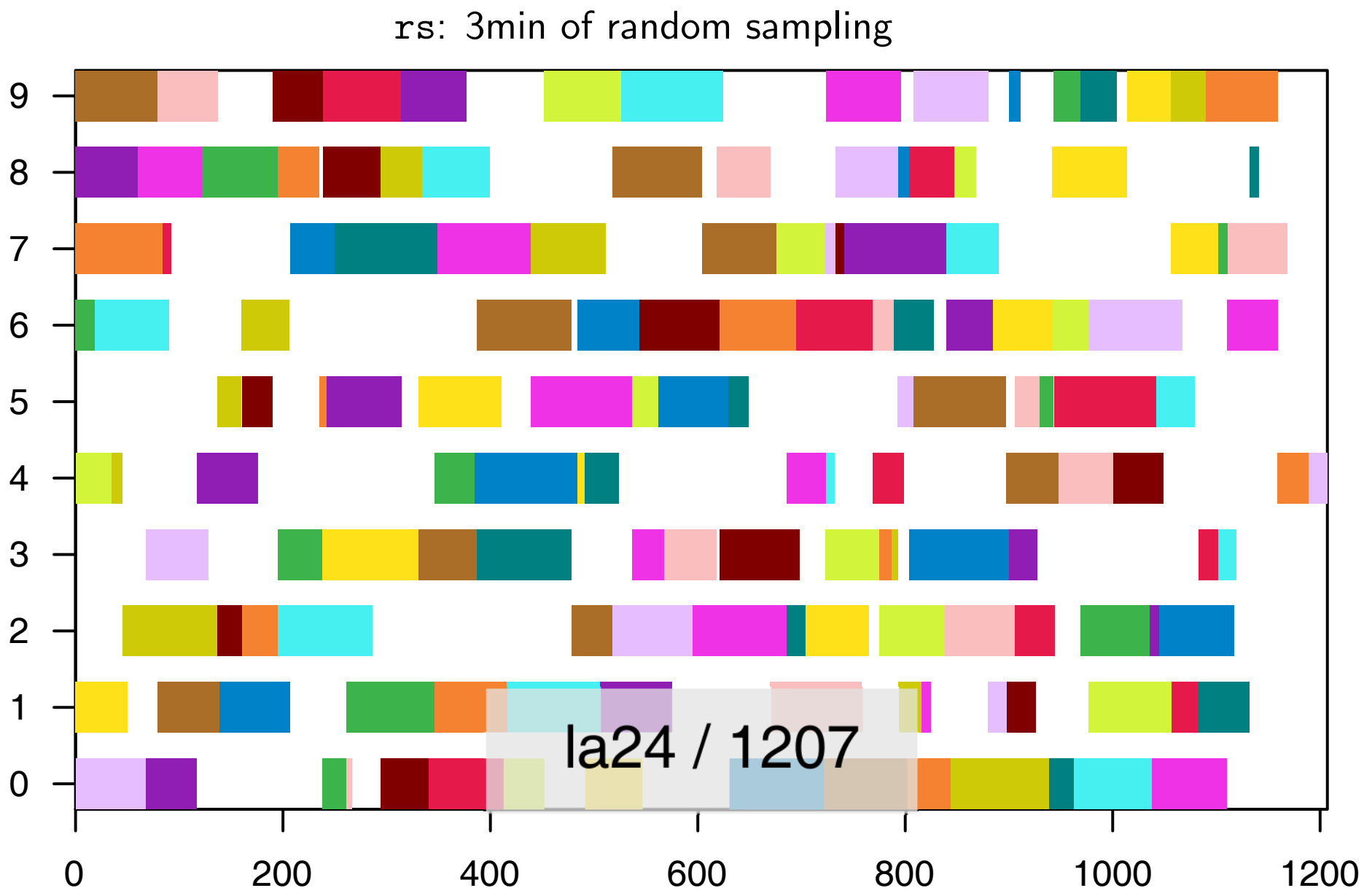

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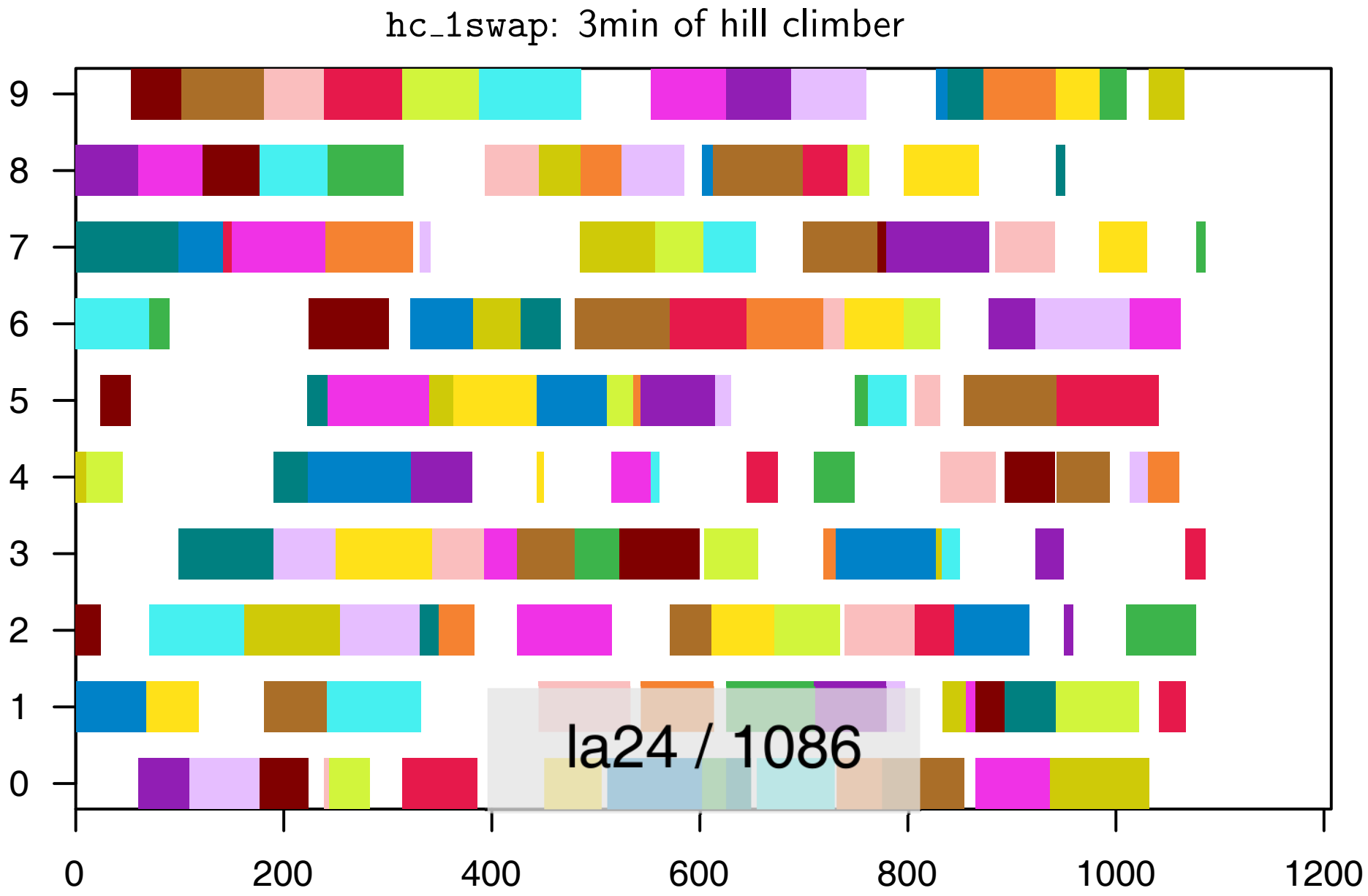

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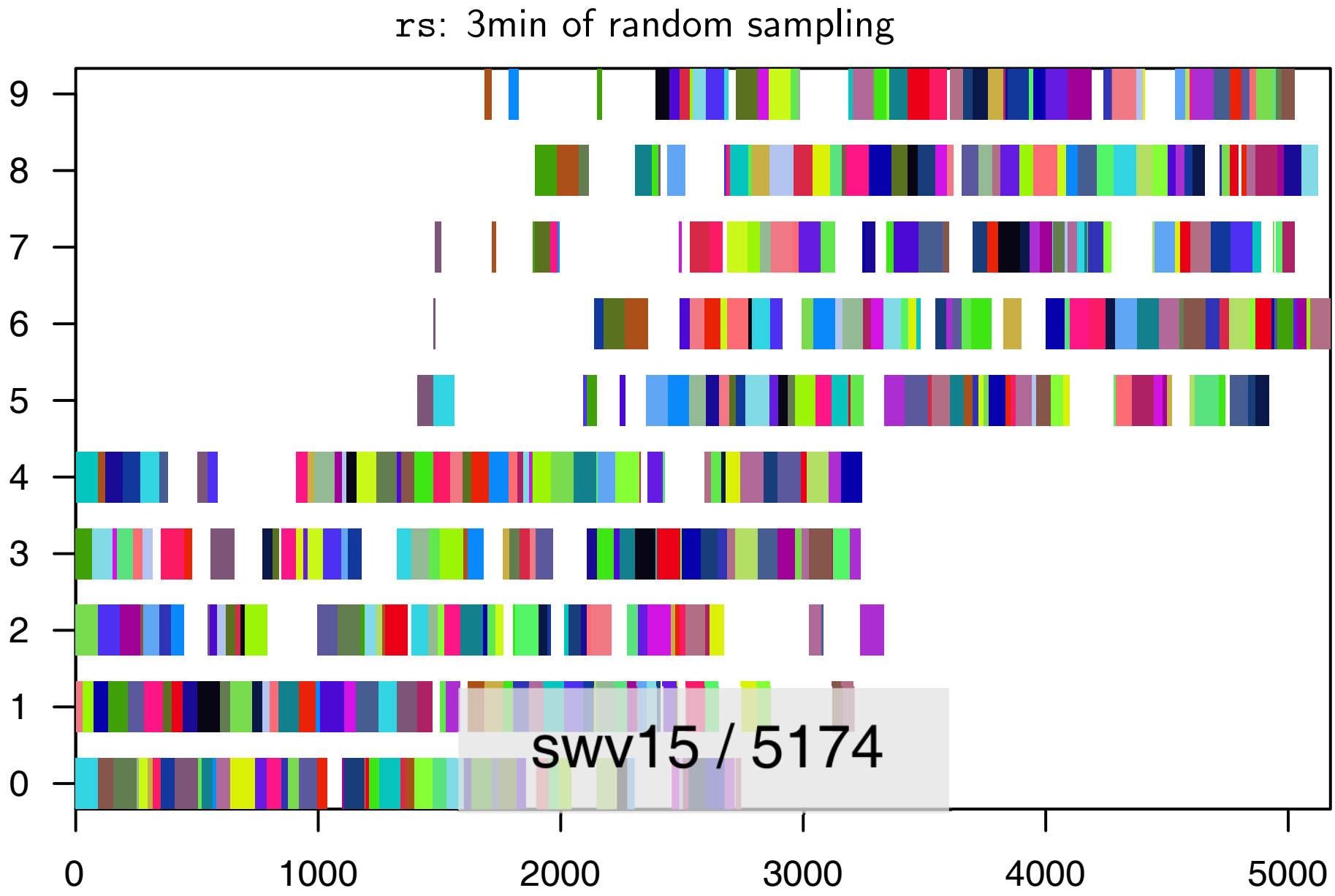

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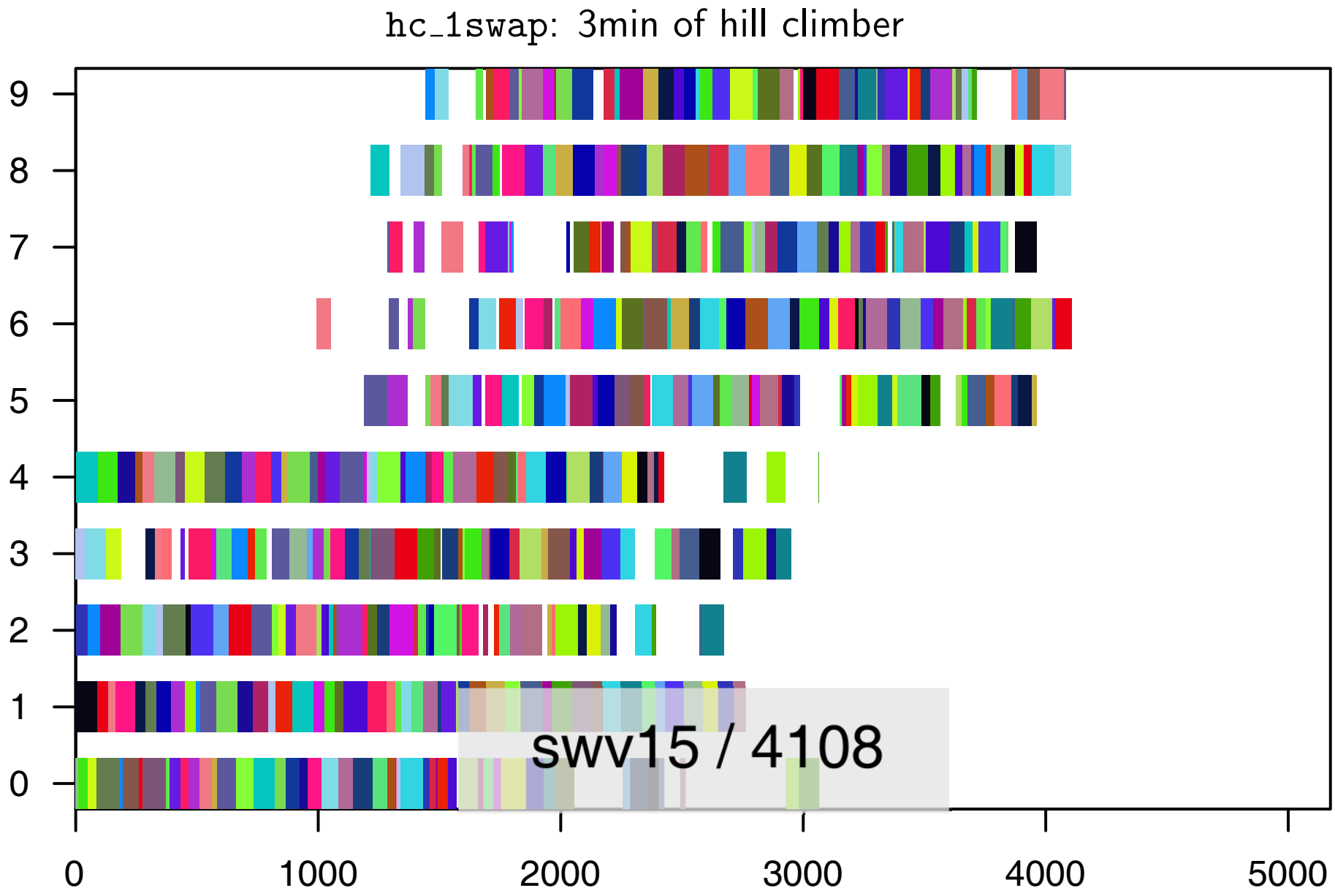

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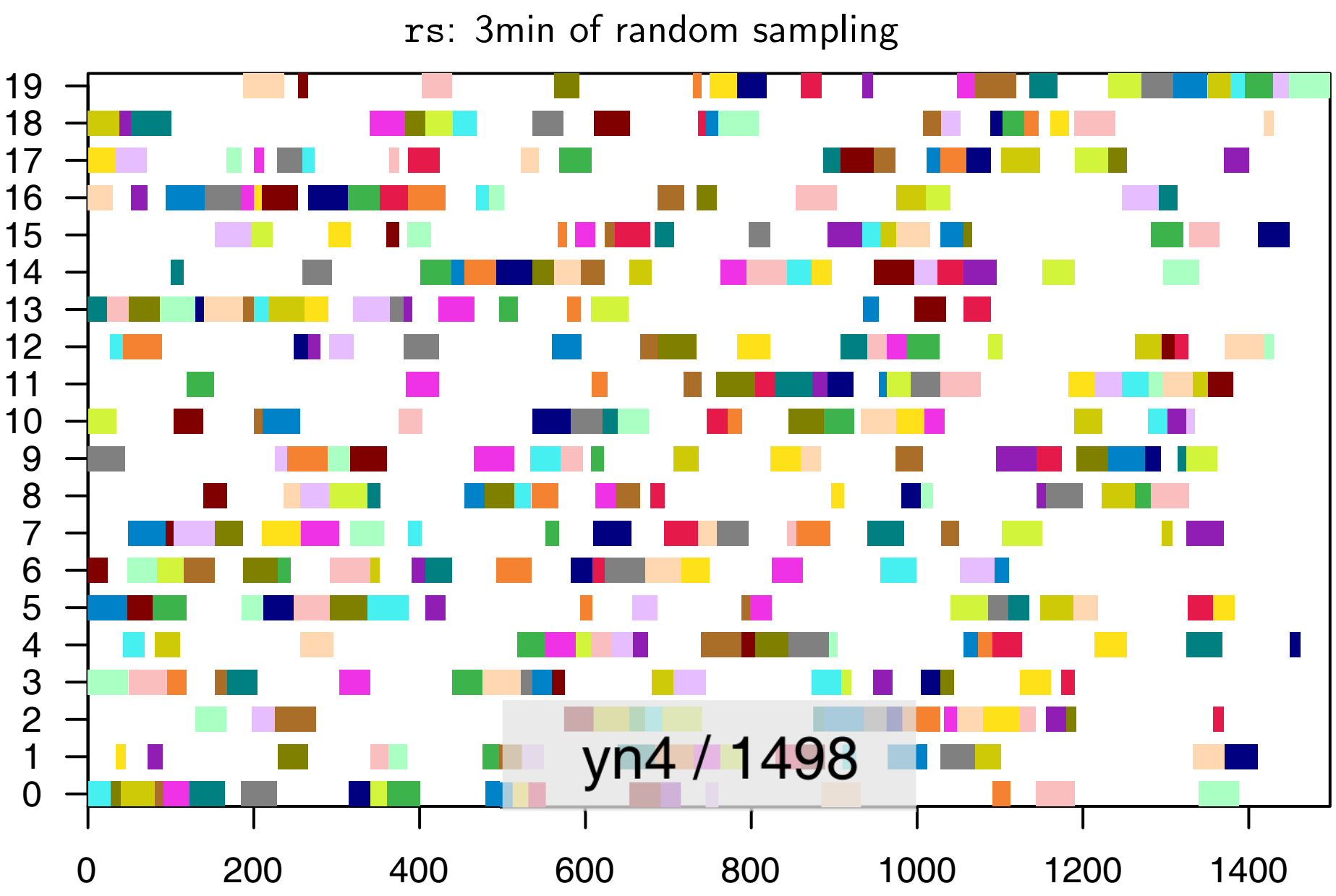

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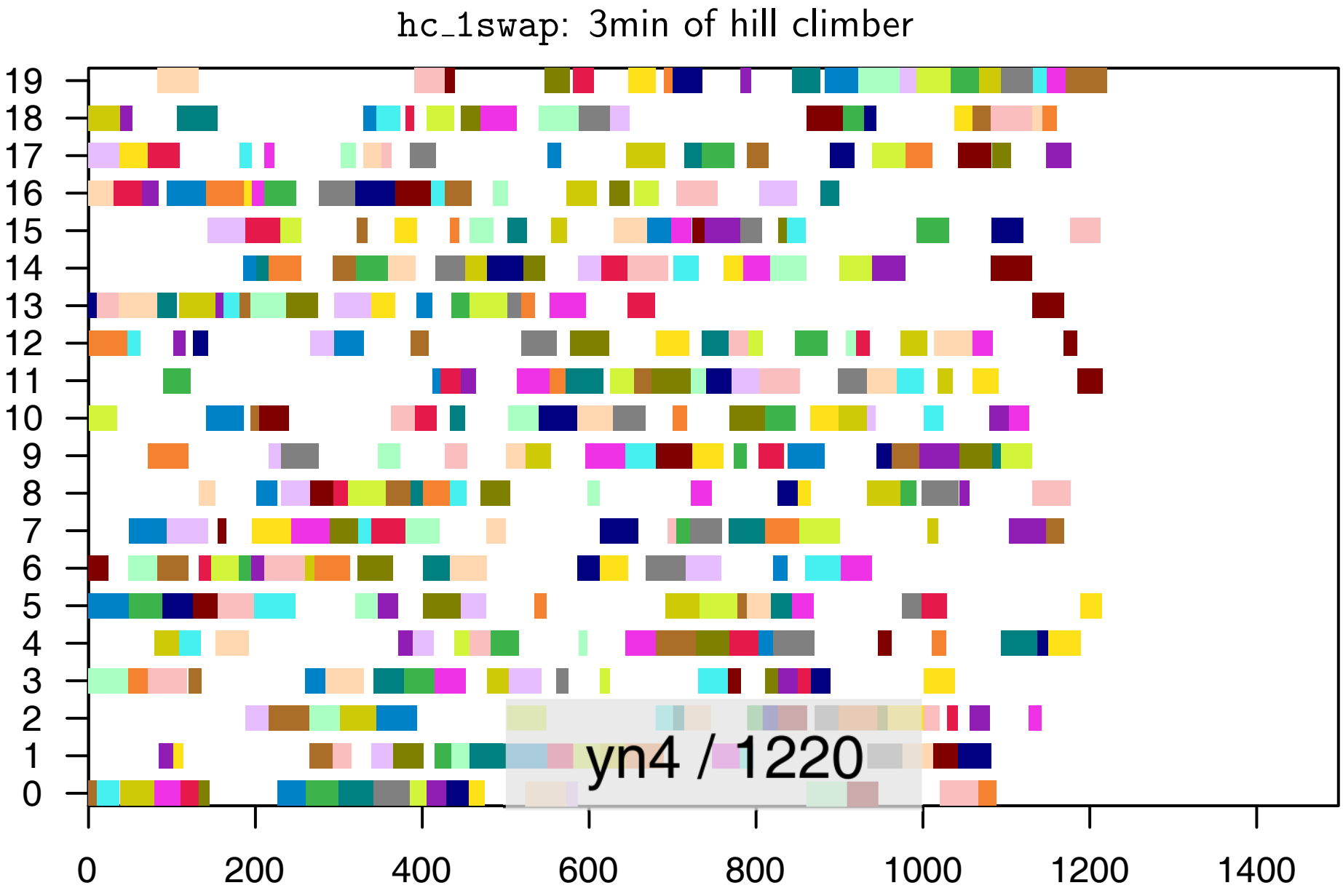

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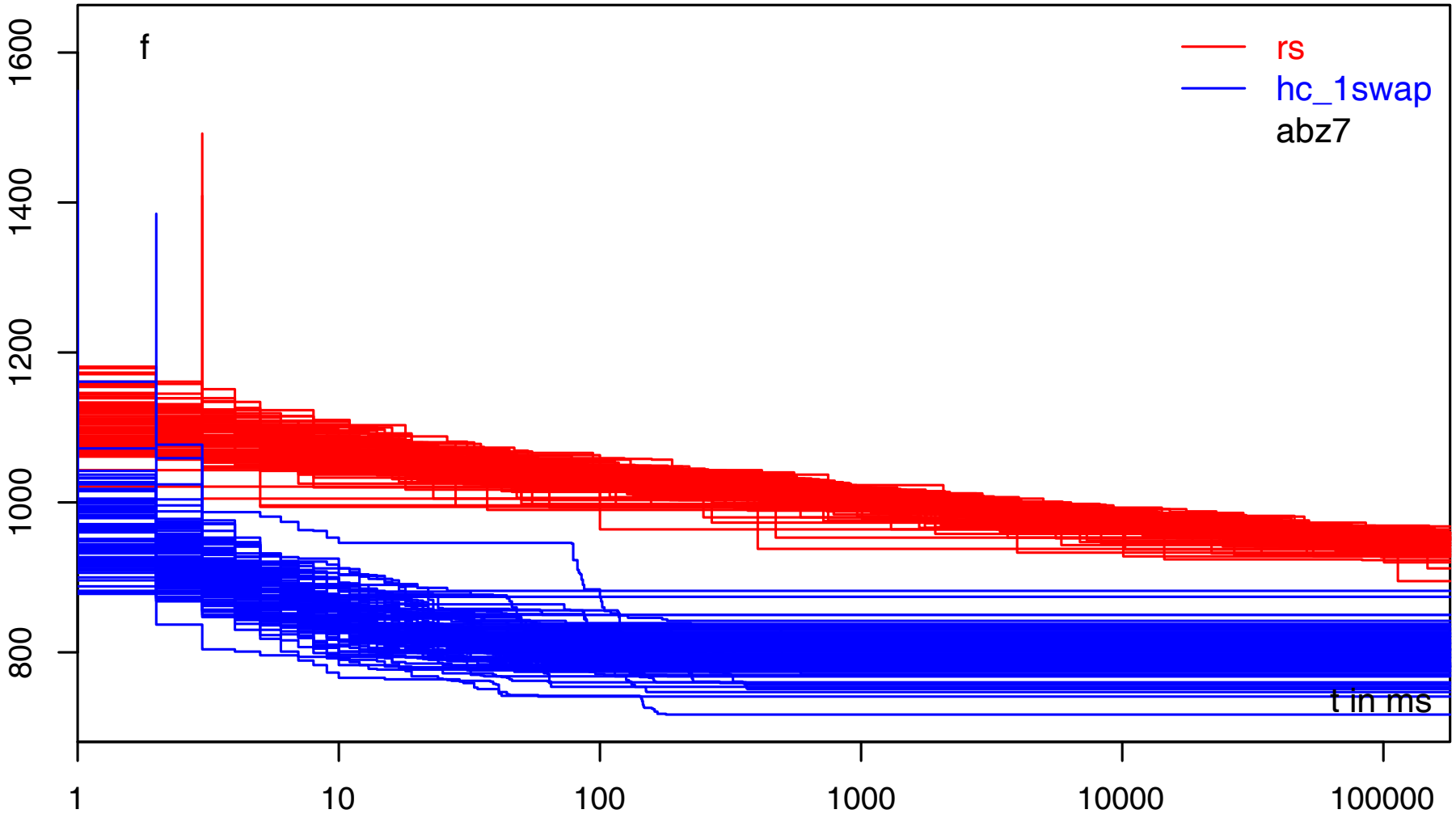

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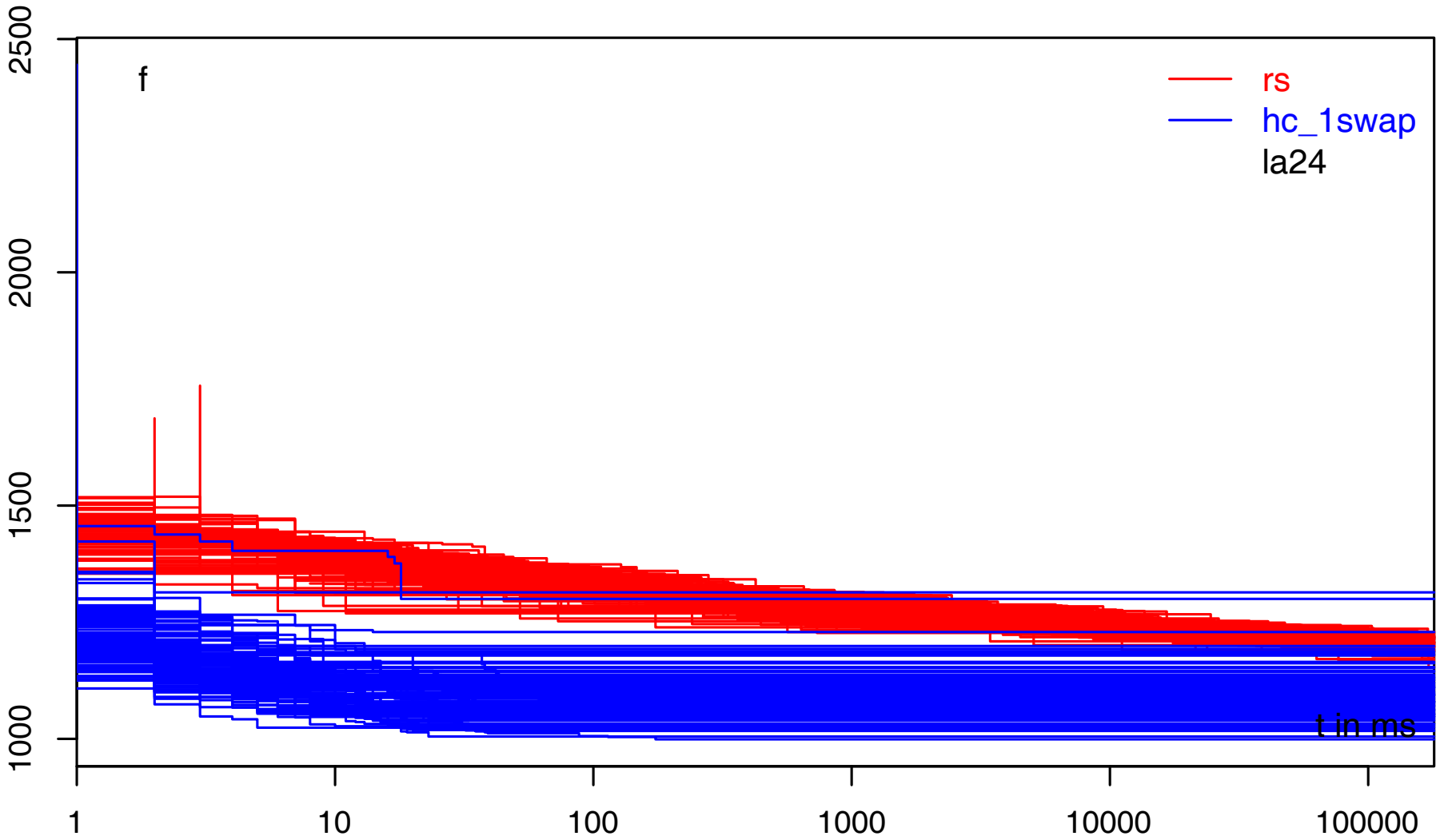

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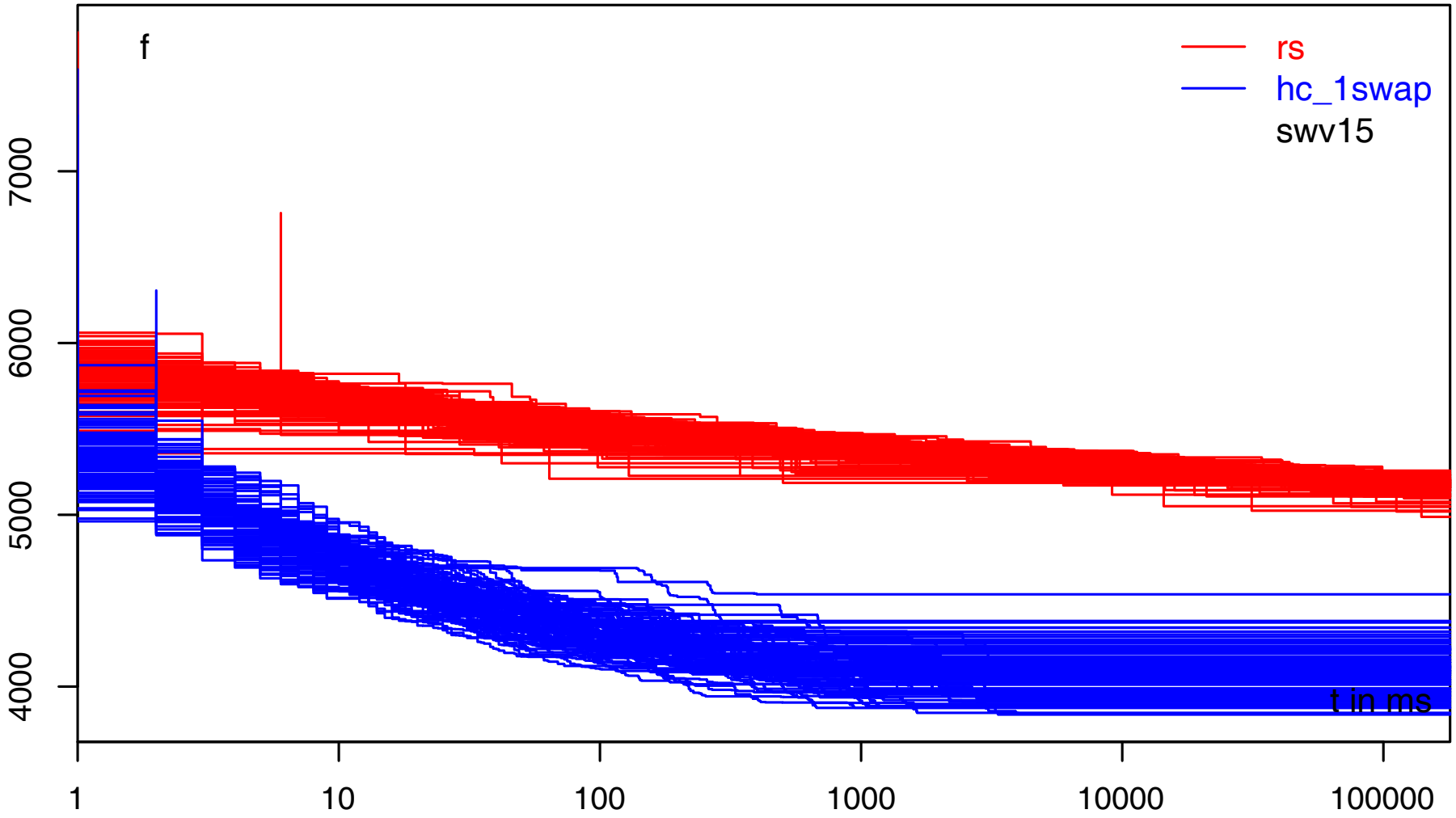

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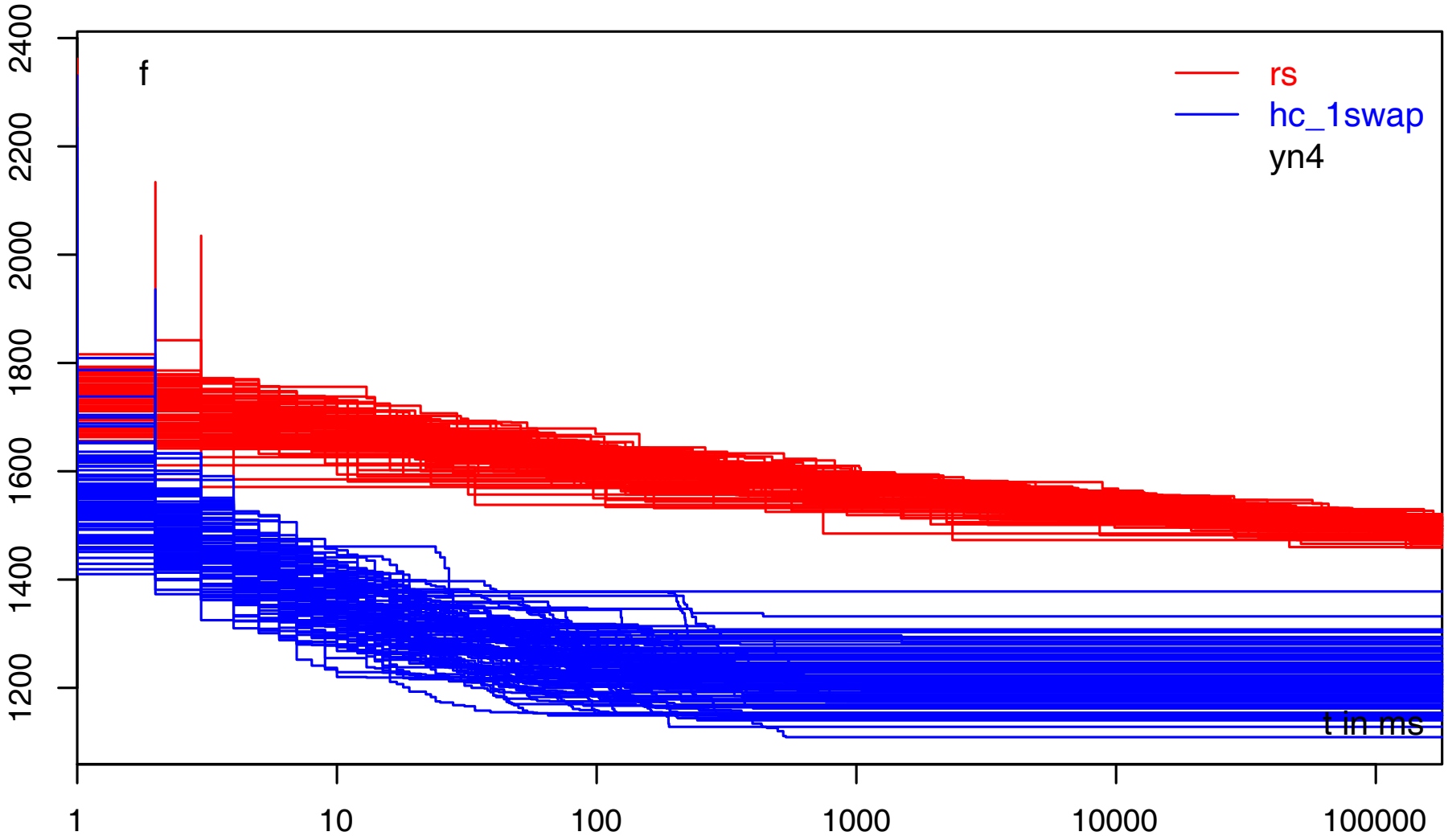

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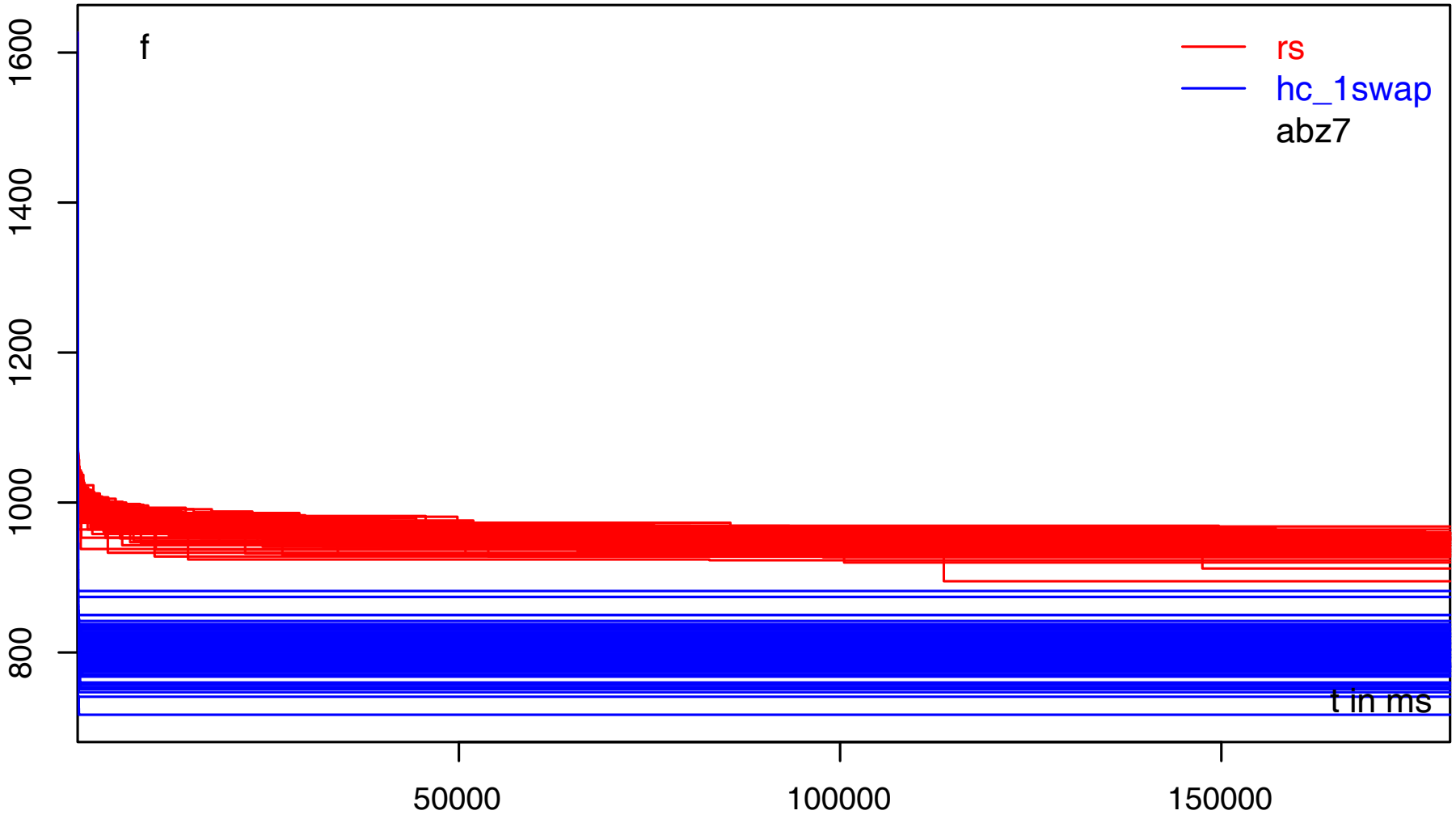

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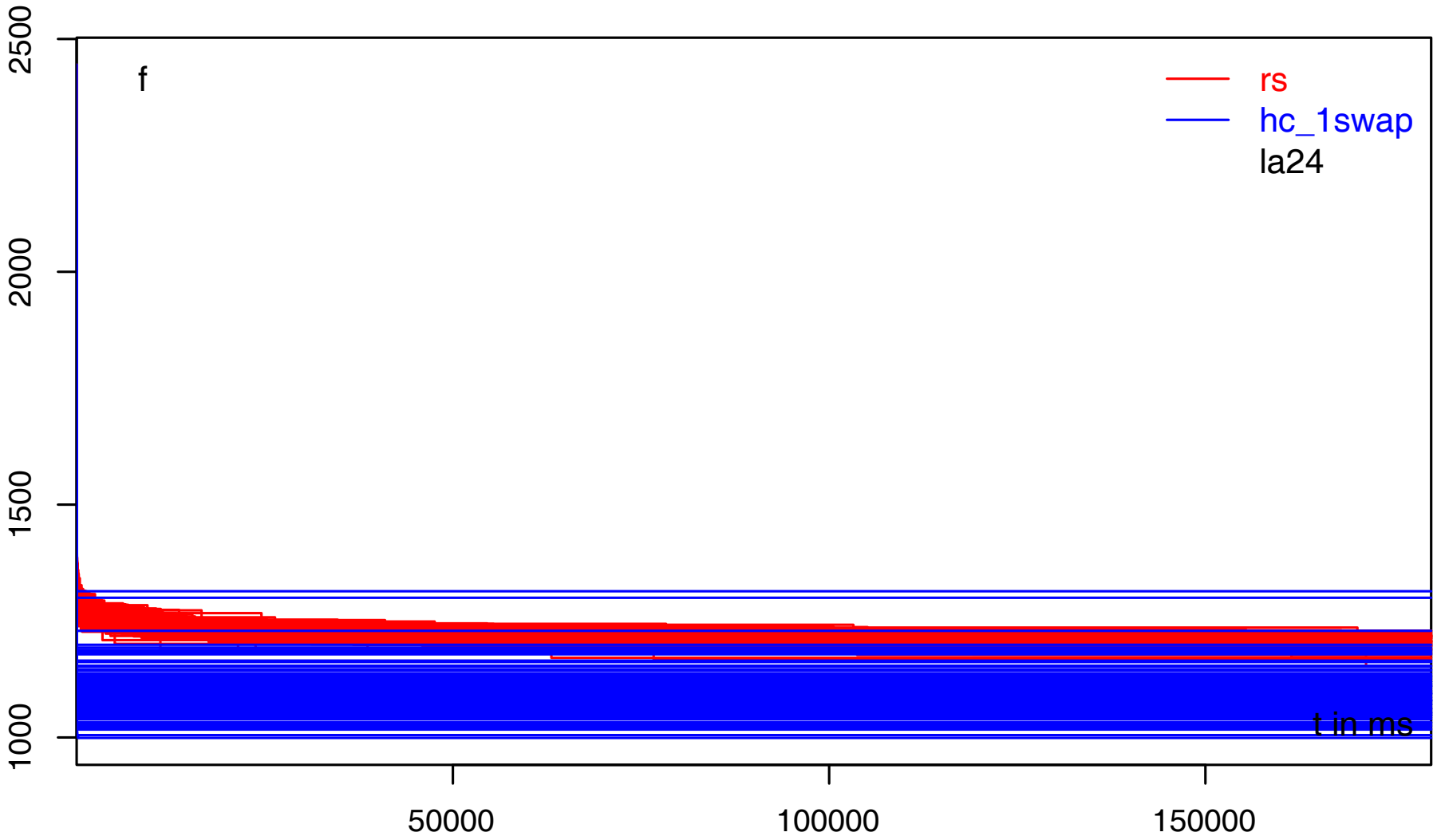

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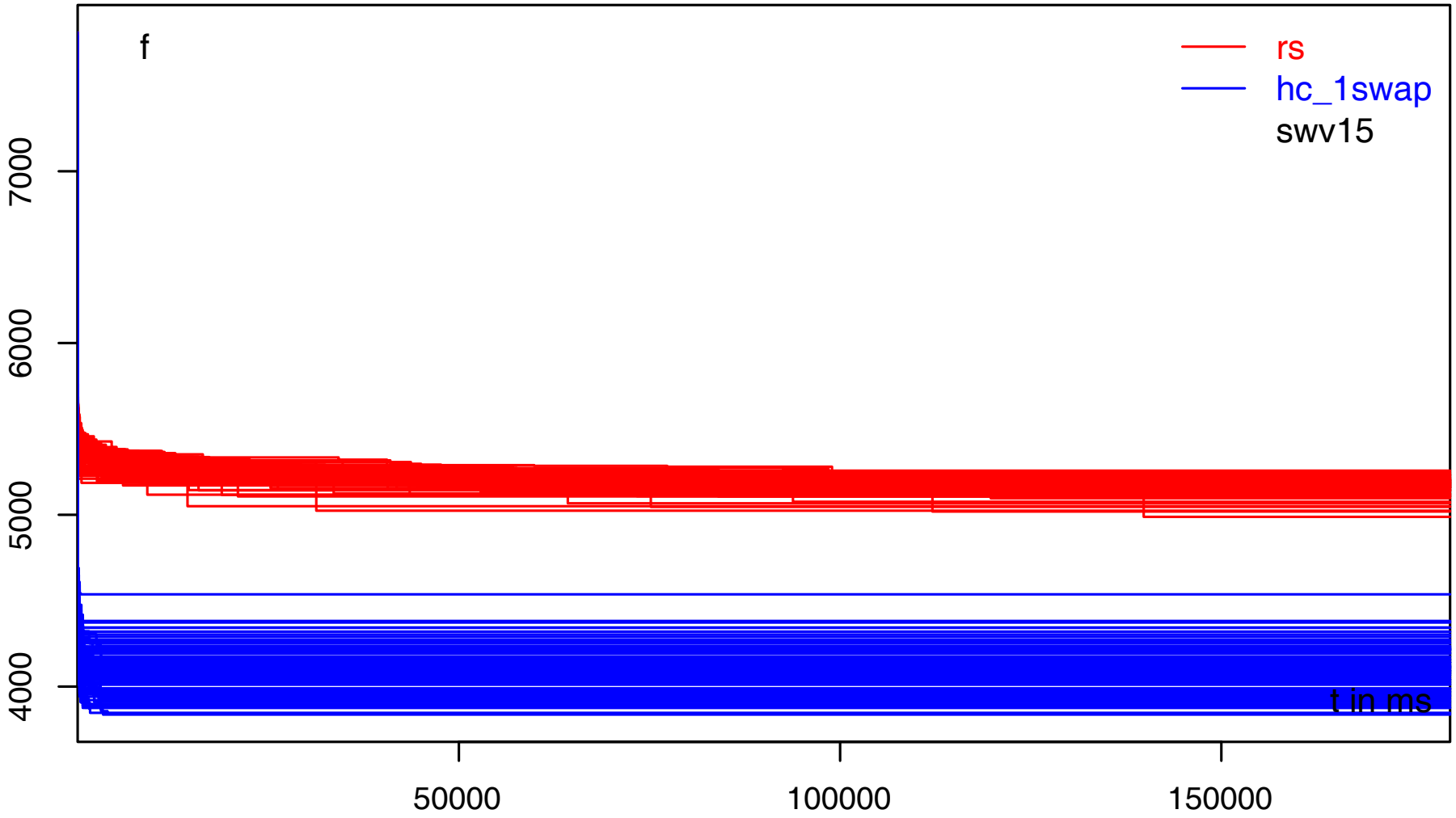

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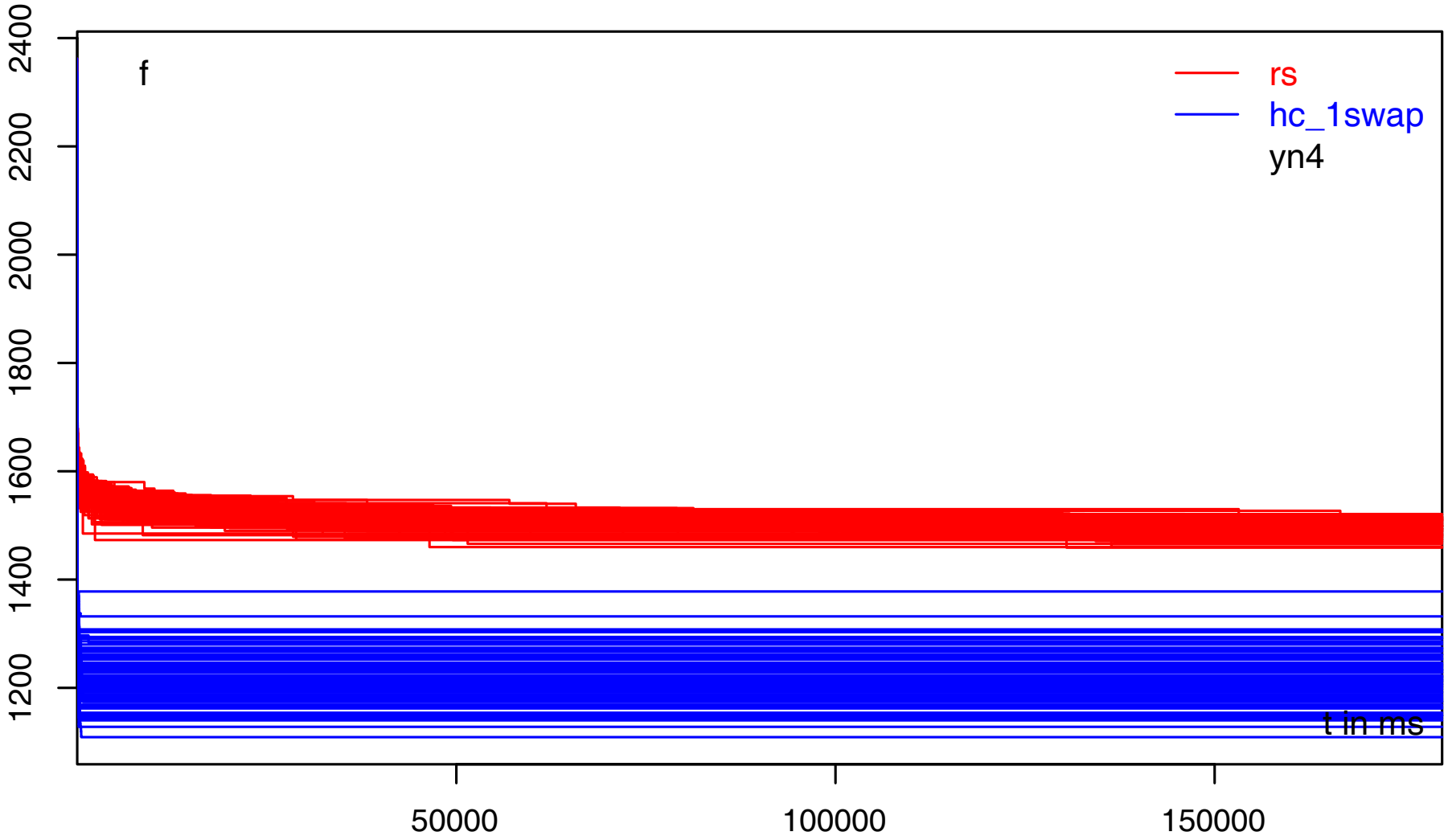

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We have 3min, but our hill climber stops improving after basically 1s!

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- This is called Premature Convergence. [8, 9]

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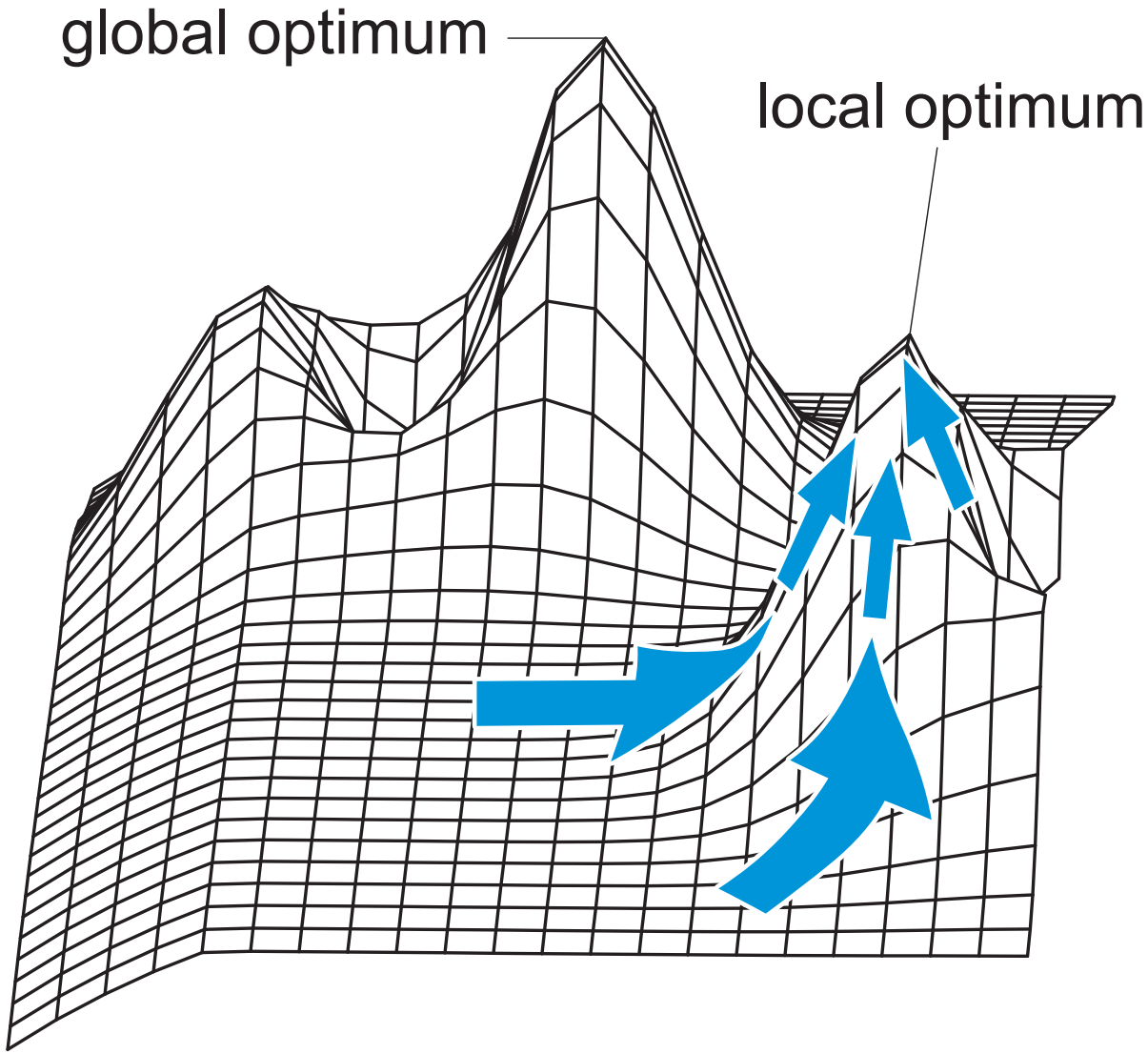

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- At the same time, a single *run* of the algorithm converges quickly.

		makespan				**last improvement**	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	rs	895	945	948	**12**	77s	8'246'019
	hc_1swap	**717**	**800**	**798**	28	0s	16'978
la24	rs	1154	1206	1207	**15**	81s	17'287'329
	hc_1swap	**999**	**1095**	**1086**	56	0s	6612
swv15	rs	4988	5165	5174	**49**	85s	5'525'082
	hc_1swap	**3837**	**4108**	**4108**	137	1s	104'598
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- Idea: If we did not make any progress for some time t, we simply restart at a new random solution.
- Of course, we will always remember the overall best solution we ever had (in another variable).
- Since we do not know which value of t is good, start small and increase it after each restart.

So what do we get?

- I execute the program 101 times for each of the datasets `abz7`, `la24`, `swv15`, and `yn4`

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	hc_1swap	999	1095	1086	56	0s	6612
	hcr_256+5%_1swap	**970**	**997**	**998**	**9**	6s	3'470'368
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	hcr_256+5%_1swap	**3701**	**3850**	**3857**	**40**	60s	9'874'102
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So what do we get?

- It is still the *same* algorithm. Restarting will not always make it better (e.g., if I restart too early) – but it will often do.

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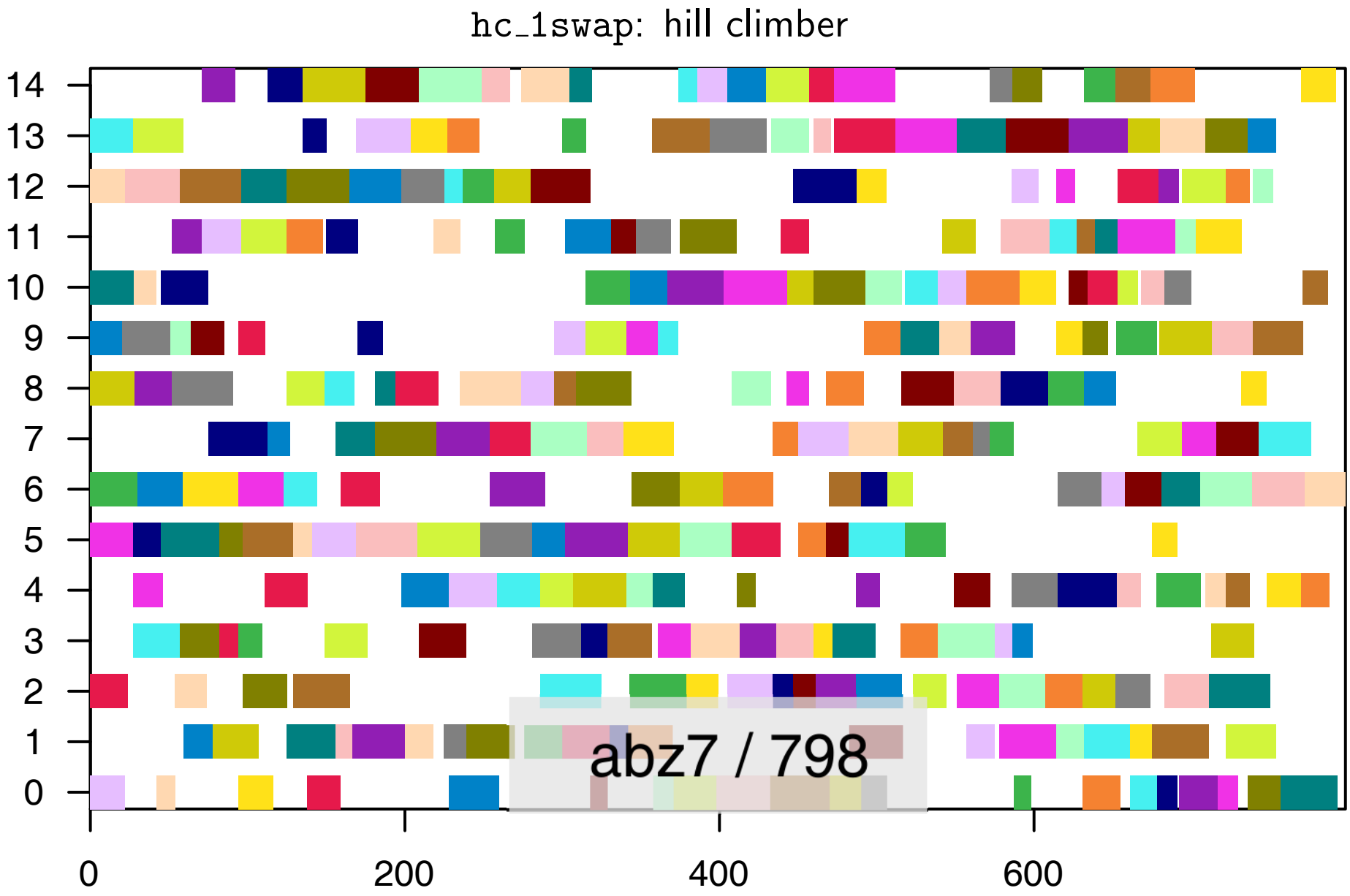

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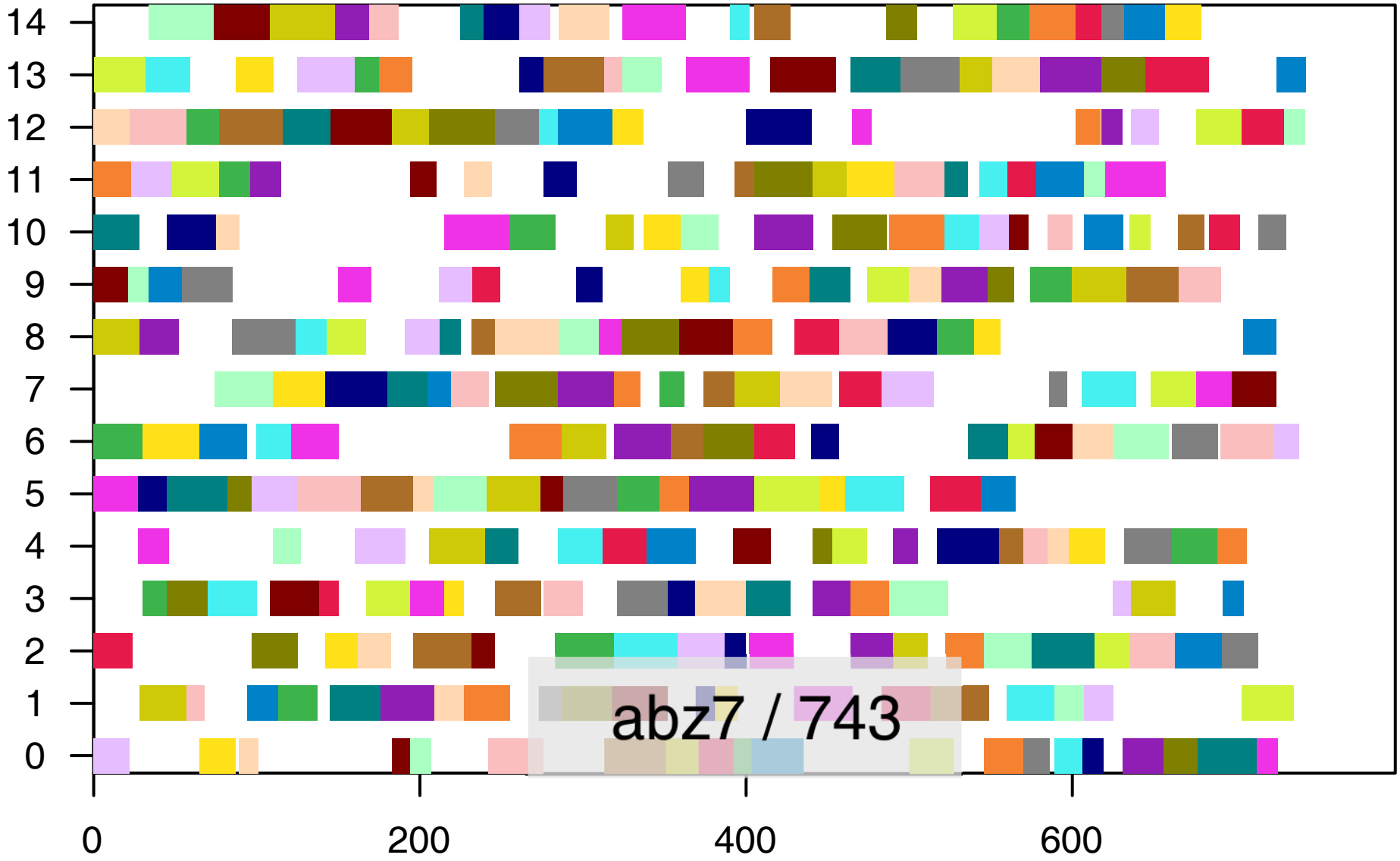
`hcr_256+5%_1swap`: hill climber with restarts after 256+5% non-improv steps



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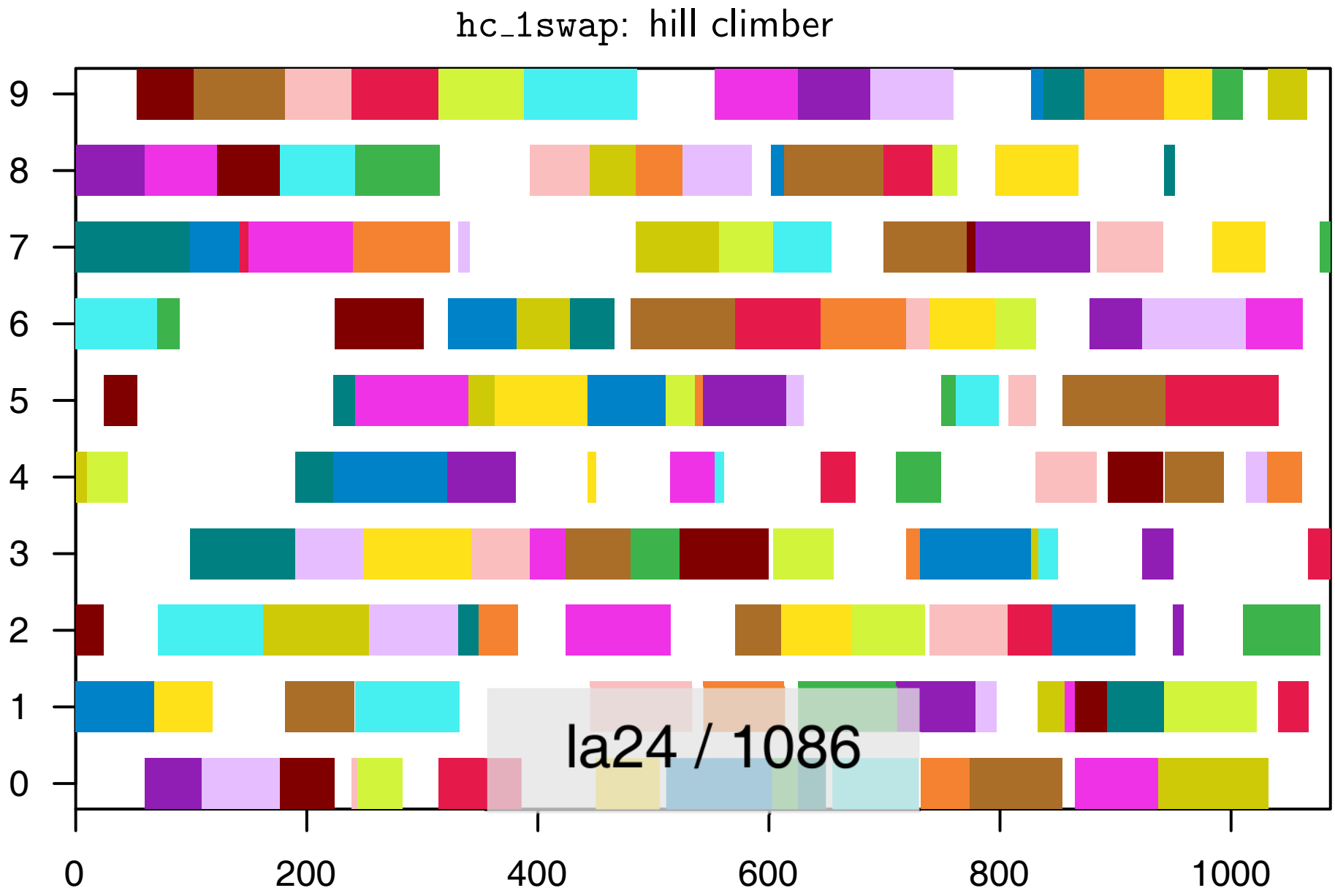

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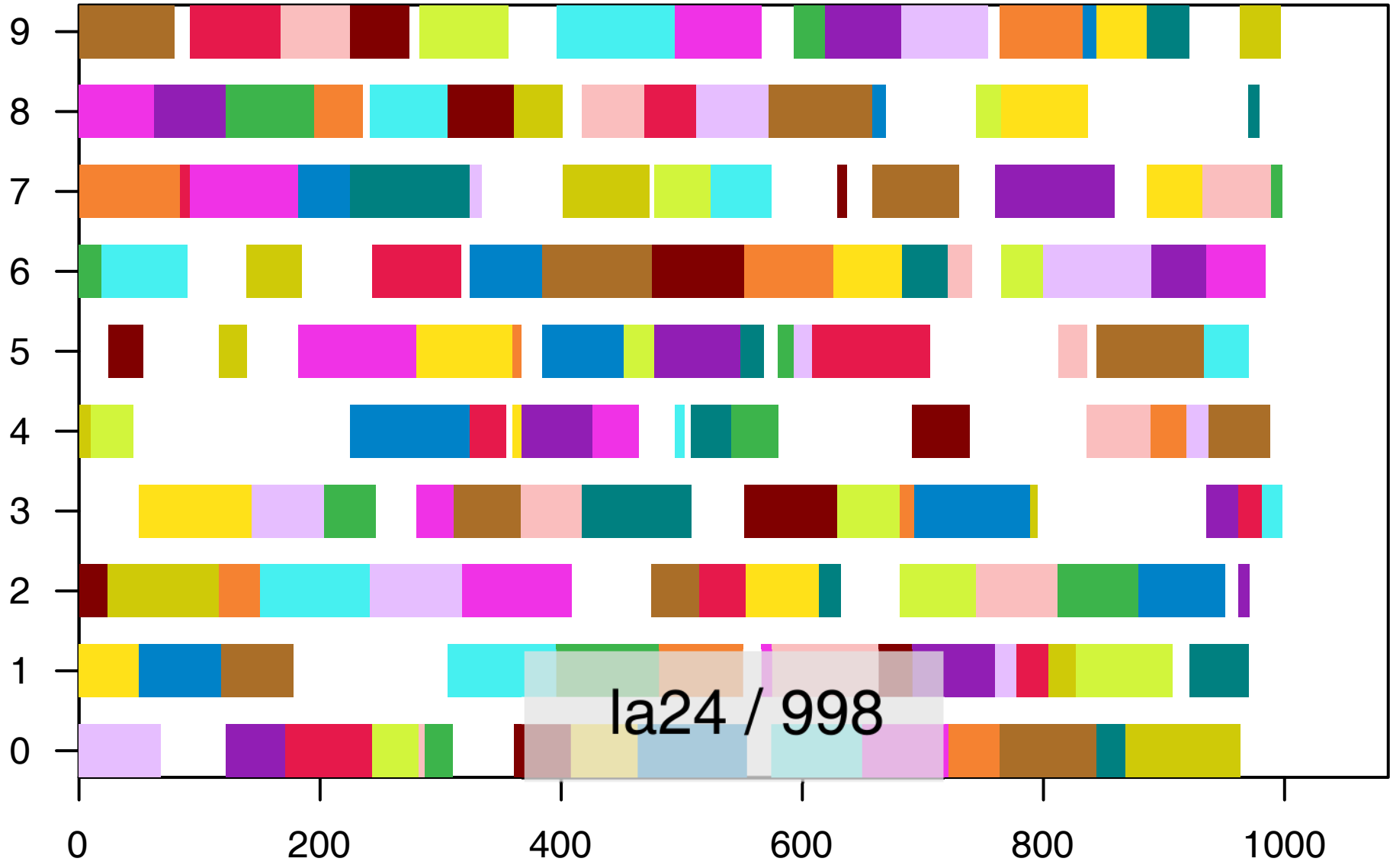

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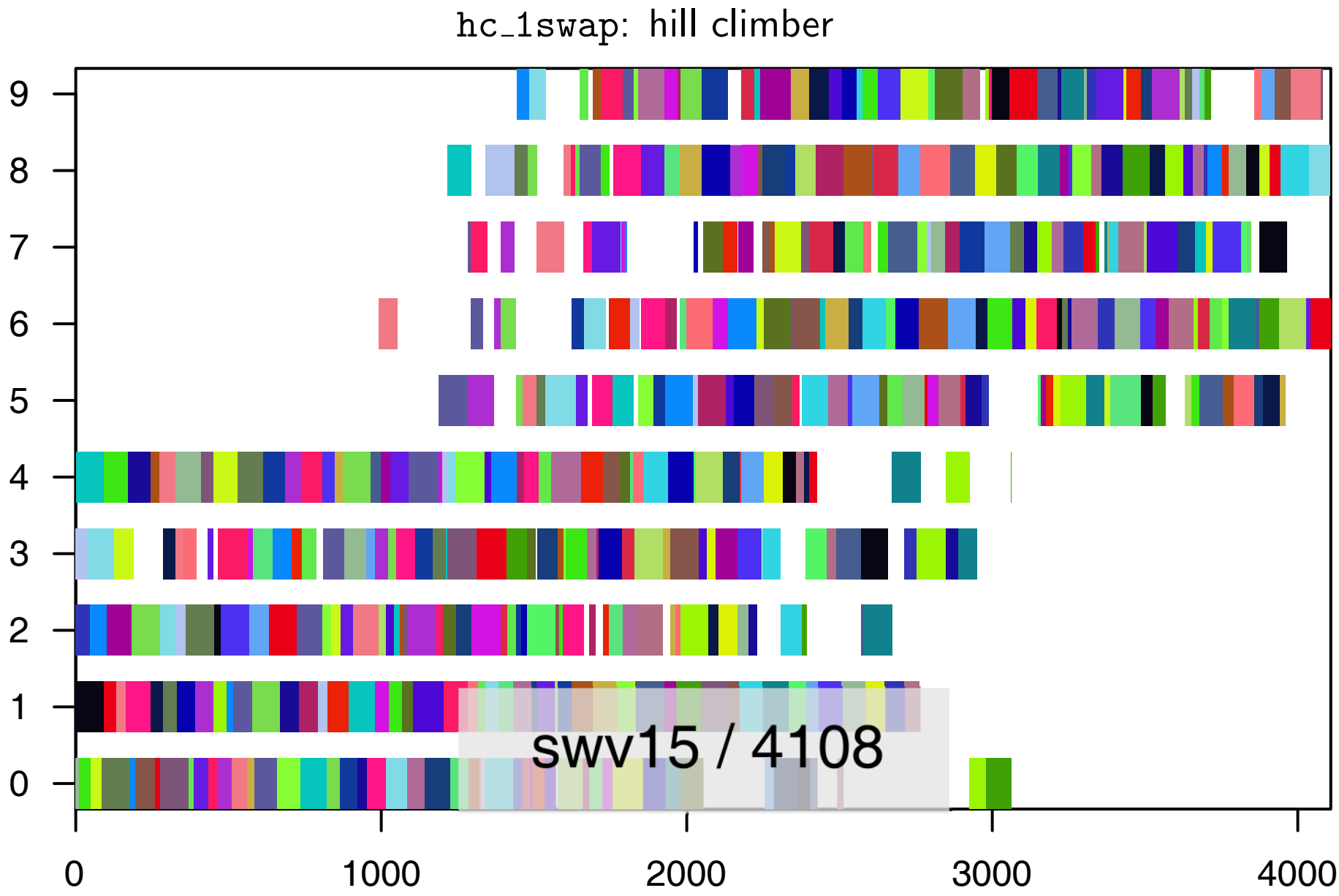

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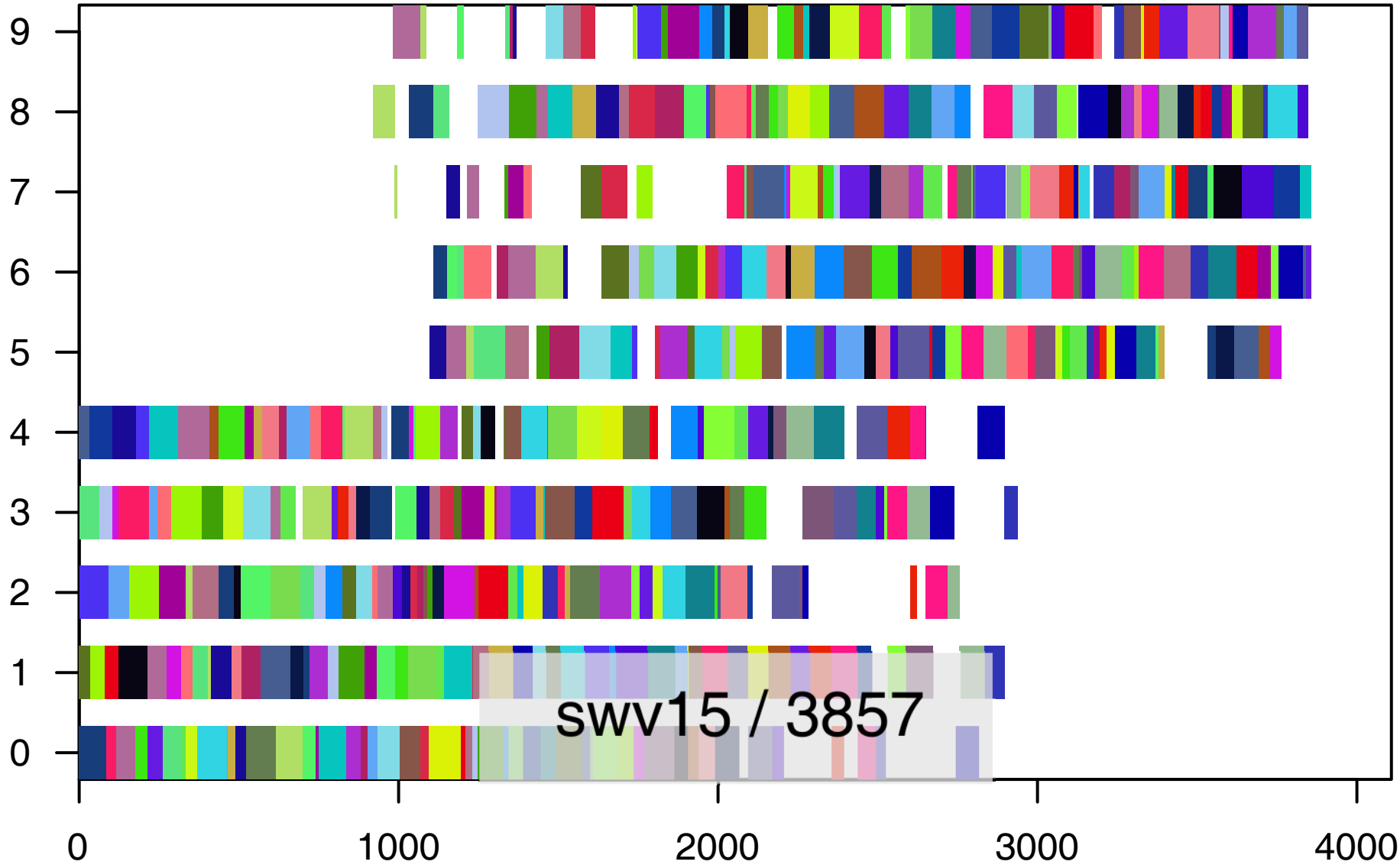

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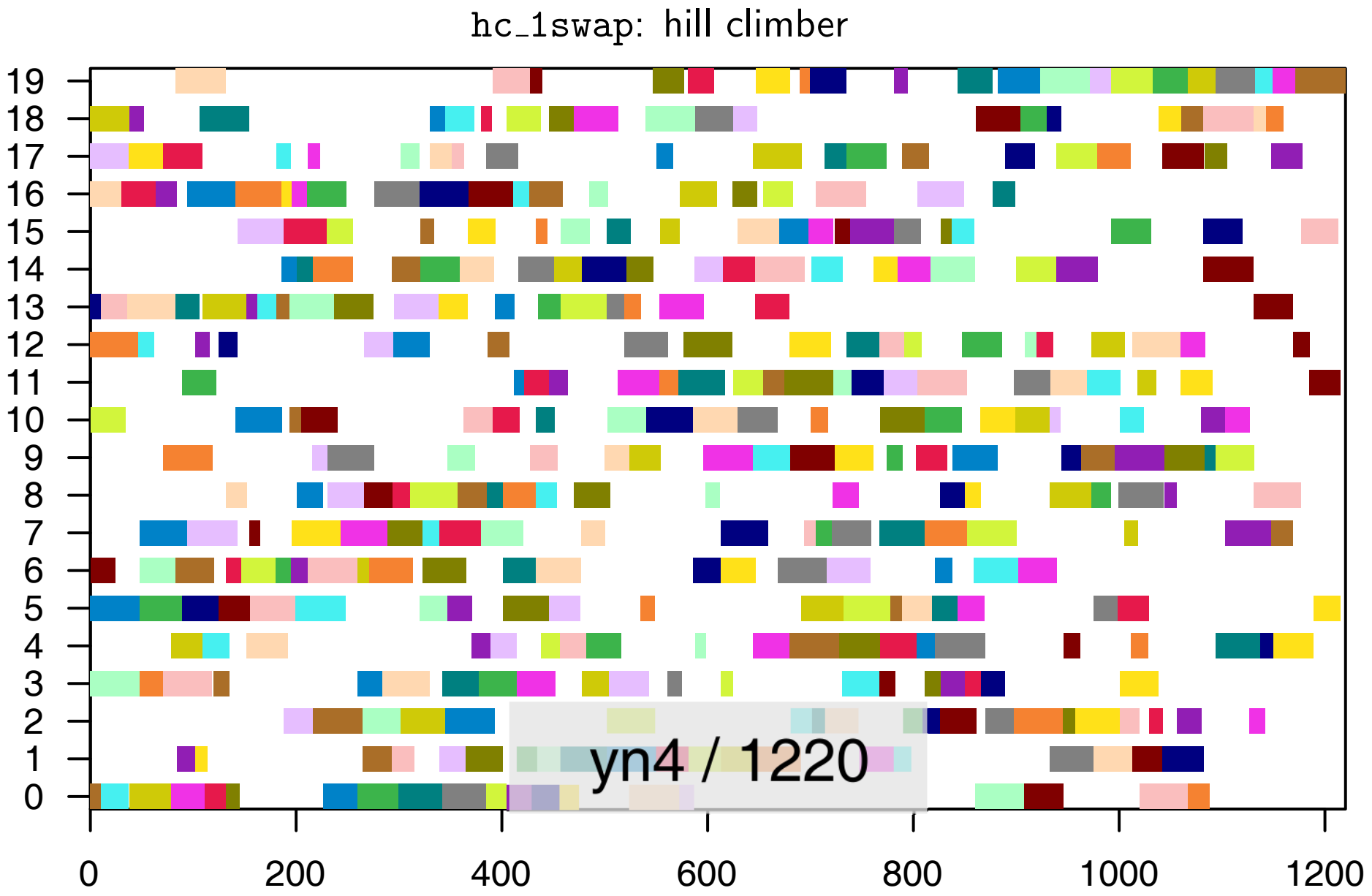

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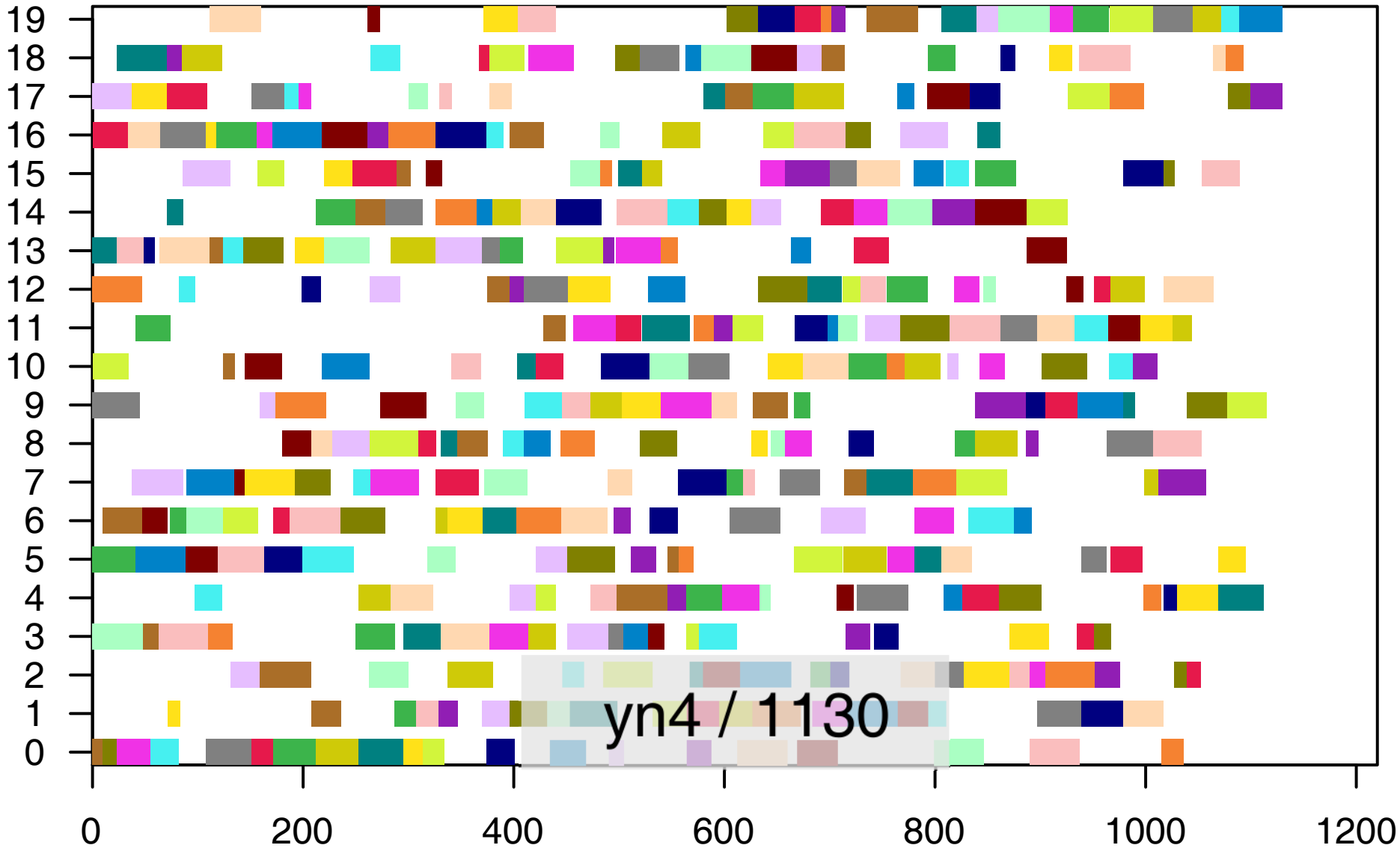

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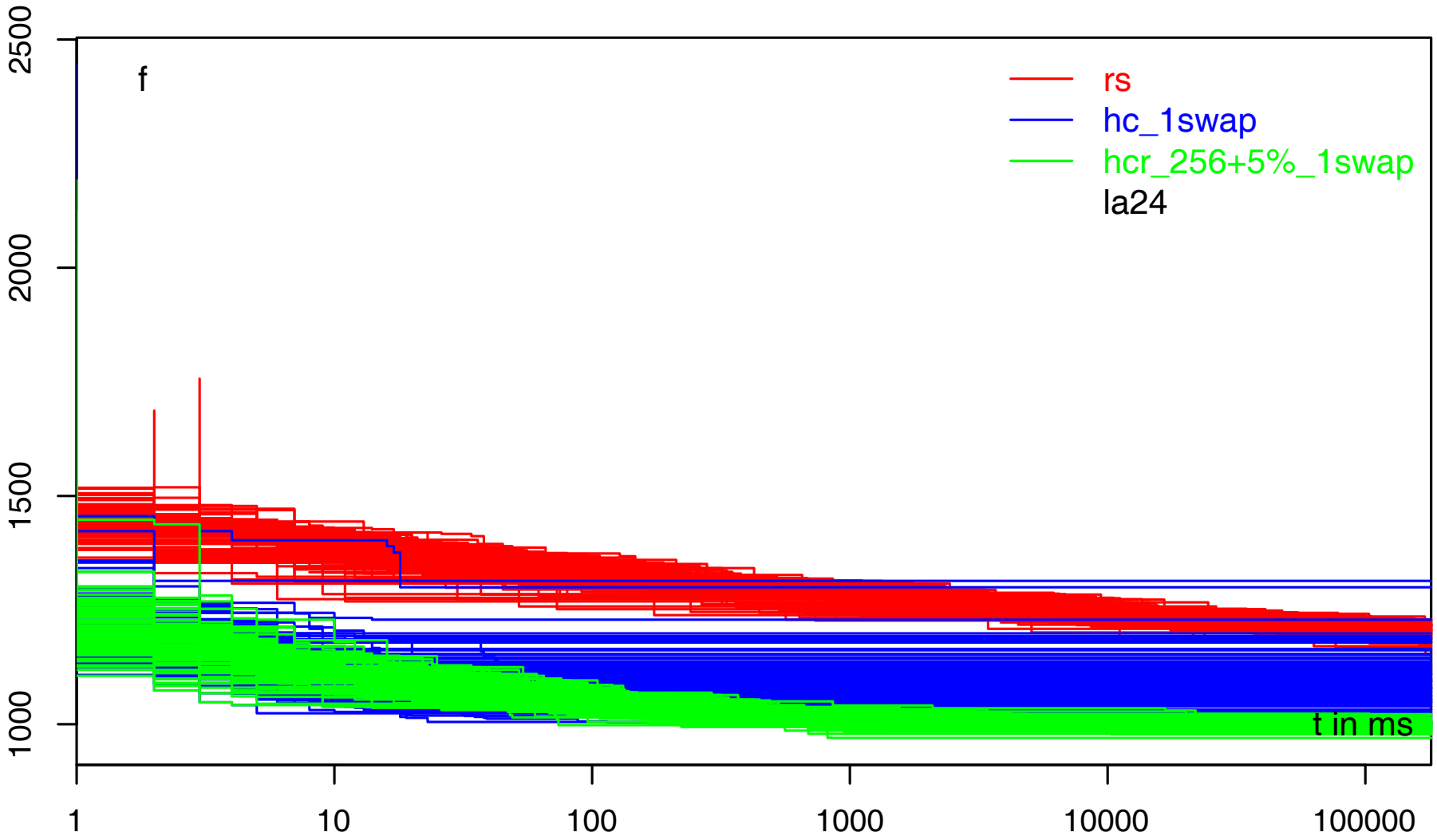

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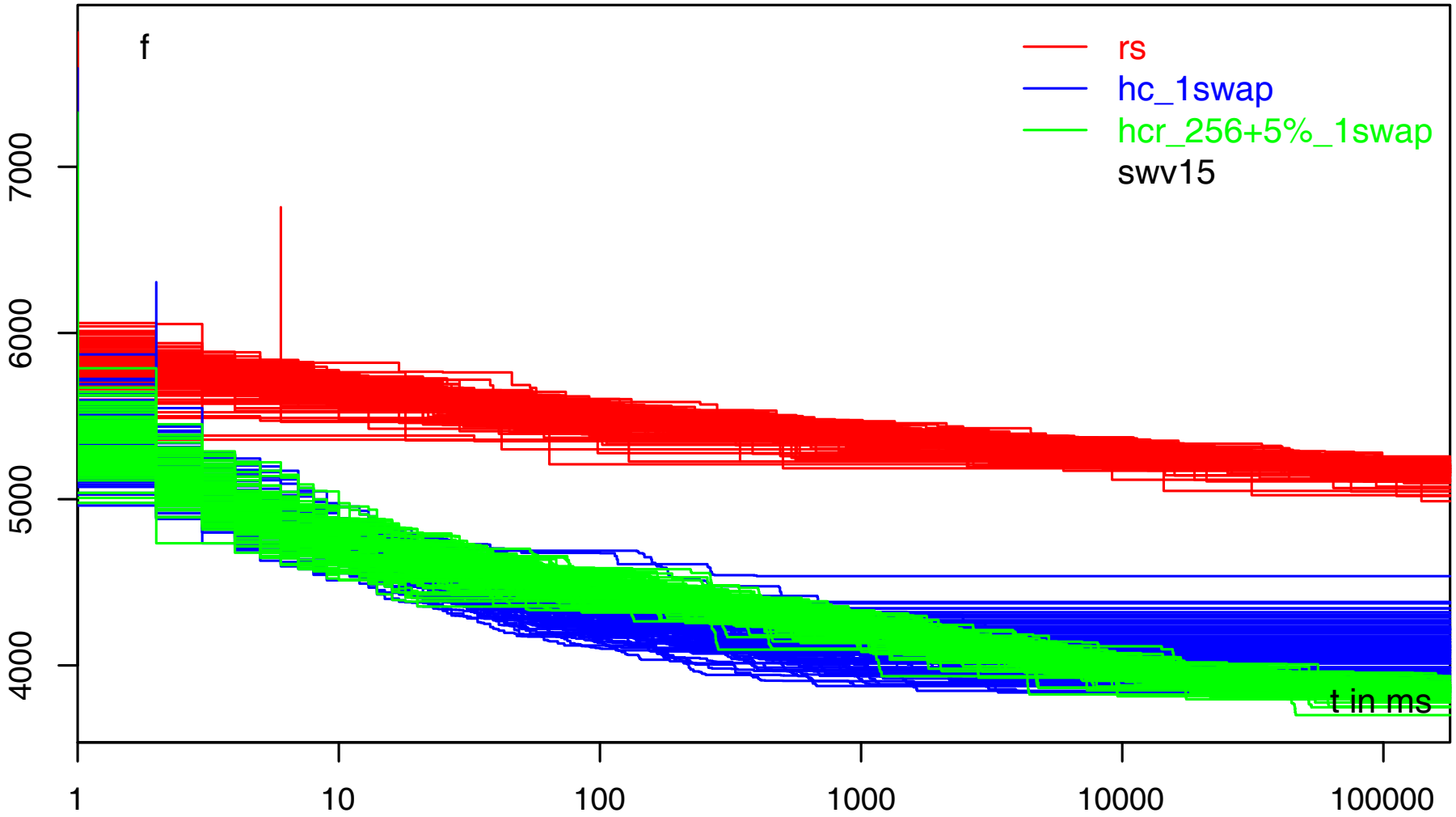

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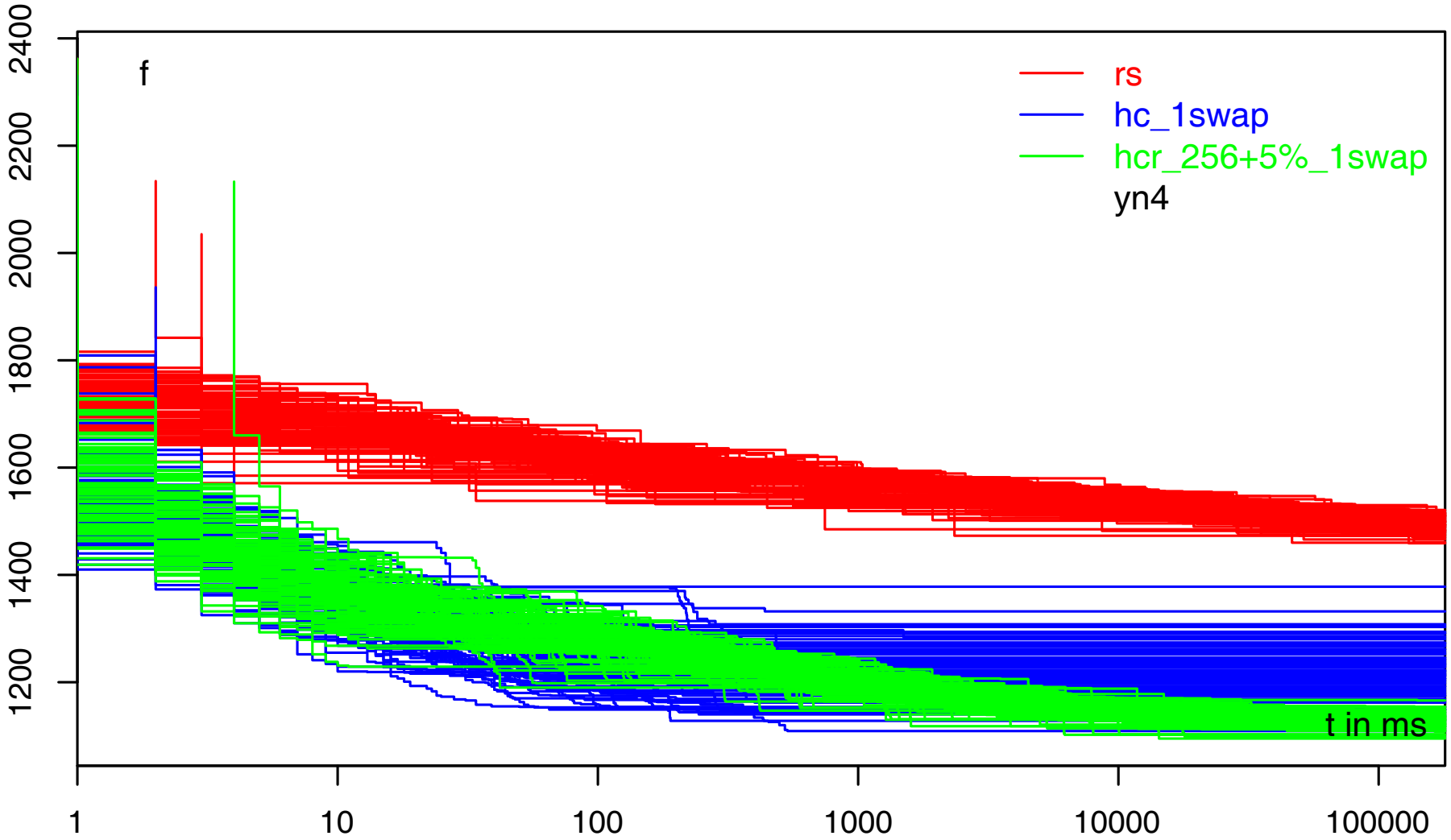

Section Outline

How else can we stop premature convergence?

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- It does not matter which two job ids I exchange in the current best string $x^\times$, the result is not better than $x^\times$.
- Notice: Which a local optimum is, is determined by the unary search operator!
- If we had a different operator with a bigger neighborhood, then maybe $x^\times$ would no longer be a local optimum and we could still improve the results after reaching it...

Making the neighborhood bigger

- Two solutions x_1 and x_2 are "neighbors" if I can reach x_2 by applying the search operator one time to x_1.

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- We could swap more than 2 job ids!
- But we should respect the causality: small changes to the solution cause small changes in the objective value – big changes will lead to unpredictable results.
- If we just change everything always, we basically have random sampling again...

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- Idea: Let's most often swap 2 jobs

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- We most often make small moves, but sometimes bigger ones.
- Thereotically, we could always escape from local any optima, but the probability may sometimes be very very small.

Create Modified Copy of Existing Solution 2

Listing: Swap a random number of sub-jobs.

```
public class JSSPUnaryOperatorNSwap
    implements IUnarySearchOperator<int[]> {

  public void apply(int[] x, int[] dest, Random random) {
    System.arraycopy(x, 0, dest, 0, x.length);
    int i = random.nextInt(dest.length);
    int first = dest[i];
    int last = first;

    boolean hasNext;
    do {
      hasNext = random.nextBoolean();
      inner: for (;;) {
        final int j = random.nextInt(dest.length);
        final int job_j = dest[j];
        if ((last != job_j) &&
            (hasNext || (first != job_j))) {
          dest[i] = job_j;
          i = j;
          last = job_j;
          break inner;
        }
      }
    } while (hasNext);
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	`hc_nswap`	**3599**	**3867**	**3859**	**113**	70s	11'559'667
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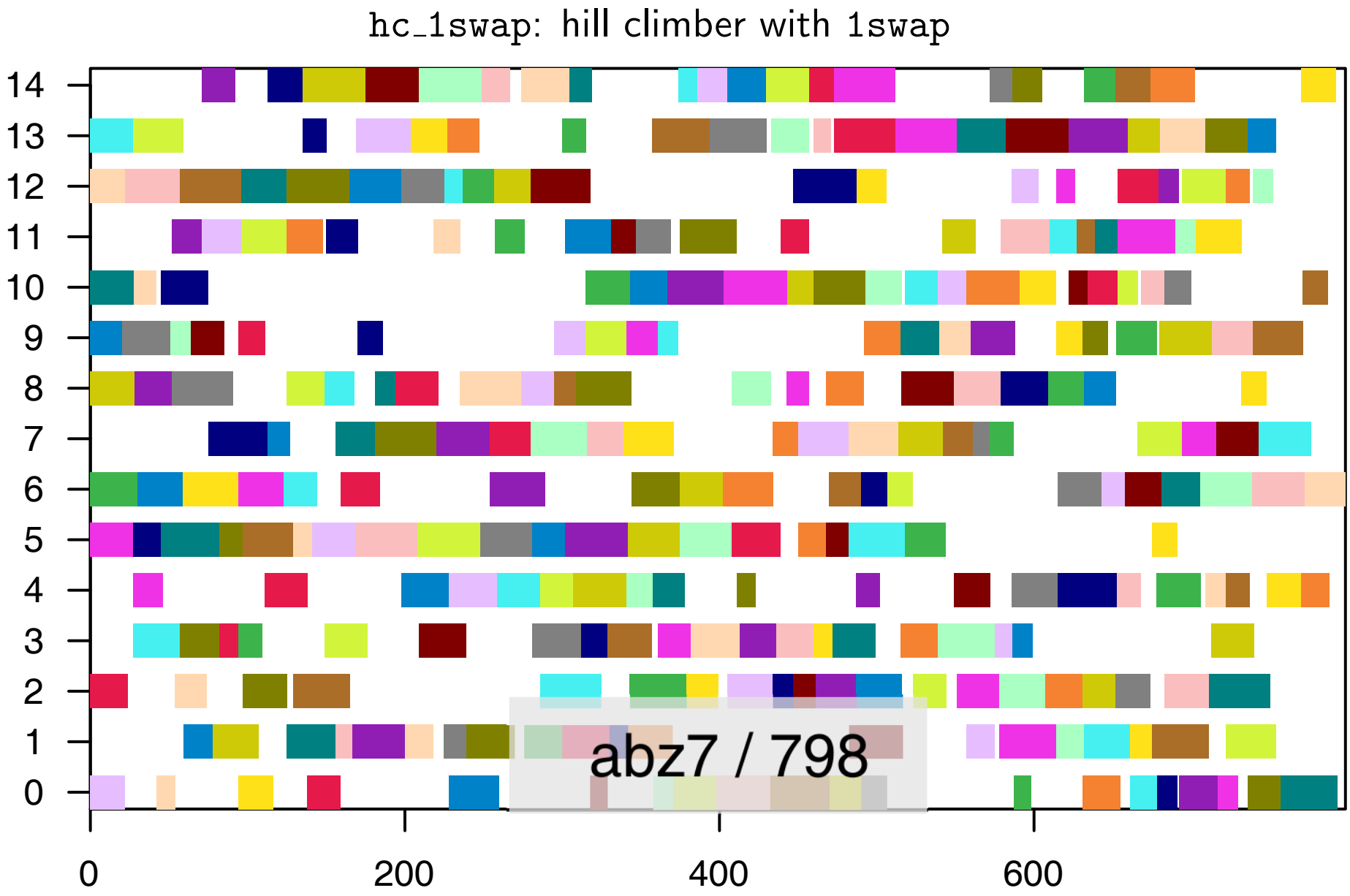

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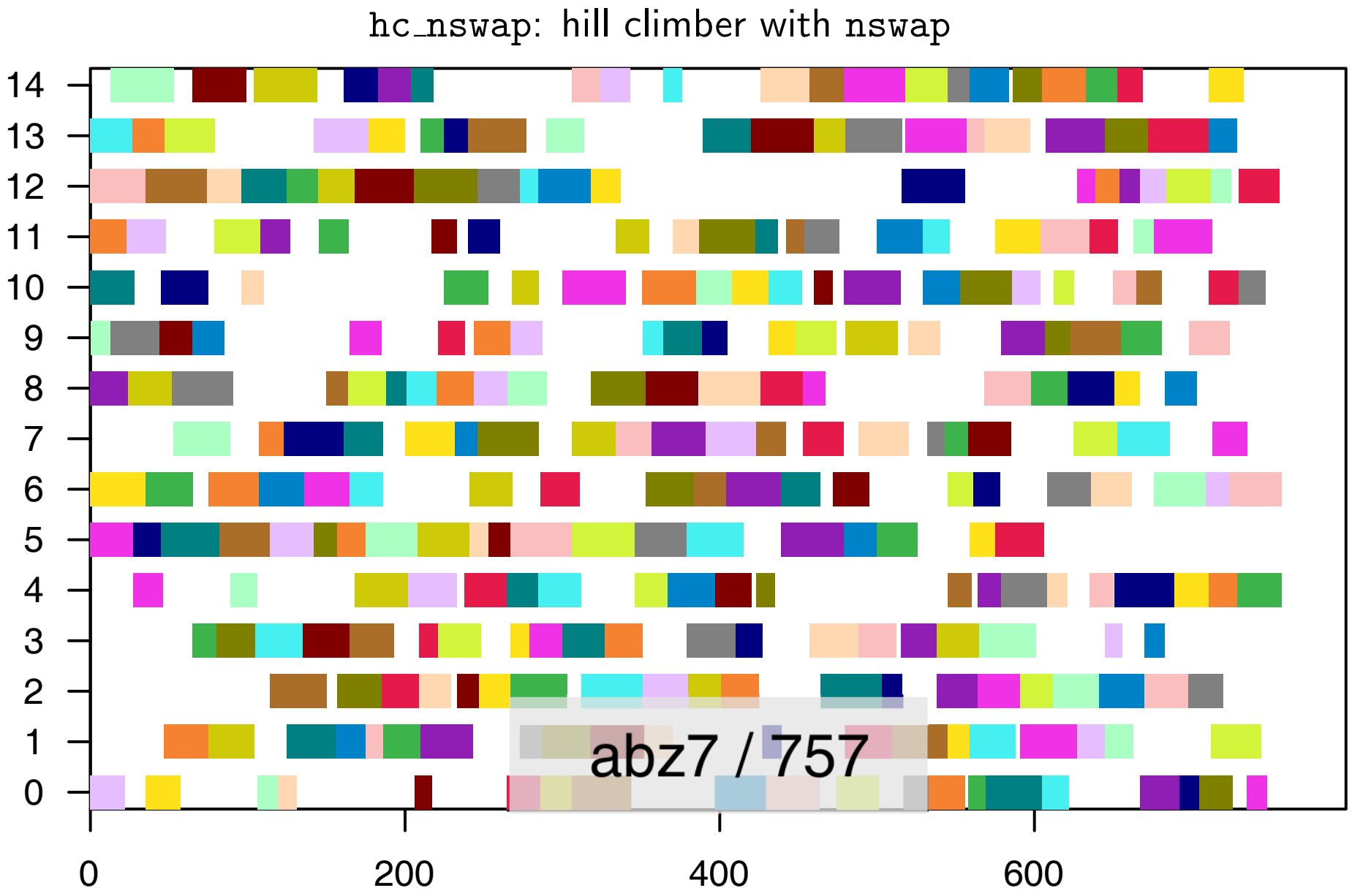

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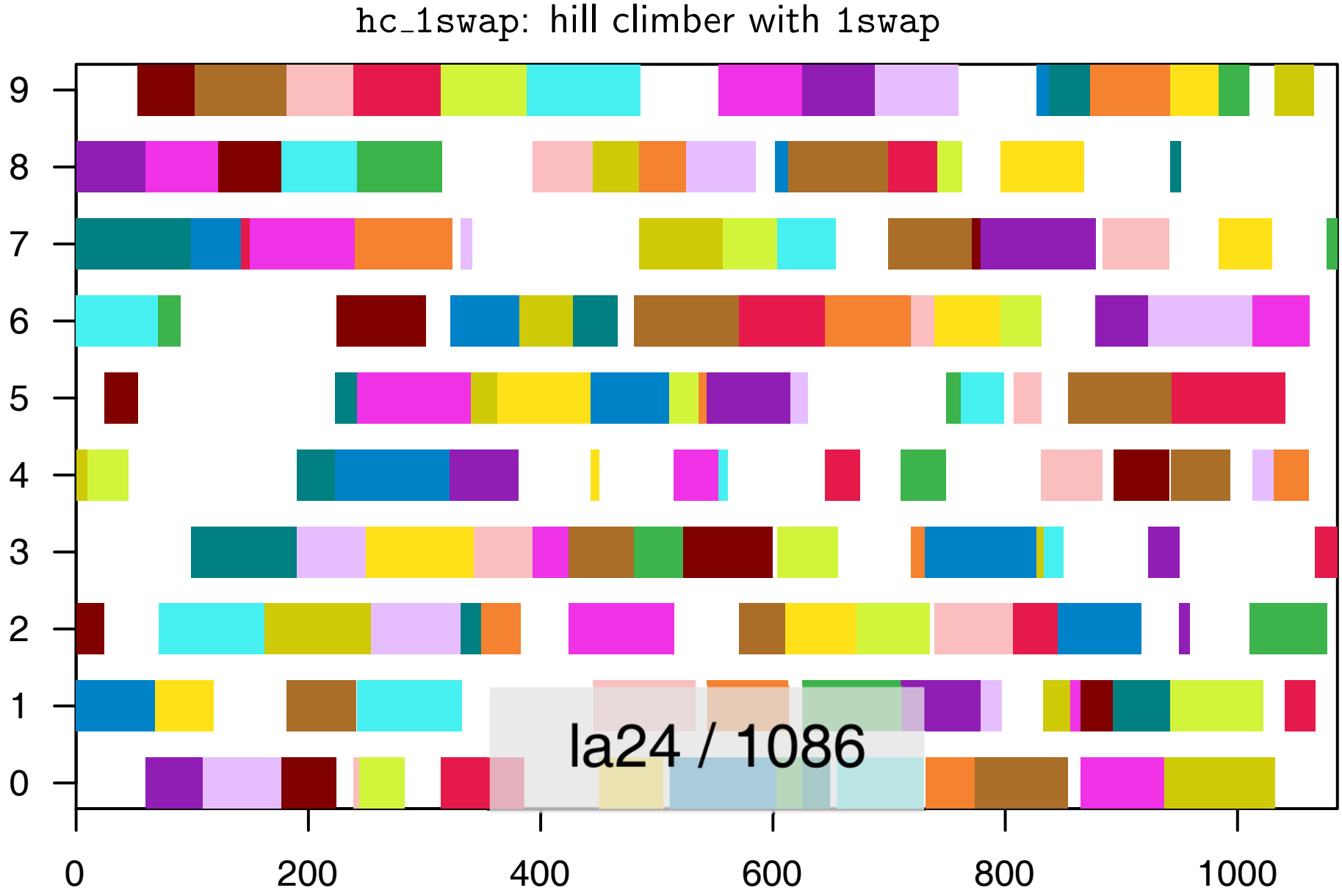

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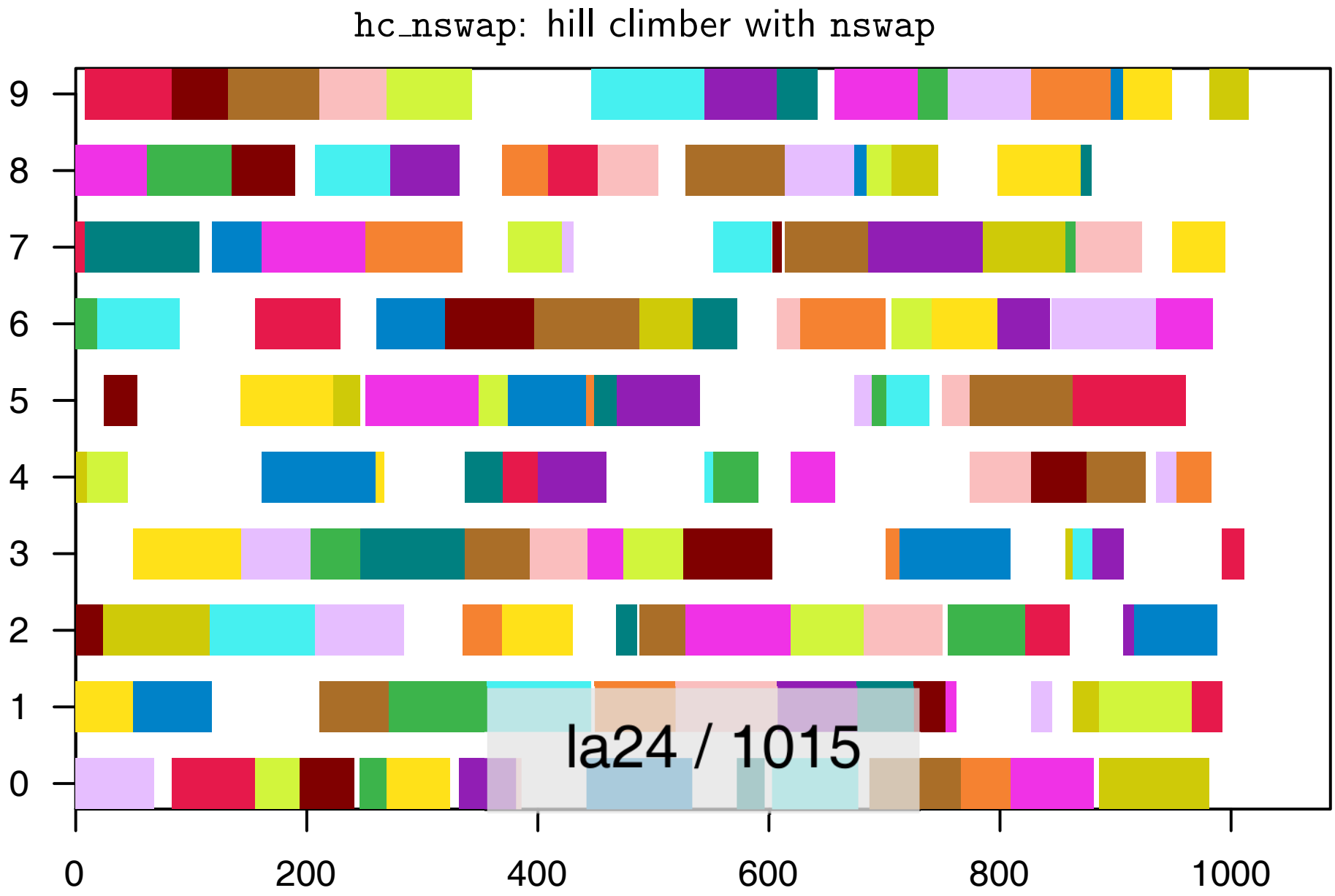

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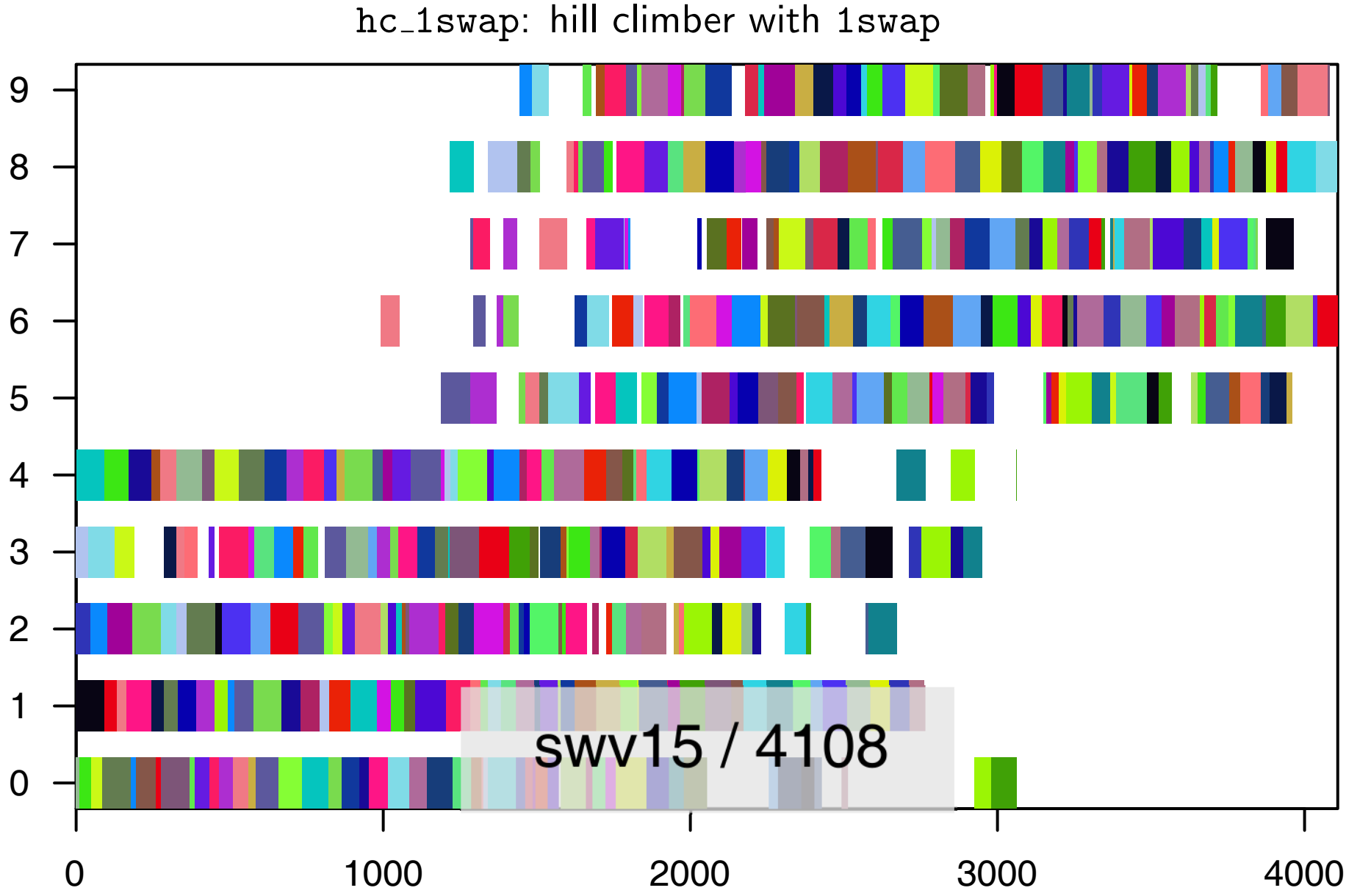

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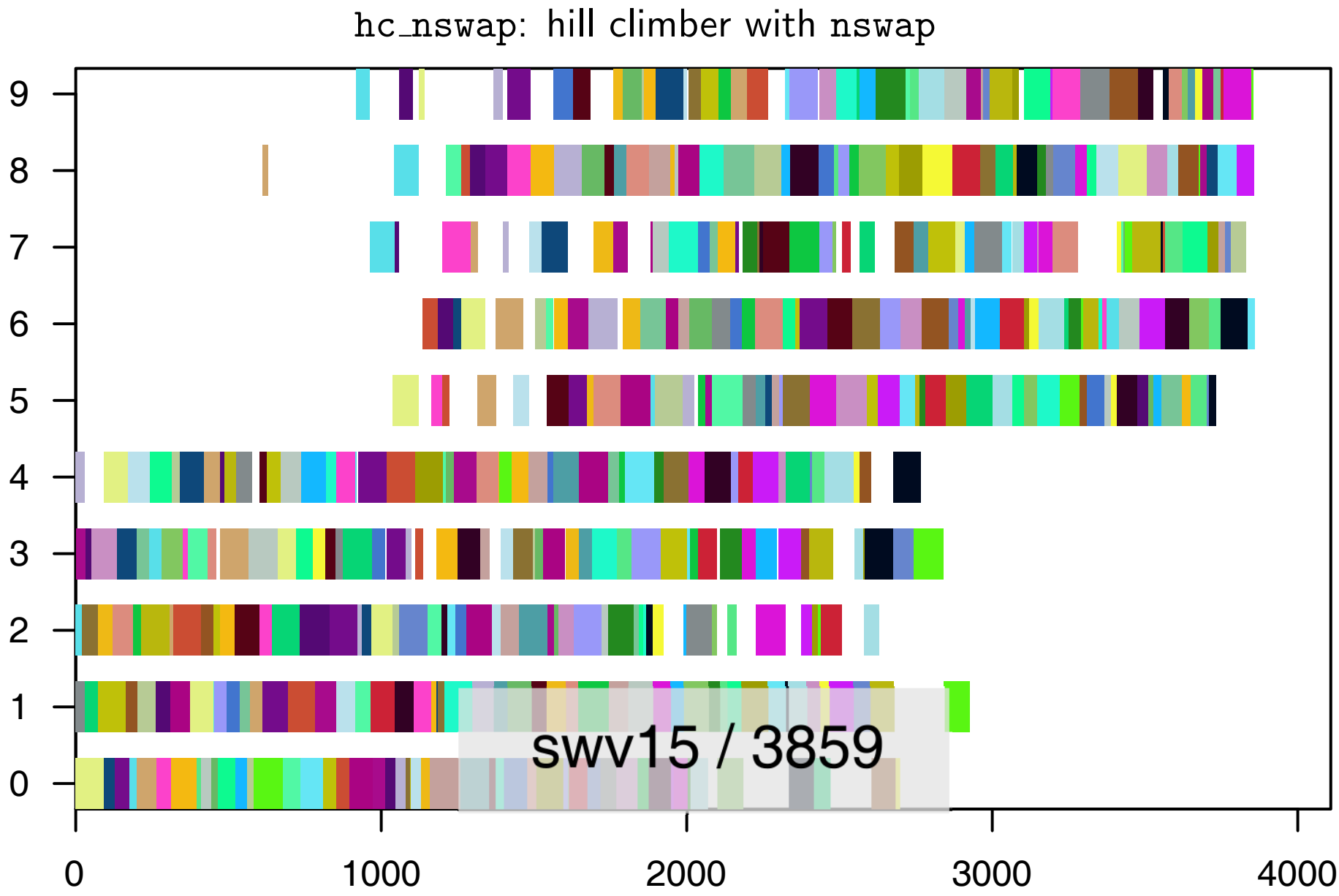

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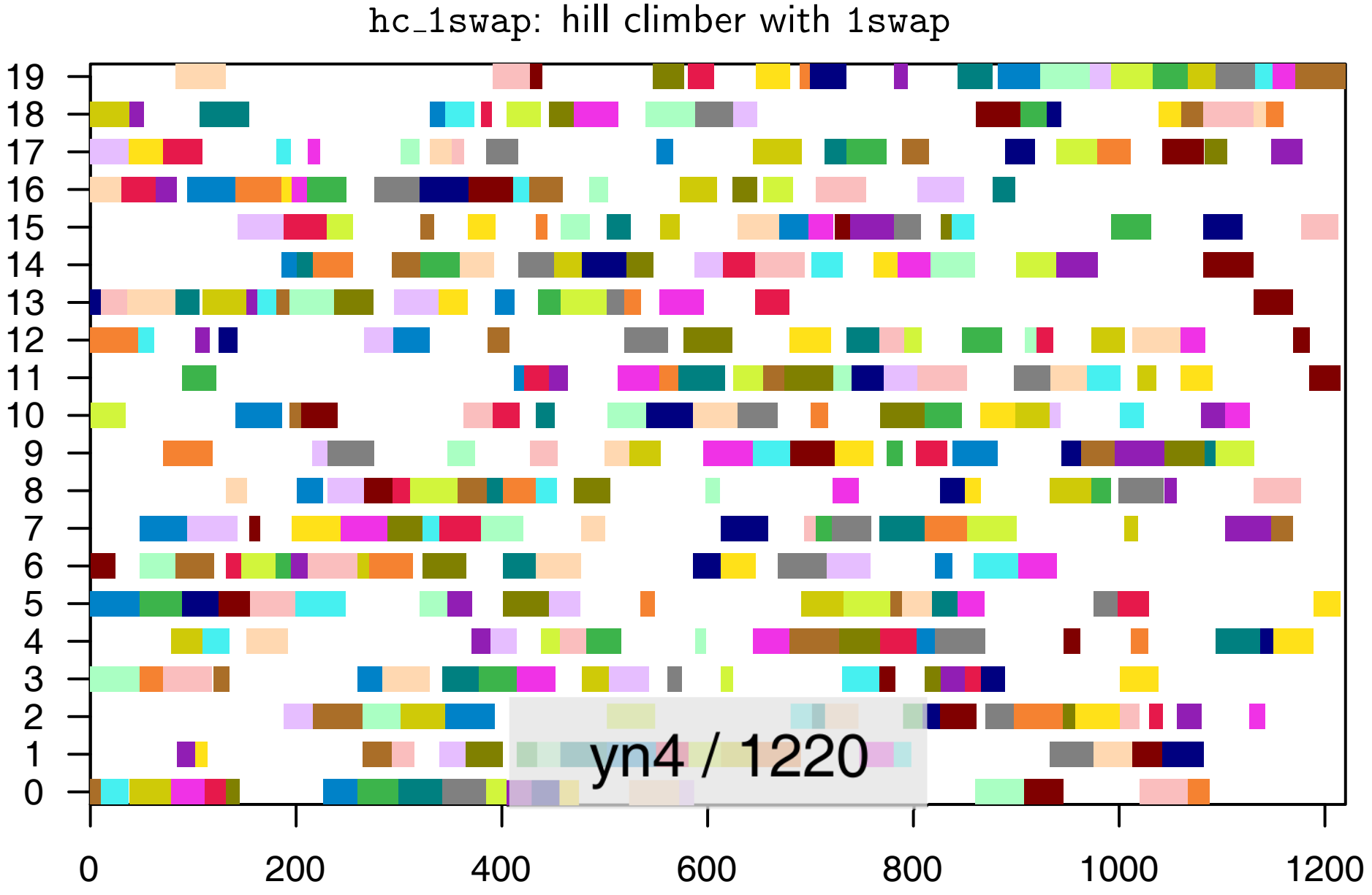

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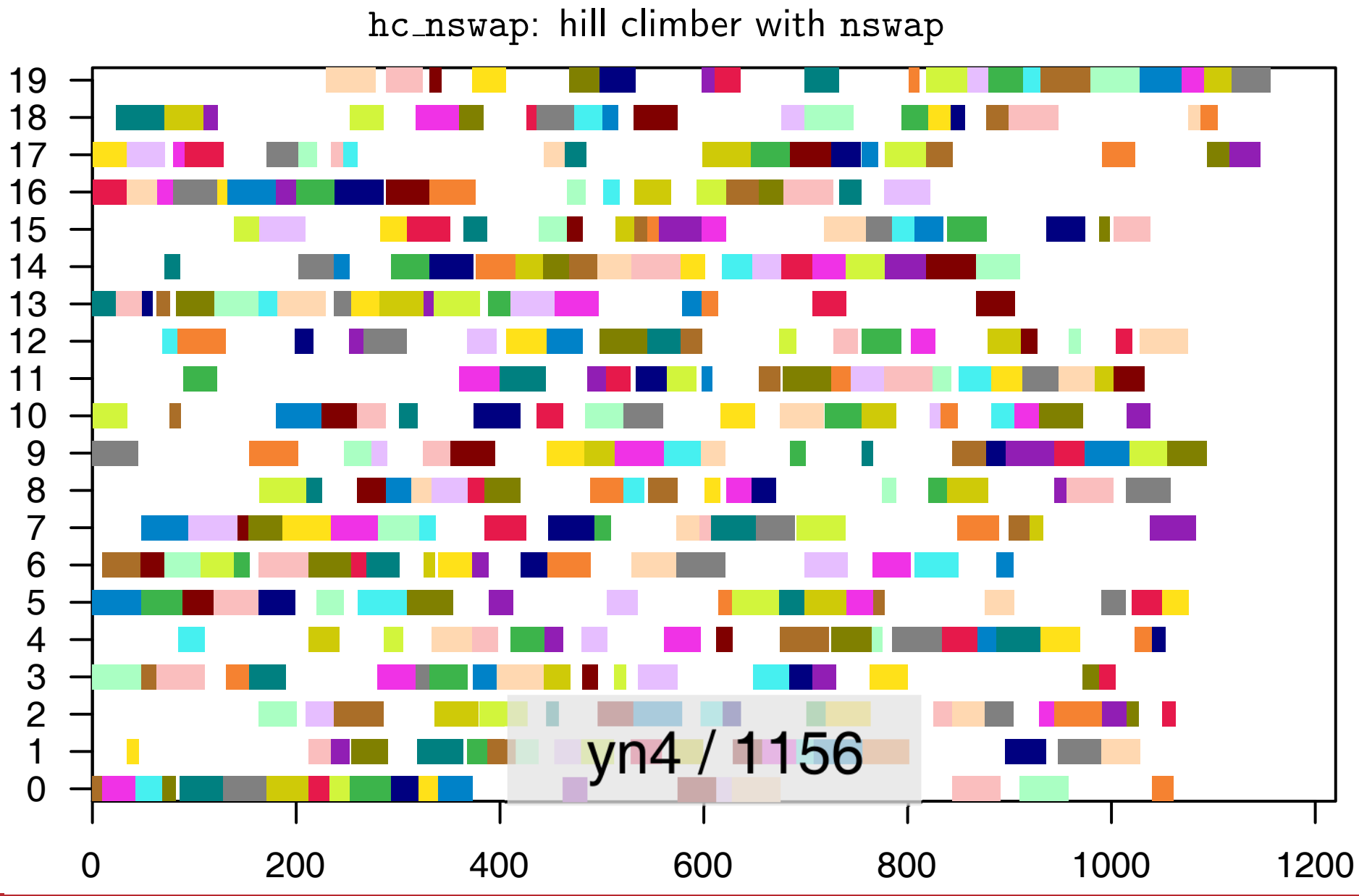

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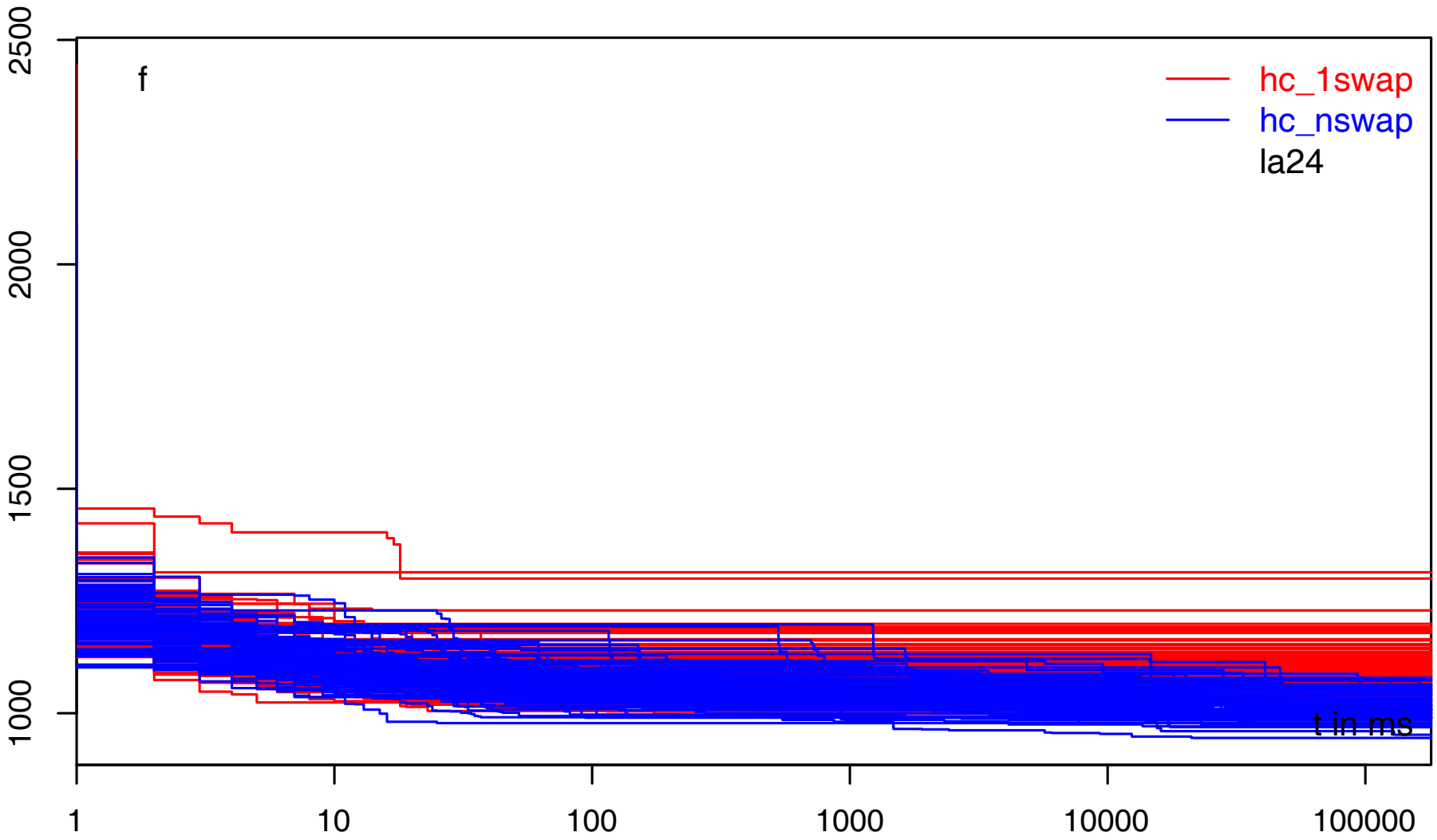

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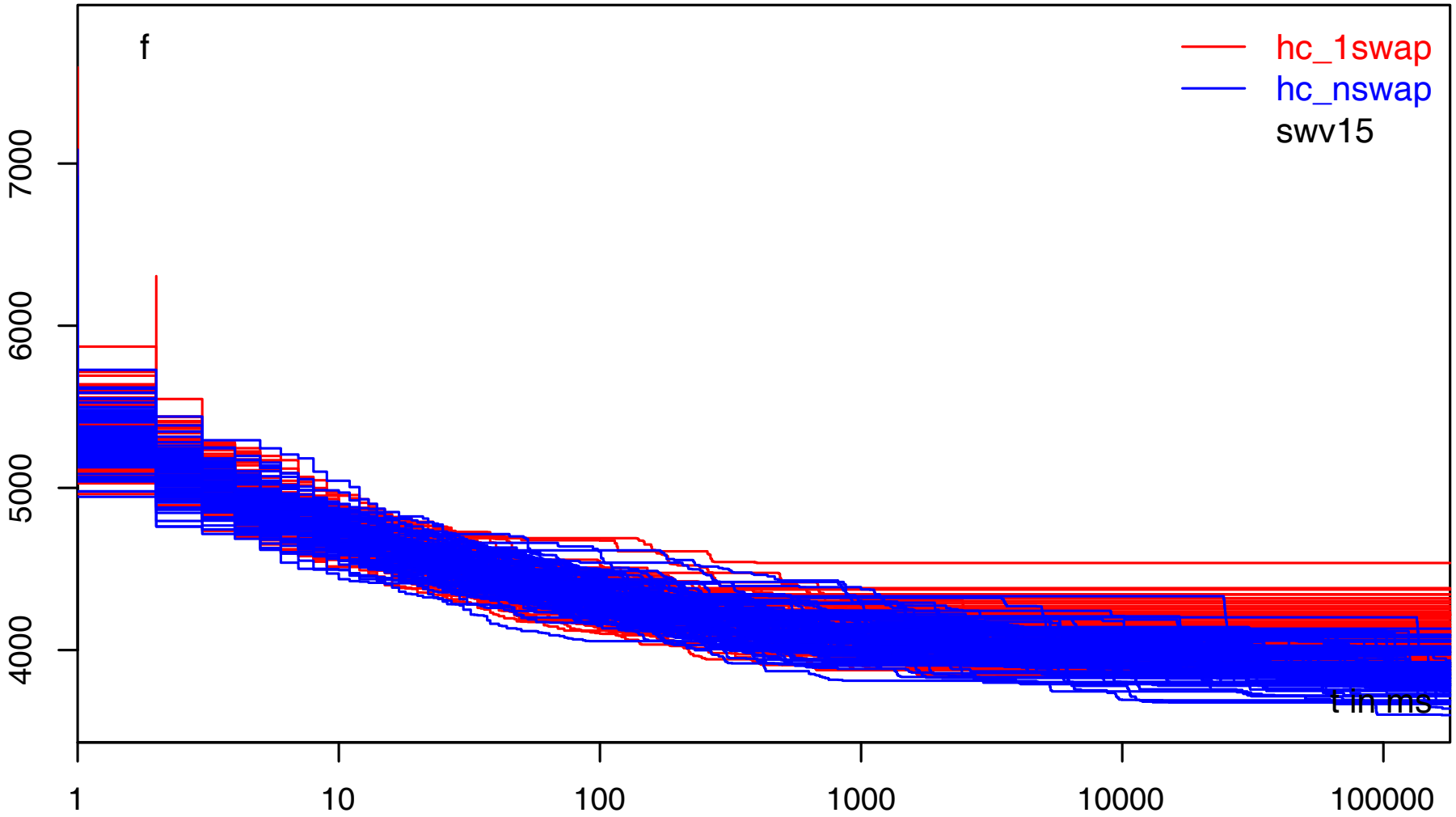

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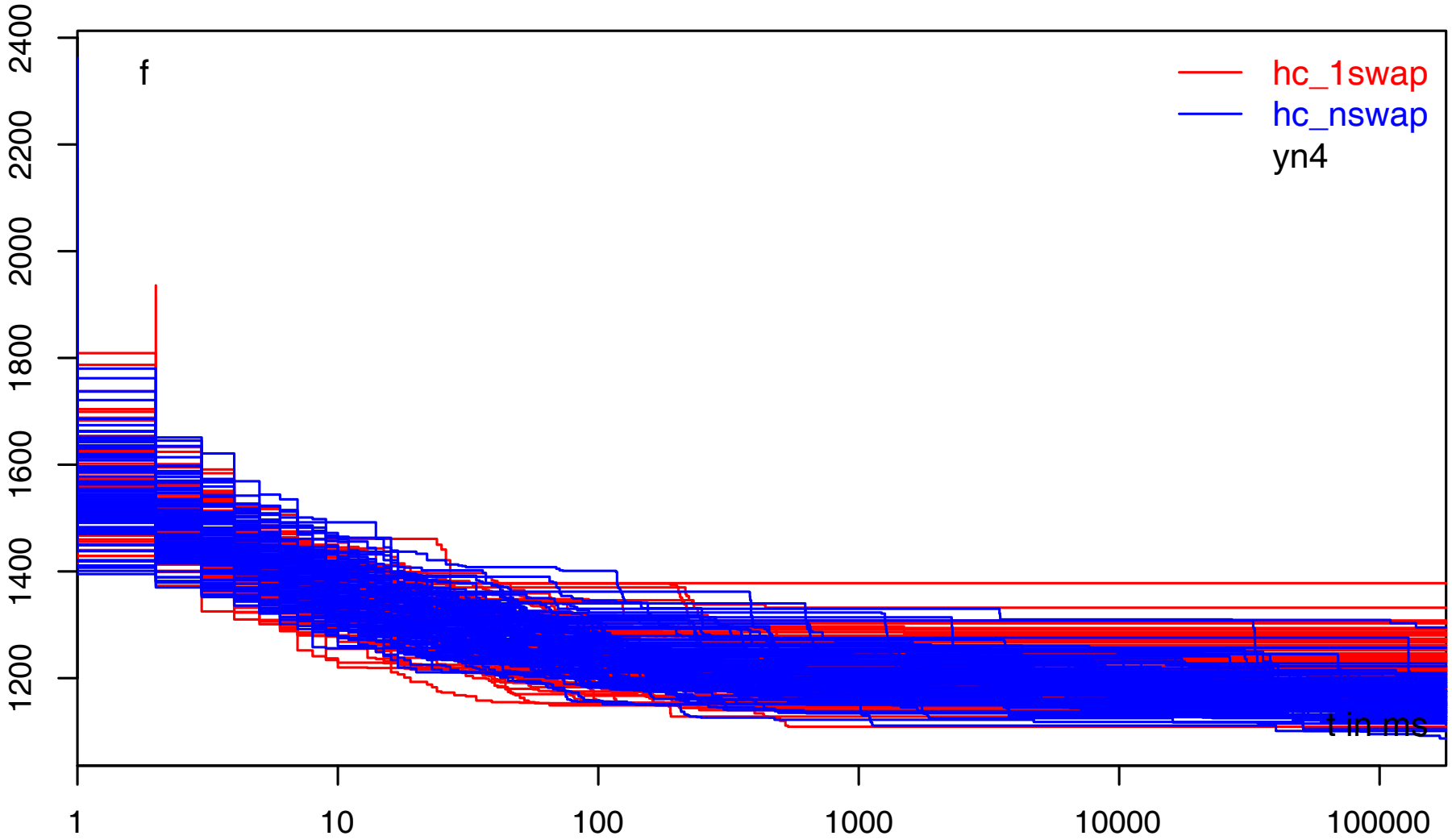

Section Outline

Combining the two ideas

- We had two entirely different ideas how to improve the hill climber.

Combining the two ideas

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- Let's see how they work together!

So what do we get?

- I execute the program 101 times for each of the datasets `abz7`, `la24`, `swv15`, and `yn4`

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		makespan				last improvement	
$\mathcal{I}$	algo	best	mean	med	sd	med(t)	med(FEs)
abz7	hc_1swap	717	800	798	28	0s	16978
	hcr_256+5%_1swap	723	742	743	**7**	21s	5681591
	hc_nswap	724	757	757	17	30s	8145596
	hcr_256+5%_nswap	**707**	**733**	**734**	7	64s	17293038
la24	hc_1swap	999	1095	1086	56	0s	6612
	hcr_256+5%_1swap	970	997	998	9	6s	3470368
	hc_nswap	**945**	1017	1015	29	21s	11123744
	hcr_256+5%_nswap	**945**	**981**	**984**	**9**	57s	29246097
swv15	hc_1swap	3837	4108	4108	137	1s	104598
	hcr_256+5%_1swap	3701	3850	3857	40	60s	9874102
	hc_nswap	**3599**	3867	3859	113	70s	11559667
	hcr_256+5%_nswap	3645	**3804**	**3811**	44	91s	14907737
yn4	hc_1swap	1109	1222	1220	48	0s	31789
	hcr_256+5%_1swap	1095	1129	1130	14	22s	4676669
	hc_nswap	1087	1160	1156	33	63s	13111115
	hcr_256+5%_nswap	**1081**	**1117**	**1119**	**14**	55s	11299461

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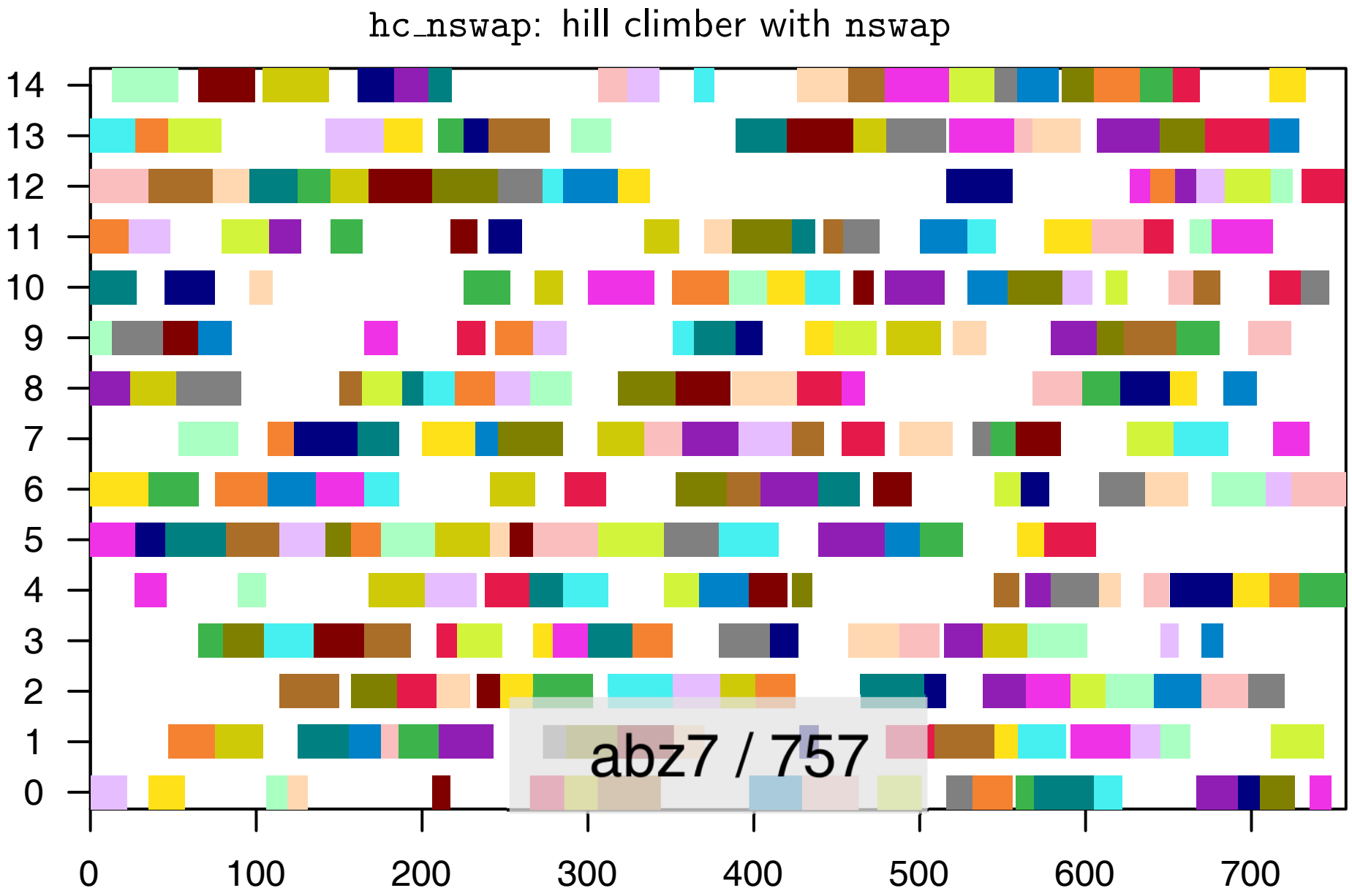

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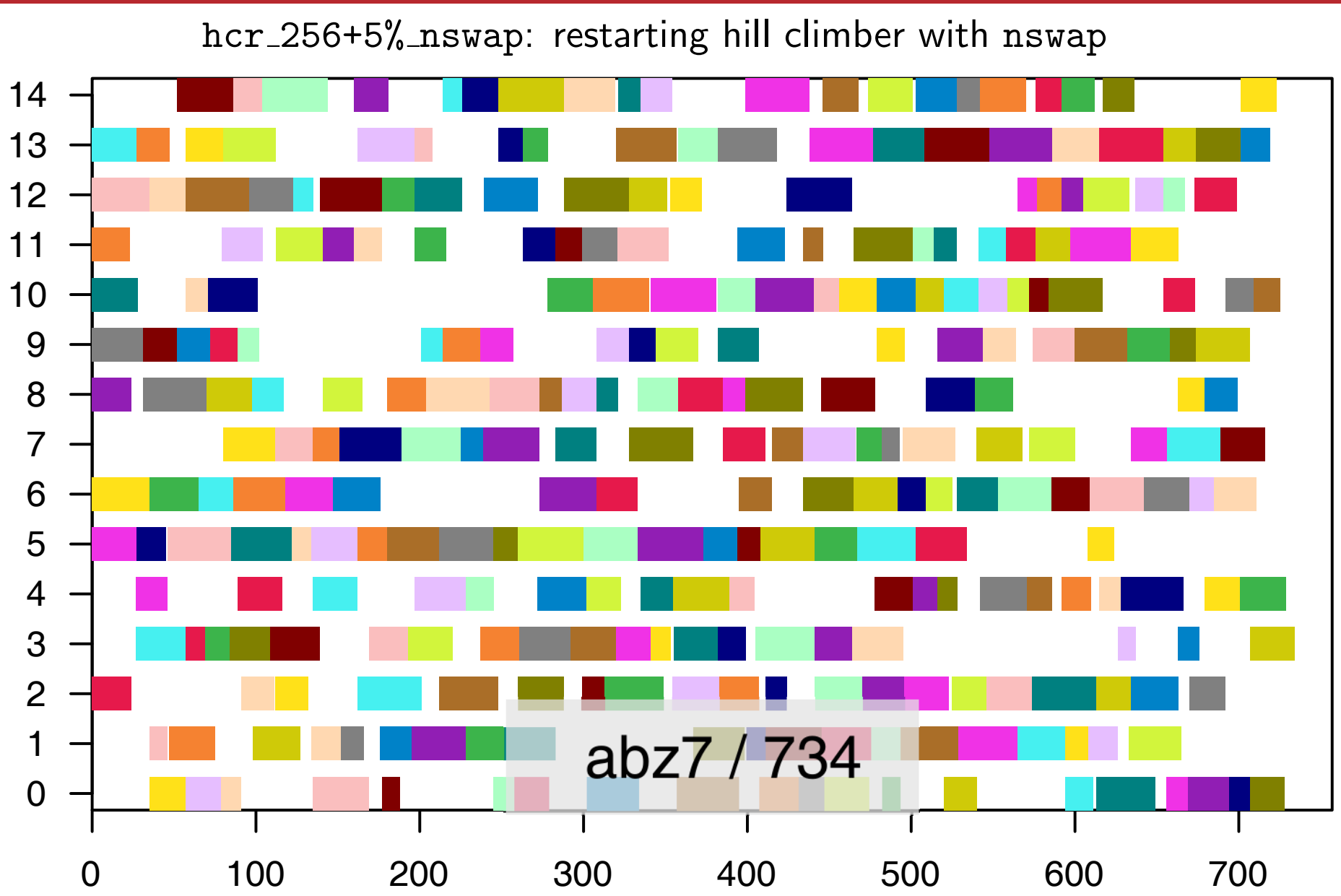

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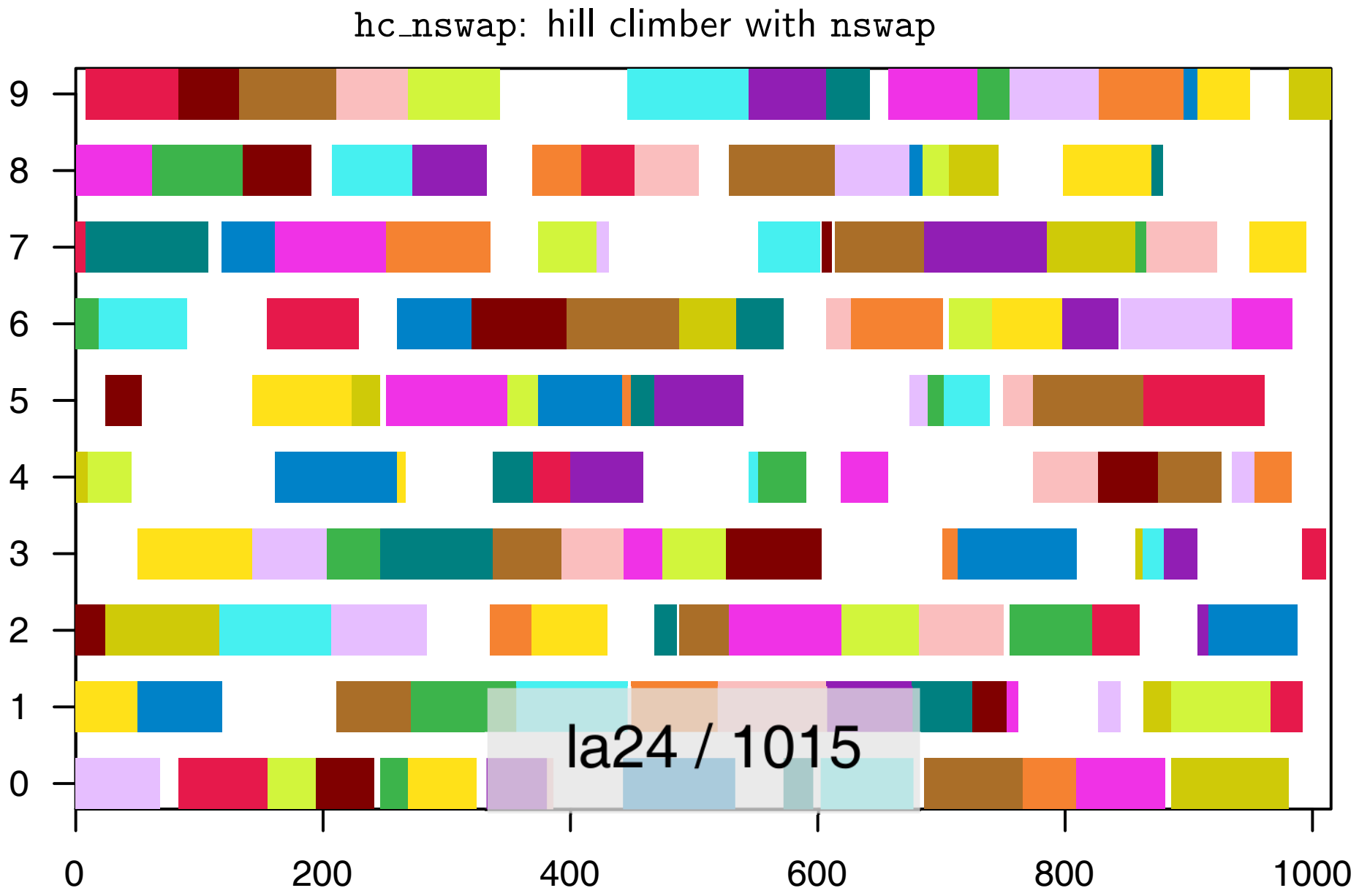

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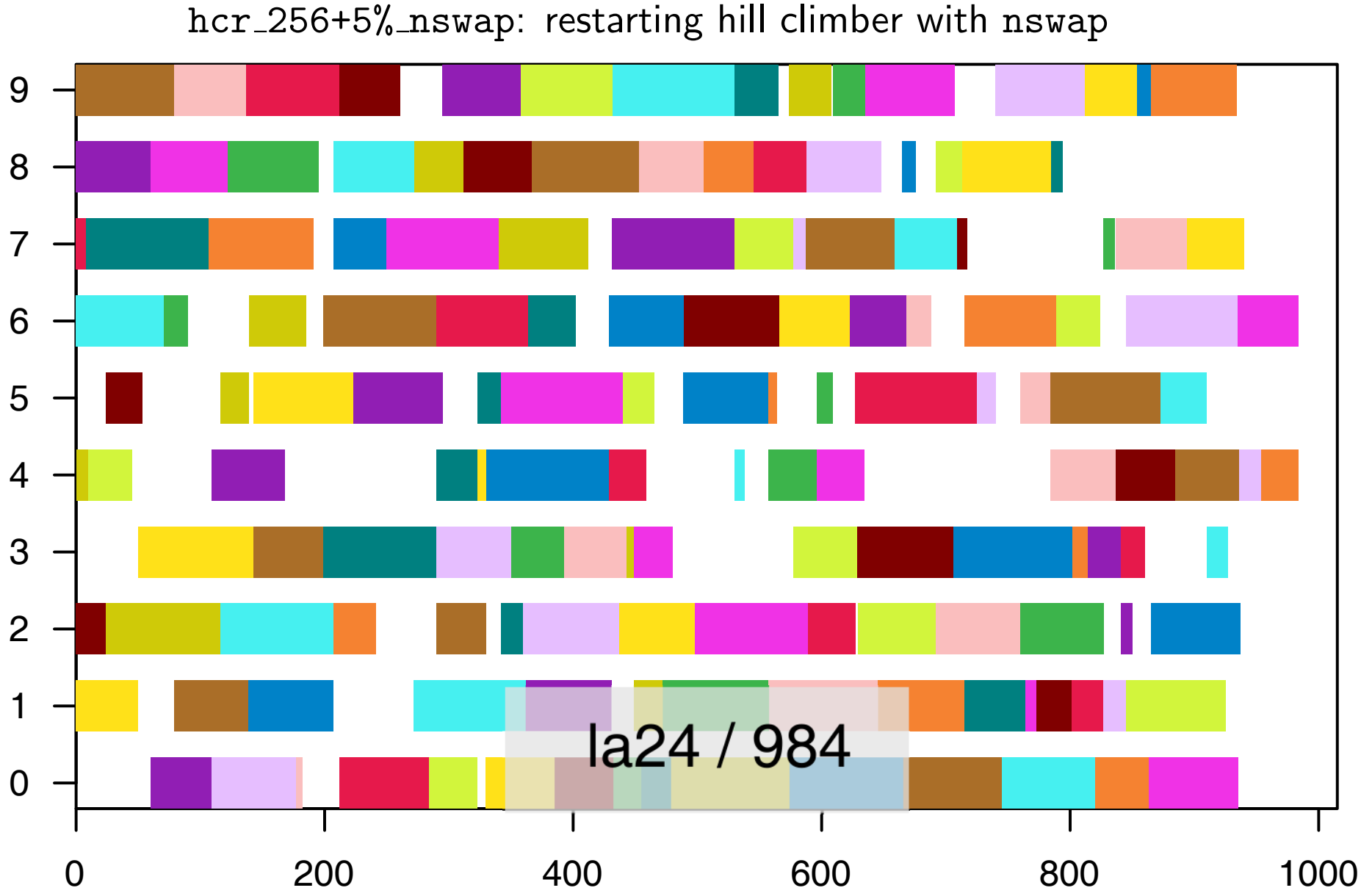

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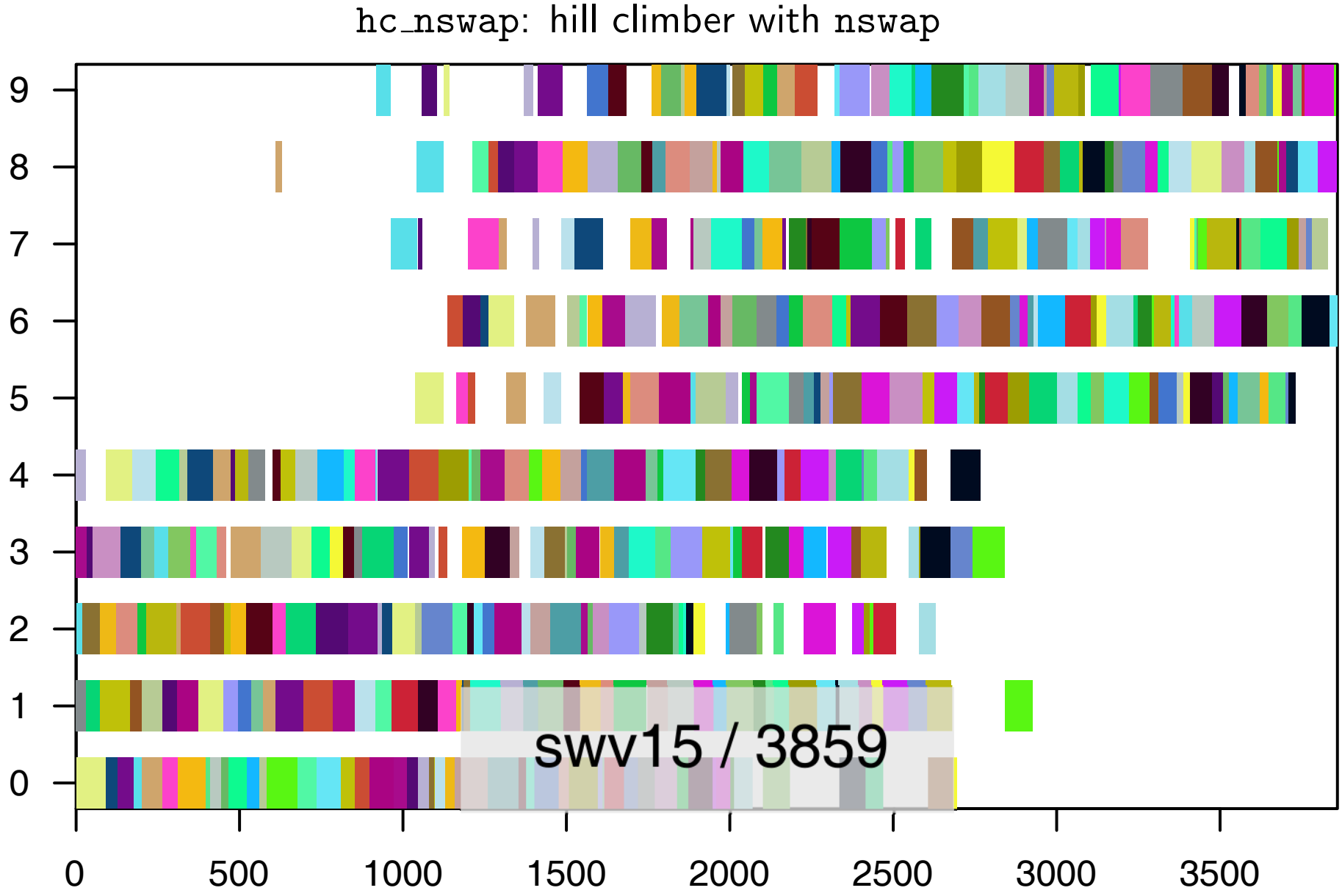

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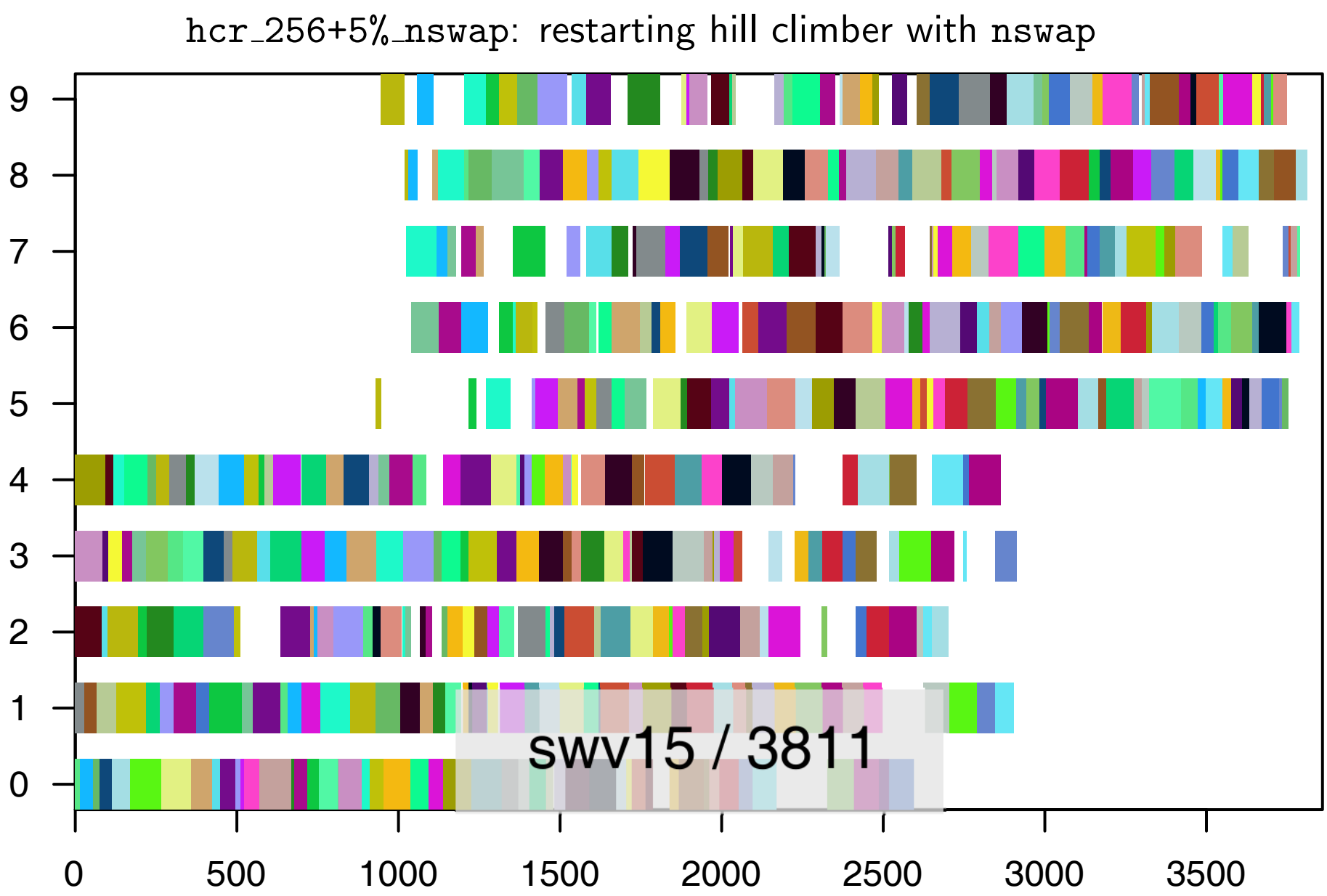

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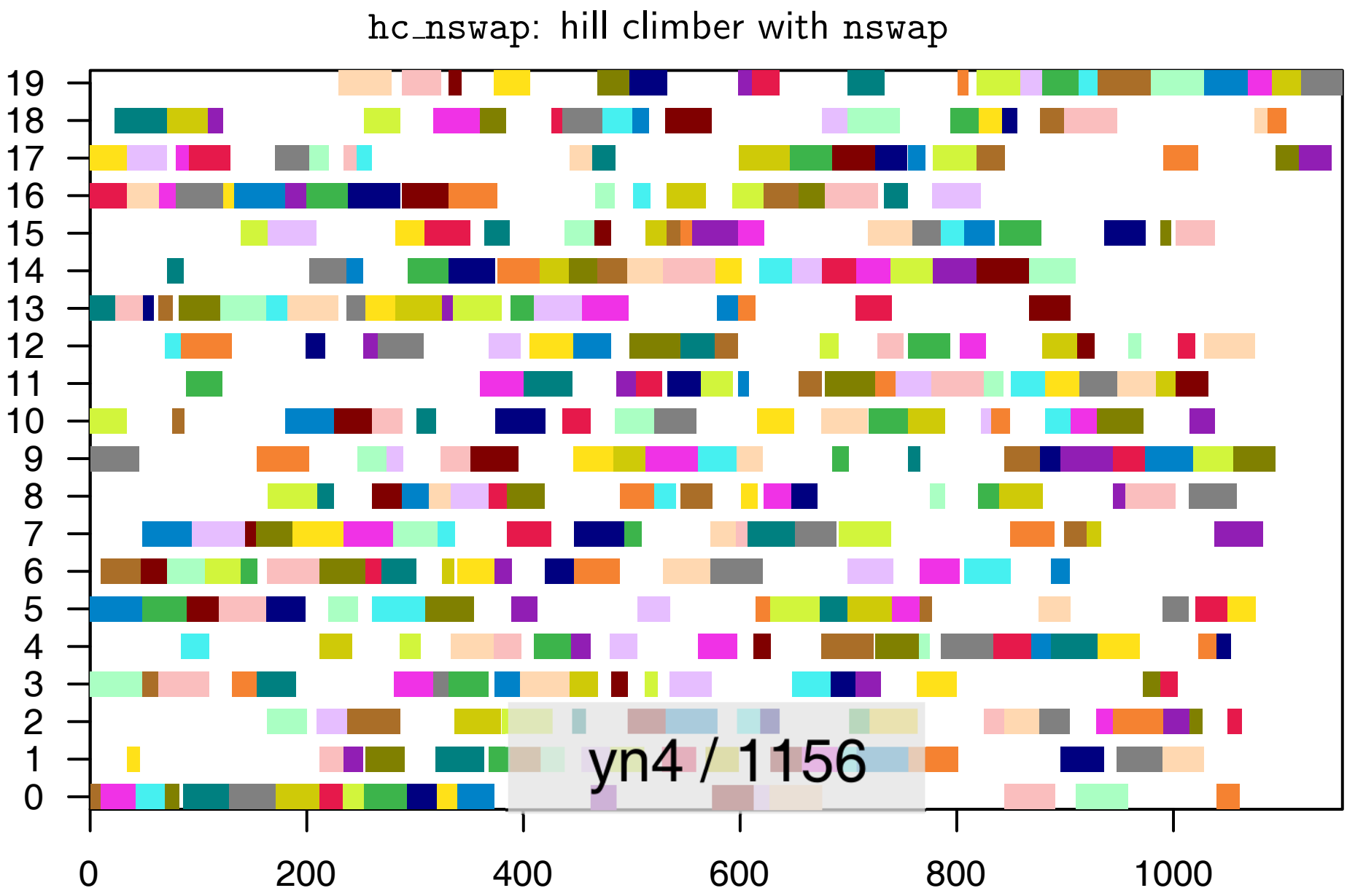

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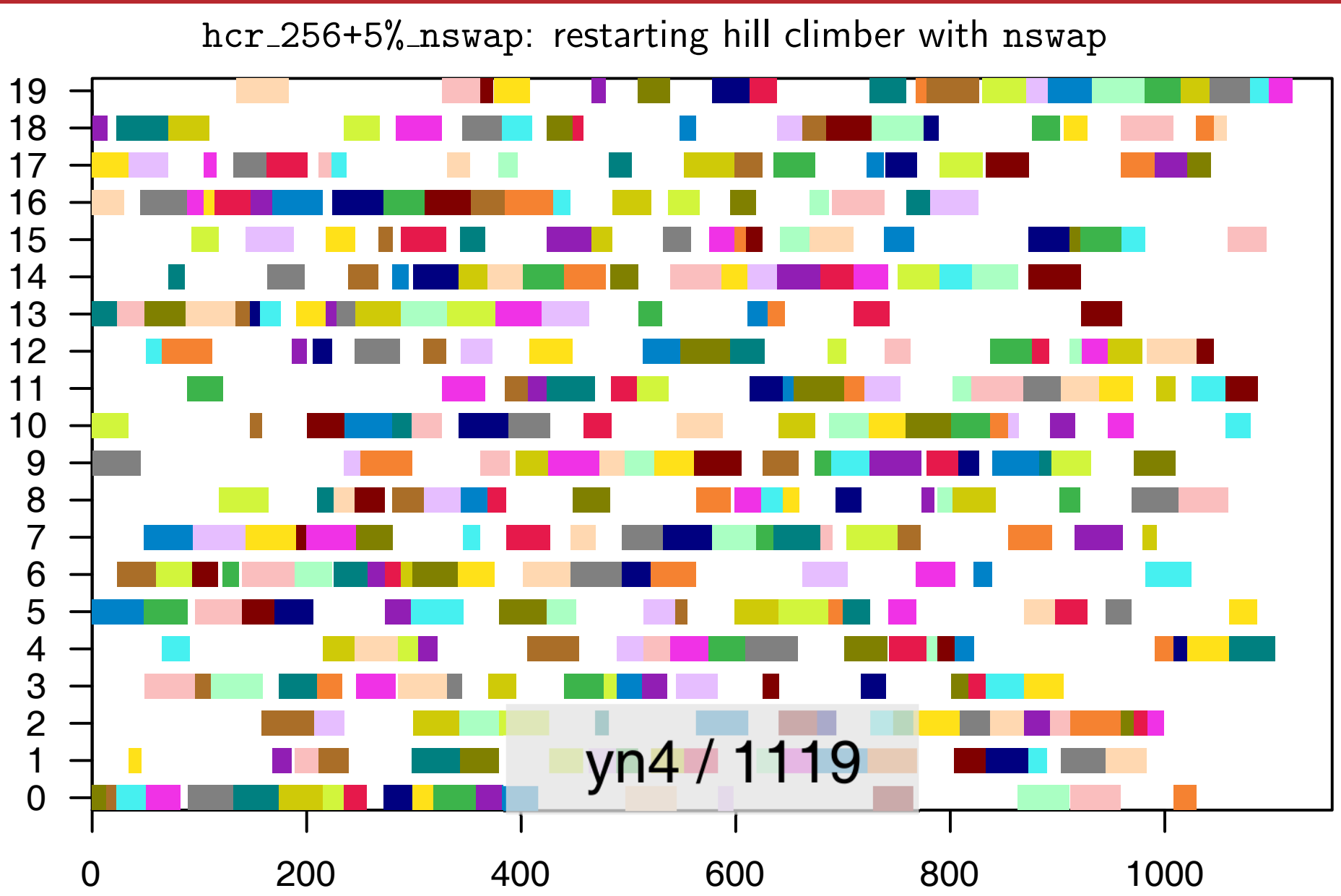

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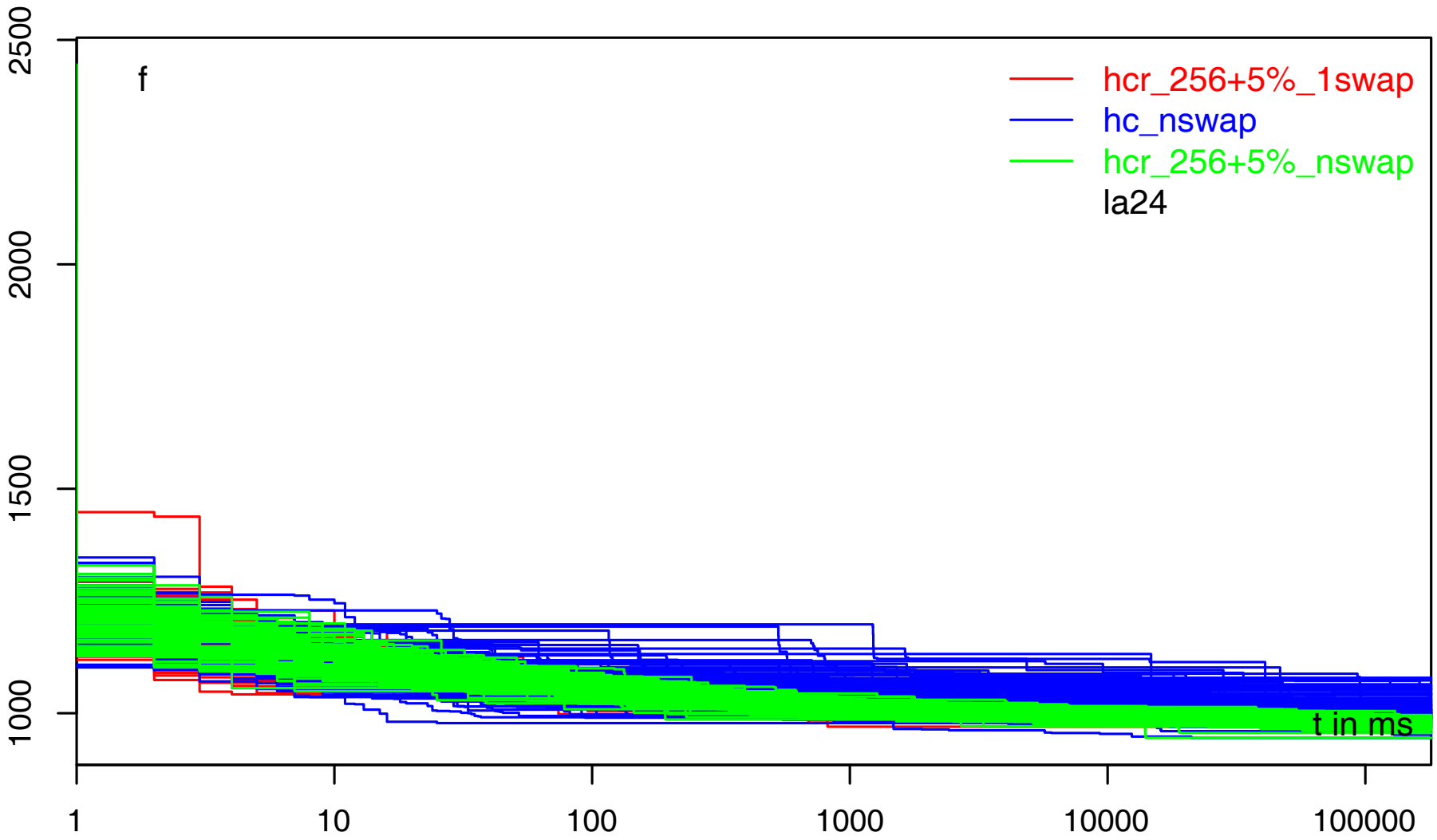

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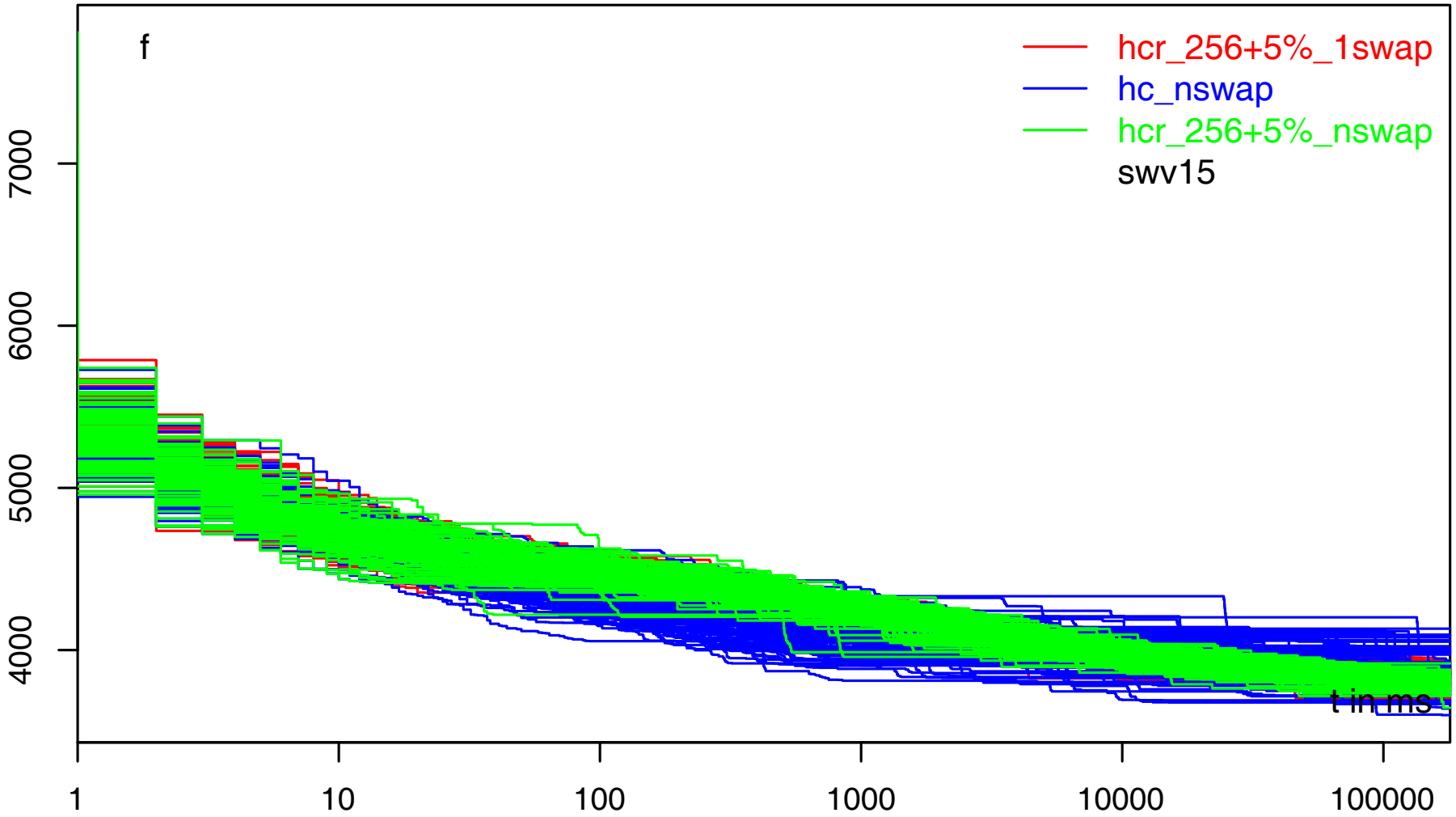

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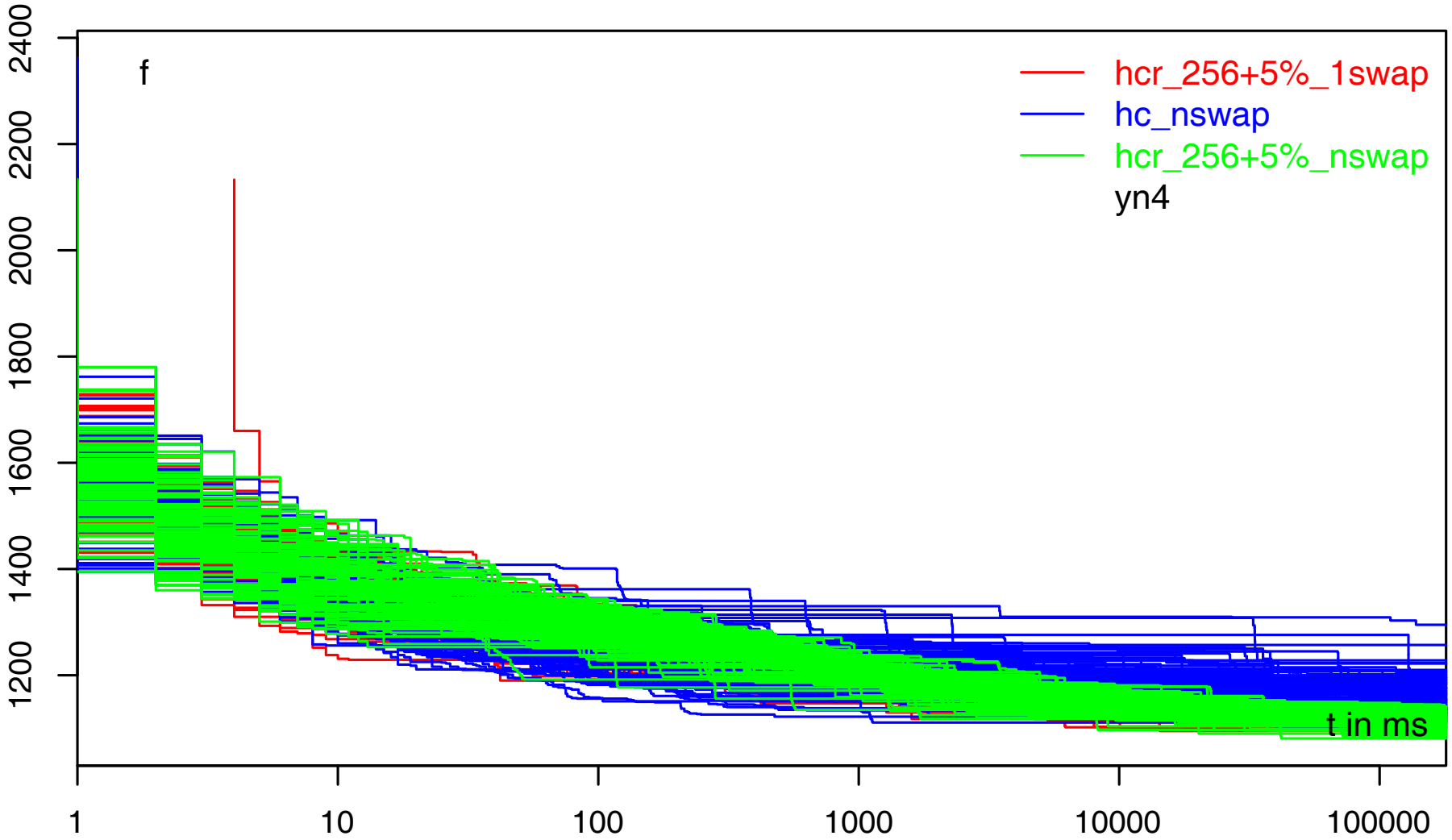

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- But we still sometimes get bad results.
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- A hill climber can always get trapped in a local optimum, even with restarts. . . . . . if the basins of attraction of the local optima are larger than those of the global optimum.

谢谢

Thank you

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Caspar David Friedrich, “Der Wanderer über dem Nebelmeer”, 1818
http://en.wikipedia.org/wiki/Wanderer_above_the_Sea_of_Fog

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