Metaheuristics for Smart Manufacturing

2. The Structure of Optimization

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经济技术开发区 锦绣大道99号

Outline

1. Introduction
2. Smart Manufacturing Example Problem
3. Solution Space and Objective Function
4. From Solution Space to Search Space
5. Number of Solutions and Termination
6. Summary

The slides are available at http://iao.hfuu.edu.cn/155, the book at http://thomasweise.github.io/aitoa, and the source code at http://www.github.com/thomasWeise/aitoa-code

course book

course material

An Introduction to Optimization Algorithms

The contents of this course are available as free electronic book *"An Introduction to Optimization Algorithms"* [1] at http://thomasweise.github.io/aitoa in pdf, html, azw3, and epub format, created with our bookbuildeR tool chain.

An Introduction to Optimization Algorithms

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2019-07-26

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- We will approach this topic based on an example from the field of Smart Manufacturing.
- We will first learn about the basic ingredients that make up an optimization task.
- Then we will step-by-step work our way from stupid to good metaheuristics for solving it.

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  4. a search space $\mathbb{X}$, i.e., a simpler data structure for internal use, which can more efficiently be processed by an optimization algorithm than $\mathbb{Y}$

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  7. a termination criterion, which tells the optimization process when to stop.

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- Looks complicated, but don't worry. We will do this one-by-one.
- We want to get an understanding of the structure of optimization problems from the metaheuristic perspective by looking at one concrete problem from production planning.

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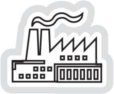

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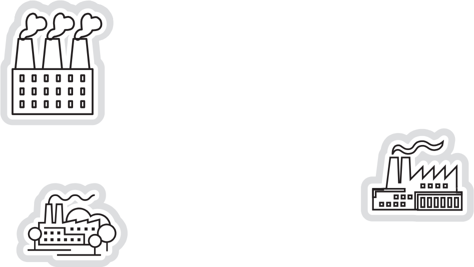

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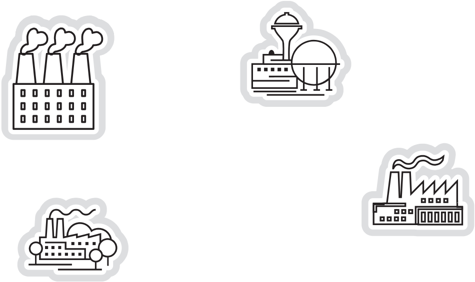

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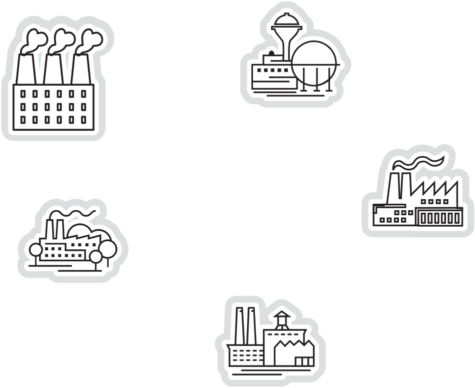

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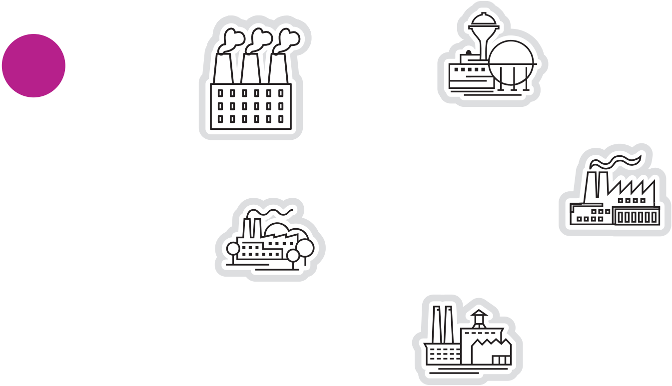

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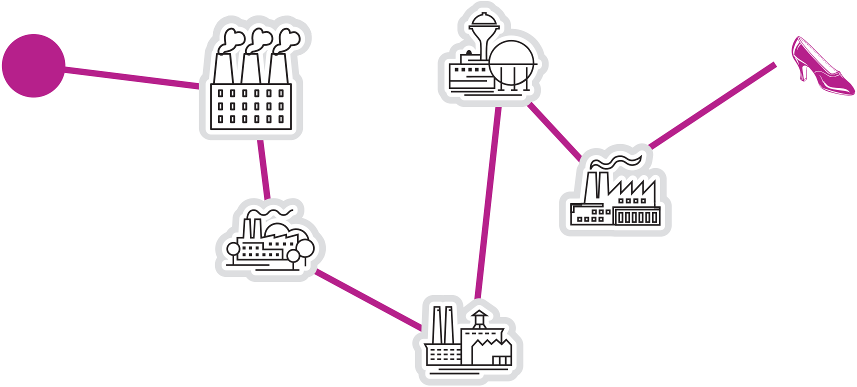

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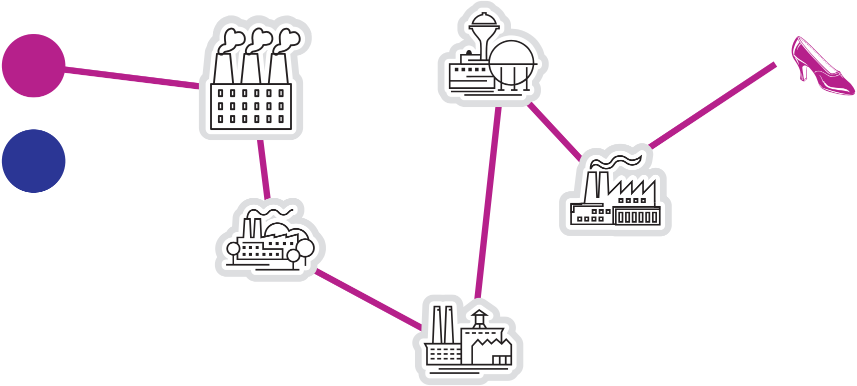

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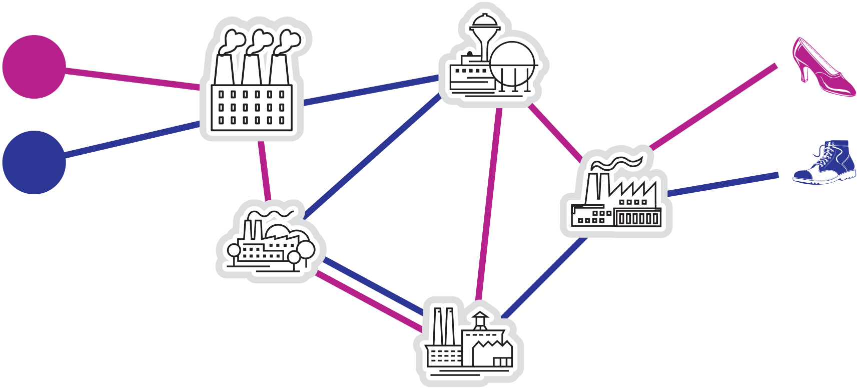

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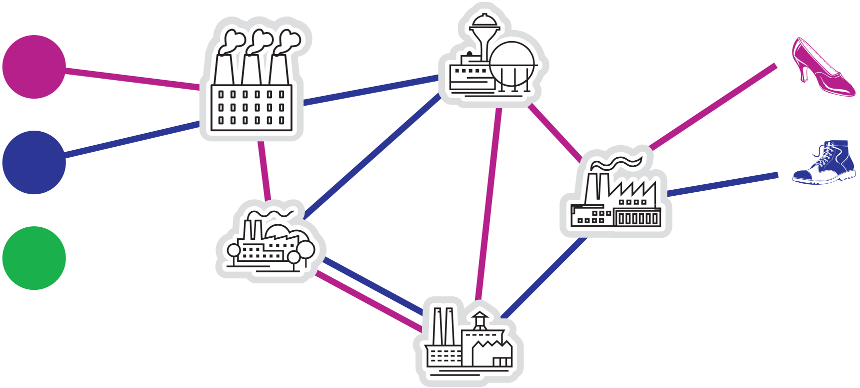

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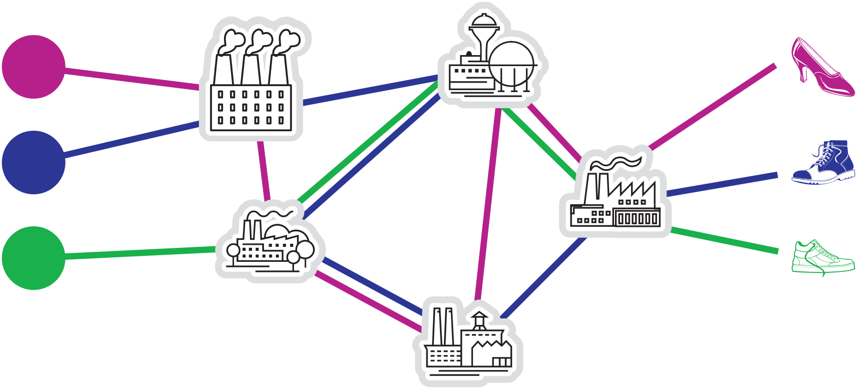

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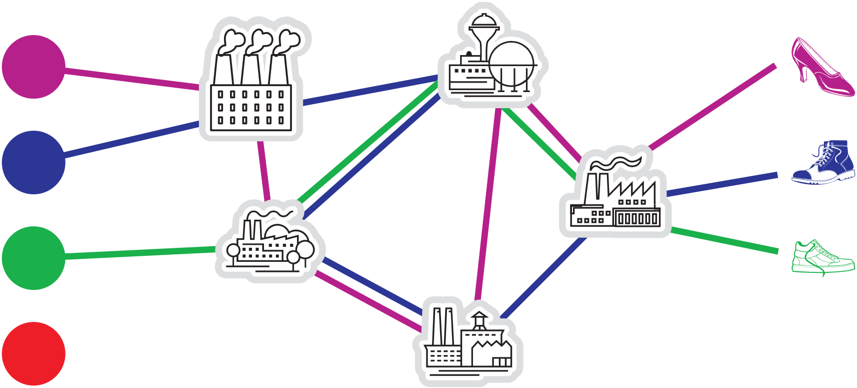

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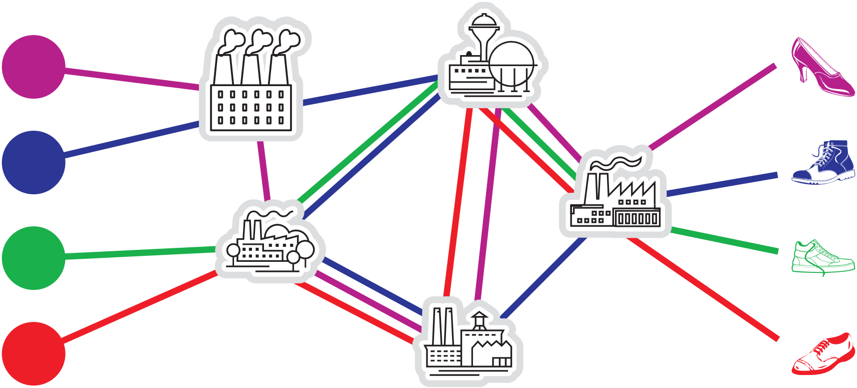

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- This problem is $\mathcal{NP}$-hard. [10, 11]

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- What do such JSSP instances look like?

Demo Instance

```
+++++++++++++++++++++++++++++
A simple demo
4 5
0 10 1 20 2 20 3 40 4 10
1 20 0 10 3 30 2 50 4 30
2 30 1 20 4 12 3 40 0 10
4 50 3 30 2 15 0 20 1 15
+++++++++++++++++++++++++++++
```

Demo Instance

number of jobs

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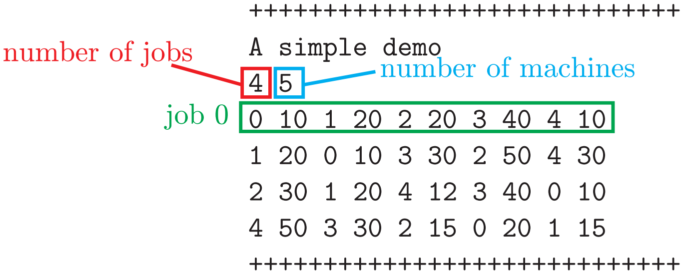

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number of jobs

number of machines

job 0

Demo Instance

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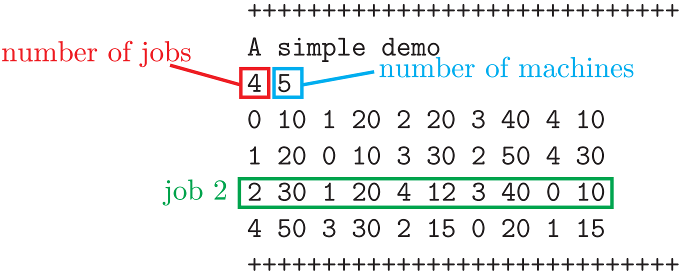
```

number of jobs

number of machines

job 1

Demo Instance



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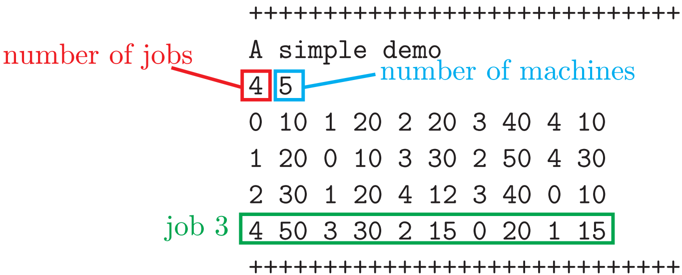

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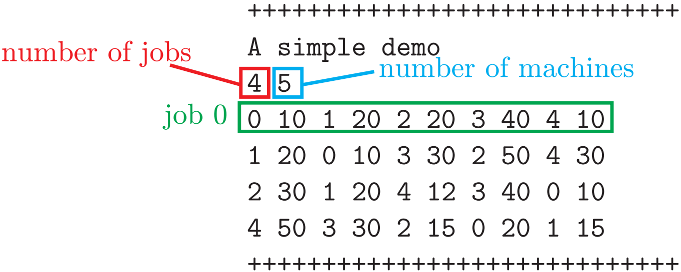
```

number of jobs

number of machines

job 3

Demo Instance



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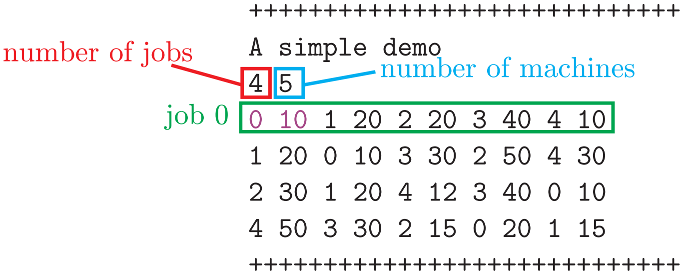

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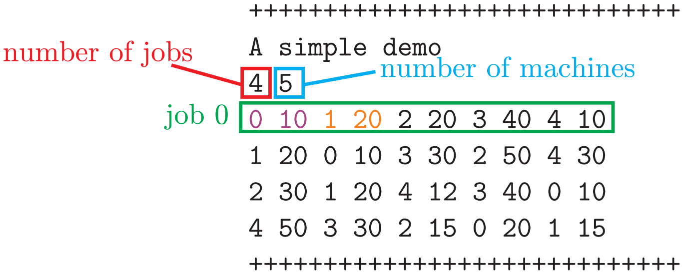
Job 0 first needs to be processed by machine 0 for 10 time units

Demo Instance



number of jobs

number of machines

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Job 0 first needs to be processed by machine 0 for 10 time units, it then goes to machine 1 for 20 time units

Demo Instance

number of jobs

number of machines

job 0

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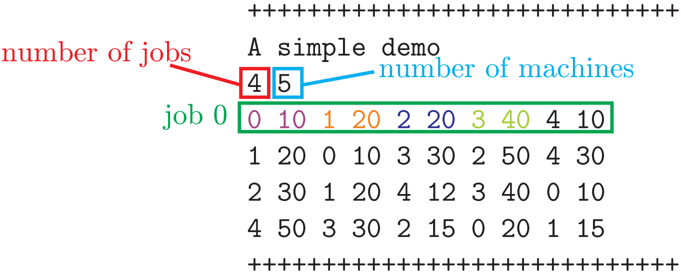

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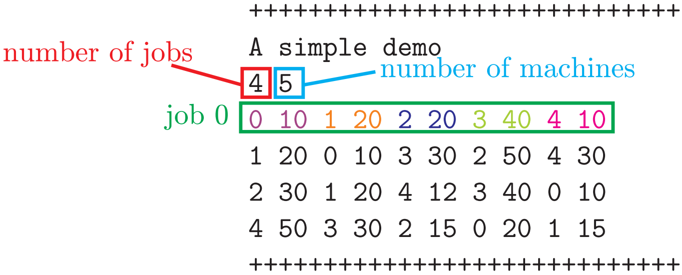

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Job 0 first needs to be processed by machine 0 for 10 time units, it then goes to machine 1 for 20 time units, it then goes to machine 2 for 20 time units, it then goes to machine 3 for 40 time units, and finally it goes to machine 4 for 10 time units.

Demo Instance

number of jobs

number of machines

job 1

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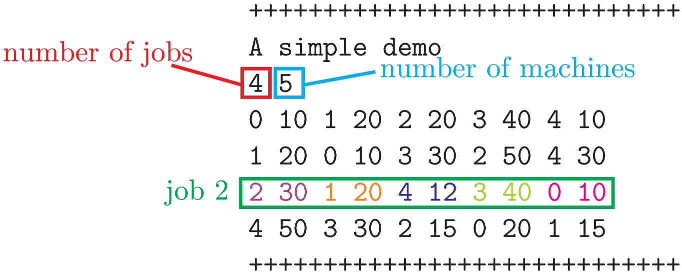
+++++++++++++++++++++++++++++
```

Similarly, Job 1 first needs to be processed by machine 1 for 20 time units, it then goes to machine 0 for 10 time units, it then goes to machine 3 for 30 time units, it then goes to machine 2 for 50 time units, and finally it goes to machine 4 for 30 time units.

Demo Instance



```
+++++++++++++++++++++++++++++
A simple demo
4 5
0 10 1 20 2 20 3 40 4 10
1 20 0 10 3 30 2 50 4 30
2 30 1 20 4 12 3 40 0 10
4 50 3 30 2 15 0 20 1 15
+++++++++++++++++++++++++++++
```

Job 2 first needs to be processed by machine 2 for 30 time units, it then goes to machine 1 for 20 time units, it then goes to machine 4 for 12 time units, it then goes to machine 3 for 40 time units, and finally it goes to machine 0 for 10 time units.

Demo Instance

number of jobs

number of machines

job 3

```
+++++++++++++++++++++++++++++
A simple demo
4 5
0 10 1 20 2 20 3 40 4 10
1 20 0 10 3 30 2 50 4 30
2 30 1 20 4 12 3 40 0 10
4 50 3 30 2 15 0 20 1 15
+++++++++++++++++++++++++++++
```

And Job 3 first needs to be processed by machine 4 for 50 time units, it then goes to machine 3 for 30 time units, it then goes to machine 2 for 15 time units, it then goes to machine 0 for 20 time units, and finally it goes to machine 1 for 15 time units.

Instance abz7

Instance abz7 by Adams et al. [15].

20 jobs

15 machines

```
 +++++++++++++++++++++++++++++
 Adams, Balas, and Zawack 15 x 20 instance (Table 1, instance 7)
 20 15
   2 24  3 12  9 17  4 27  0 21  6 25  8 27  7 26  1 30  5 31 11 18 14 16 13 39 10 19 12 26
   6 30  3 15 12 20 11 19  1 24 13 15 10 28  2 36  5 26  7 15  0 11  8 23 14 20  9 26  4 28
   6 35  0 22 13 23  7 32  2 20  3 12 12 19 10 23  9 17  1 14  5 16 11 29  8 16  4 22 14 22
   9 20  6 29  1 19  7 14 12 33  4 30  0 32  5 21 11 29 10 24 14 25  2 29  3 13  8 20 13 18
  11 23 13 20  1 28  6 32  7 16  5 18  8 24  9 23  3 24 10 34  2 24  0 24 14 28 12 15  4 18
   8 24 11 19 14 21  1 33  7 34  6 35  5 40 10 36  3 23  2 26  4 15  9 28 13 38 12 13  0 25
  13 27  3 30  6 21  8 19 12 12  4 27  2 39  9 13 14 12  5 36 10 21 11 17  1 29  0 17  7 33
   5 27  4 19  6 29  9 20  3 21 10 40  8 14 14 39 13 39  2 27  1 36 12 12 11 37  7 22  0 13
  13 32 11 29  8 24  3 27  5 40  4 21  9 26  0 27 14 27  6 16  2 21 10 13  7 28 12 28  1 32
  12 35  1 11  5 39 14 18  7 23  0 34  3 24 13 11  8 30 11 31  4 15 10 15  2 28  9 26  6 33
  10 28  5 37 12 29  1 31  7 25  8 13 14 14  4 20  3 27  9 25 13 31 11 14  6 25  2 39  0 36
   0 22 11 25  5 28 13 35  4 31  8 21  9 20 14 19  2 29  7 32 10 18  1 18  3 11 12 17  6 15
  12 39  5 32  2 36  8 14  3 28 13 37  0 38  6 20  7 19 11 12 14 22  1 36  4 15  9 32 10 16
   8 28  1 29 14 40 12 23  4 34  5 33  6 27 10 17  0 20  7 28 11 21  2 21 13 20  9 33  3 27
   9 21 14 34  3 30 12 38  0 11 11 16  2 14  5 14  1 34  8 33  4 23 13 40 10 12  6 23  7 27
   9 13 14 40  7 36  4 17  0 13  5 33  8 25 13 24 10 23  3 36  2 29  1 18 11 13  6 33 12 13
   3 25  5 15  2 28 12 40  7 39  1 31  8 35  6 31 11 36  4 12 10 33 14 19  9 16 13 27  0 21
  12 22 10 14  0 12  2 20  5 12  1 18 11 17  8 39 14 31  3 31  7 32  9 20 13 29  4 13  6 26
   5 18 10 30  7 38 14 22 13 15 11 20  9 16  3 17  1 12  2 13 12 40  6 17  8 30  4 38  0 13
   9 31  8 39 12 27  1 14  5 33  3 31 11 22 13 36  0 16  7 11 14 14  4 29  6 28  2 22 10 17

+++++++++++++++++++ EOF ++++++++++++++++++++++++++++++++++++
```

Instance `la24`

Instance `la24` by Lawrence [16].

15 jobs

10 machines

```
+++++++++++++++++++++++++++++
Lawrence 15x10 instance (Table 7, instance 4)
15 10
7  8 9 75 0 72 6 74 4 30 8 43 2 38 5 98 1 26 3 19
6 19 8 73 3 43 0 23 1 85 4 39 5 13 9 26 2 67 7  9
1 50 3 93 5 80 4  7 0 55 2 61 6 57 8 72 9 42 7 46
1 68 7 43 4 99 6 60 5 68 0 91 8 11 3 96 9 11 2 72
7 84 2 34 8 40 5  7 1 70 6 74 3 12 0 43 9 69 4 30
8 60 0 49 4 59 5 72 9 63 1 69 7 99 6 45 3 27 2  9
6 71 2 91 8 65 1 90 9 98 4  8 7 50 0 75 5 37 3 17
8 62 7 90 5 98 3 31 2 91 4 38 9 72 1  9 0 72 6 49
4 35 0 39 9 74 5 25 7 47 3 52 2 63 8 21 6 35 1 80
9 58 0  5 3 50 8 52 1 88 6 20 2 68 5 24 4 53 7 57
7 99 3 91 4 33 5 19 2 18 6 38 0 24 9 35 1 49 8  9
0 68 3 60 2 77 7 10 8 60 5 15 9 72 1 18 6 90 4 18
9 79 1 60 3 56 6 91 2 40 8 86 7 72 0 80 5 89 4 51
4 10 2 92 5 23 6 46 8 40 7 72 3  6 1 23 0 95 9 34
2 24 5 29 9 49 8 55 0 47 6 77 3 77 7  8 1 28 4 48
+++++++++++++++++++++++++++++
```

Instance `swv15` by Storer et al. [17].

50 jobs

10 machines

```
+++++++++++++++++++++++++++++
Storer, Wu, and Vaccari hard 50x10 instance (Table 2, instance 15)
50 10
 2  93 4  40 0   1 3  77 1  77 5  16 9  74 8  11 6  51 7  92
 0  92 4  80 1  76 3  59 2  70 5  86 9  17 6  78 7  30 8  93
 1  44 2  92 3  96 4  77 0  53 9  10 7  49 5  84 8  59 6  14
 1  60 2  19 3  76 0  73 4  85 7  13 8  93 5  68 9  50 6  78
 2  20 0  24 3  41 1   2 4   4 9  44 7  79 8  81 5  16 6  39
 3  41 2  35 1  32 4  18 0  15 8  98 6  29 5  19 7  14 9  26
 1  59 0  45 4  53 3  44 2  98 5  84 6  23 7  45 8  39 9  89
 1  30 4  51 3  25 0  51 2  84 6  60 5  45 7  89 8  25 9  97
 0  47 3  18 2  40 4  62 1  58 5  36 7  93 8  77 9  90 6  15
 3  33 1  68 0  41 4  72 2  20 6  69 7  47 5  22 9  47 8  22
 2  28 1 100 4  20 0  35 3  26 5  24 9  41 6  42 7 100 8  32
 0  65 2  12 4  53 3  93 1  40 8  18 7  23 5  60 6  89 9  53
 0  58 1  60 4  97 3  31 2  50 9  85 5  64 7  38 6  85 8  35
 3  64 0  58 1  49 2  45 4   9 8  49 6  22 5  99 9  15 7   7
 0  10 4  85 3  72 2  37 1  77 5  70 7  45 9   8 6  83 8  57
 4  93 0  87 1  87 2  18 3   4 8  78 5  67 9  20 6  17 7  35
 4  72 0  56 3  57 2  15 1  45 6  41 5  40 9  85 8  32 7  81
 0  36 3  63 4  79 2  32 1   5 6  25 7  86 9  91 5  21 8  35
 2  83 4  29 0   9 1  38 3  73 7  50 9  99 5  18 8  29 6  41
 0 100 3  29 2  60 4  63 1  64 8  71 6  35 5  26 9   9 7  22
 1  81 0  60 3  62 4  48 2  68 7  28 5  69 8  92 6  79 9  10
 0  40 4  80 1  41 2  10 3  68 8  28 9  51 7  33 6  82 5  25
 4  30 2  12 0  35 3  17 1  70 9  29 7  18 8  93 6  94 5  37
 1  36 2  41 3  27 4  36 0  78 7  64 6  88 5  25 9  92 8  66
 2  65 3  27 4  74 0  32 1  40 5  88 8  73 6  92 7  83 9  42
 0  48 1  85 2  92 4  95 3  61 8  72 9  76 5  58 7  11 6  89
 3  84 2  50 0  70 4  24 1  42 9  55 5 100 6  70 7   4 8  68
 0  95 4  41 2  11 3  98 1  85 5  64 6   8 7  26 8   6 9   6
 0  84 2  49 1  17 3  69 4  55 8  75 6  45 9  38 7  59 5  28
 2  48 0  29 4   1 1  64 3  41 5  23 7  64 9  31 6  56 8  12
 2  81 4  25 3  33 0  22 1  50 5  74 9  56 8  33 7  85 6  83
 1  62 4  25 0  21 2  20 3   8 6  36 9   9 5  91 8  90 7  49
 1  43 0  16 2  91 3  96 4  24 5  11 9  91 7  41 8  35 6  66
 1  91 2  20 4  44 0  42 3  87 9  57 6  15 5  38 8  42 7  89
 0  33 3  95 4  68 2  22 1  80 7  53 8  13 9  70 5  22 6  69
 0  15 3  47 1  24 2  31 4  41 8  14 9  28 7  59 5  52 6  39
 2  95 0  42 4   5 1  57 3  67 6  30 9  21 8  70 5   9 7  20
 2  54 0  15 1  20 3  64 4  83 9  40 7   6 5  89 6  91 8  48
 0  22 4  27 1  77 3  25 2  16 8  72 9  61 6  75 7  4  5  19
 3  68 1  82 2  16 0  83 4   2 7  10 8  88 5  41 9  21 6  66
 1  64 0  76 2  85 3  71 4  97 5  97 7   8 6  40 8  70 9  35
 0  94 1  45 2  94 4  84 3  44 8  41 5  30 7  47 6  19 9  22
 2  23 1  10 0  82 3  93 4  90 8  67 7   9 9  18 5  22 6  87
 0  75 2  27 4  97 3   9 1  57 9  14 5  50 7  31 8  62 6  23
 1  42 3  41 2  35 0  75 4  18 9  65 7  38 6  38 8  51 5  56
 4  72 1  63 0  33 2  27 3  41 5  52 7  42 9  10 6  14 8  71
 2  91 1  89 0  44 4  91 3  26 6  49 5  22 8  31 9  69 7   5
 3  42 1  34 0   4 4  34 2  16 6  86 7  25 8  99 5  67 9  25
 4  34 1  93 0  26 3  81 2   9 7  96 8  79 9  68 5  76 6  10
 3  19 1  47 4  13 2  98 0  32 7  12 9  45 6  52 8  49 5  34
+++++++++++++++++++++++++++++
```

Instance `yn4`

Instance `yn4` by Yamada and Nakano[18].

20 jobs

20 machines

```
+++++++++++++++++++++++++++++
Yamada and Nakano 20x20 instance (Table 4, instance 4)
20 20
16 34 17 38  0 21  6 15 15 42  8 17  7 41 18 10 10 26 11 24  1 31 19 25 14 31 13 33  4 35  9 30  3 16 12 16  5 30  2 13
 5 41 11 33  6 15 16 38  0 40 14 38  3 37  1 20 13 22  4 34  7 16 17 39  9 15  2 19 10 36 12 39 18 26  8 19 15 39 19 34
17 34  1 12 16 10  7 47 13 28 15 27  0 19  6 34 19 33 12 40  9 37 14 24  8 15 10 34  2 44  3 37 18 22 11 31  4 39  5 26
 5 48  7 46 16 47 10 45 14 15  8 25  0 34  3 24 12 35 18 15  2 48 13 19 11 10  1 48 17 16 15 28  4 18  6 17  9 44 19 41
12 47  3 23  9 48 16 45 14 39  6 42  8 32 15 11 13 16  5 14 11 19  1 46 19 10 10 17  7 41  2 47 17 32  4 17  0 21 18 17
18 14 16 20  1 18 12 14 13 10  6 16  5 24  4 18  0 24 11 18 15 42 19 13  3 23 14 40  9 48  8 12  2 24 10 23  7 45 17 30
 0 27 12 15  4 26 13 19 17 14  5 49  7 16 18 28 16 16  8 20  9 36  2 21 14 30  3 36  1 17 15 22  6 43 11 32 10 23 19 17
 0 32 16 15 17 12  7 46  3 37 18 43 11 40 13 43  9 48  4 36 15 24  8 25  1 33 14 32  5 26  6 37 12 24 10 24  2 15 19 22
10 34  6 33 15 25  8 46  0 20 18 33  4 19 13 45  2 47  1 32  3 12 11 29 16 29  5 46 12 17  7 48 14 39 17 40 19 41  9 37
13 26  3 47  5 44  6 49  1 22 17 12 10 28 19 36  9 27  4 25 14 48  7 11 16 49 12 24 11 48  2 19  0 47 18 49  8 46 15 36
13 23 18 48 14 15  0 42  3 36  8 15  6 32 10 18  1 45 15 23 11 45  2 13 17 21 12 32  7 44  5 25 19 34 16 22  9 11  4 43
17 37  7 49 15 45  2 28  9 15  8 35 12 29 13 44  1 26  4 25  5 30  3 39  0 15 14 28 18 23  6 42 11 33 16 45 10 10 19 20
 0 10  6 37  3 15 13 13 10 11  2 49  1 28 14 28 15 13  8 29 12 21 16 32 11 21  4 48  5 11 17 26  9 33 18 22  7 21 19 49
18 38  0 41  4 30 13 43  6 11  2 43 14 27  3 26  9 30 15 19 16 36  1 31 17 47  5 41 10 34  8 40 12 32  7 13 11 18 19 27
 6 24  5 30  7 10 10 35  8 28 16 43 19 12  9 44 15 15  3 15  2 35 18 43  0 38  4 16  1 29 17 40 14 49 13 38 12 16 11 30
 3 48  6 35 13 43  2 37 17 18  5 27  9 27  7 41  1 22 15 28 16 18 10 37 18 48  4 10  8 14 11 18 14 43  0 48 12 12 19 49
 0 13 13 38  7 34  6 42  1 36  5 45 18 24  8 35 14 26 19 30 12 47 16 24 11 47  4 40 10 43  3 16 15 10  2 12  9 39 17 22
16 30 13 47 19 49  8 20  4 40  3 46 17 21 14 33  6 44  7 23  9 24  0 48 10 43 15 41  2 32  5 29 11 36  1 38 12 47 18 12
13 10  5 36 12 18 16 48  0 27 14 43 10 46  6 27  7 46 19 35 11 31  2 18  8 24  3 23 17 29 18 14  9 19  1 40 15 38  4 13
 9 45 16 44  0 43 17 31 14 35 13 17 12 42  3 14 18 37 10 39  6 48  7 38 15 26  4 49  2 28 11 35  1 42  5 24  8 44 19 38

+++++++++++++++++++ EOF ++++++++++++++++++++++++++++++++++++
```

Problem Instance Data in `Java`

- How can we represent such data in `Java` program code?

Problem Instance Data in `Java`

- How can we represent such data in `Java` program code?

Listing: A class `JSSPInstance` capable representing a problem instance.

```
public class JSSPInstance {

  public final int m; // number of machines

  public final int n; // number of jobs

  public final int[][] jobs; // one row per job

}
```

Section Outline

1. Introduction
2. Smart Manufacturing Example Problem
3. Solution Space and Objective Function
4. From Solution Space to Search Space
5. Number of Solutions and Termination
6. Summary

Output: Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like.

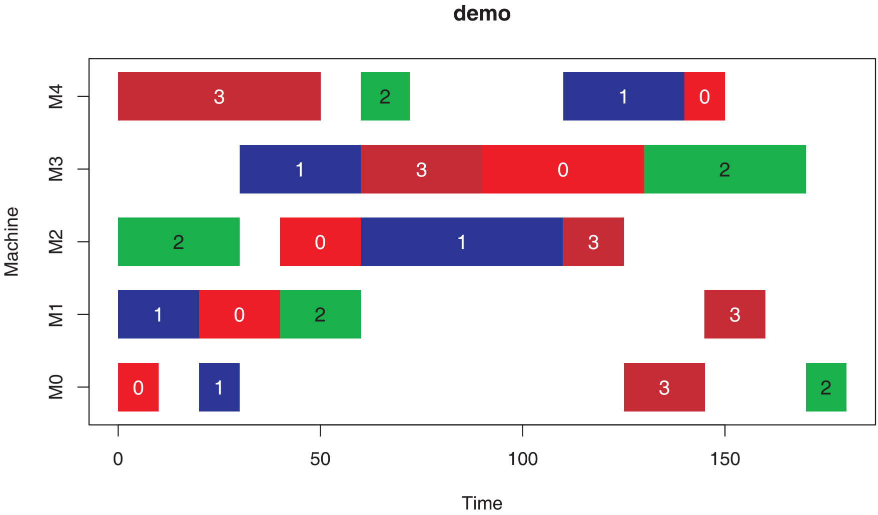
Output: Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like.
- What is a solution for the JSSP, for such an instance?

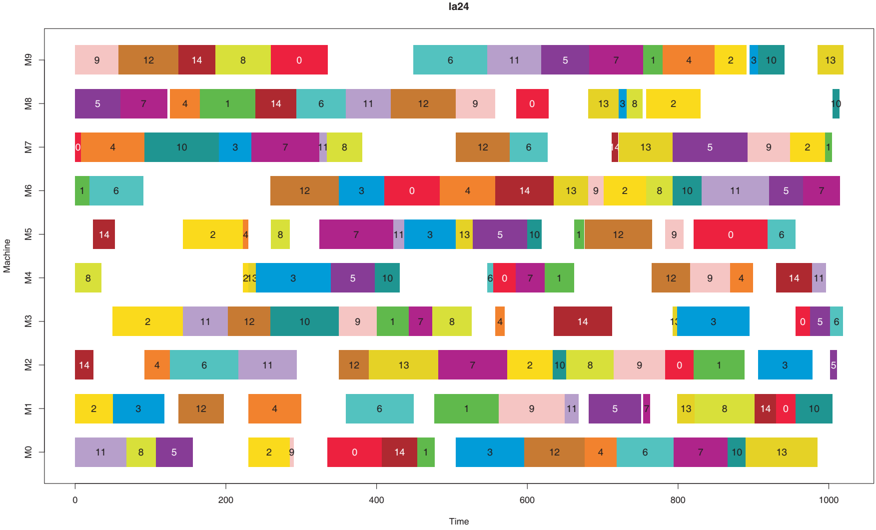
Output: Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like.
- What is a solution for the JSSP, for such an instance?
- Basically, a Gantt Chart [19, 20].

Output: Solution Space $\mathbb{Y}$

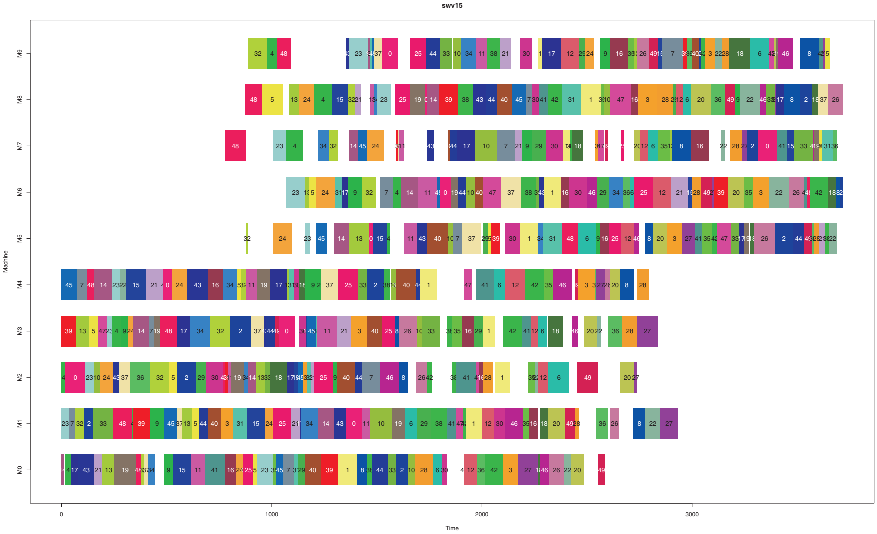


Output: Solution Space $\mathbb{Y}$



Output: Solution Space $\mathbb{Y}$

Output: Solution Space 𝕐



Output: Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like.
- What is a solution for the JSSP, for such an instance?
- A Gantt chart is a diagram which assigns each sub-job on each machine a start and end time.

Output: Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like.
- What is a solution for the JSSP, for such an instance?
- A Gantt chart is a diagram which assigns each sub-job on each machine a start and end time.
- The solution space $\mathbb{Y}$ is the set of all possible feasible Gantt charts for one problem.

Output: Solution Space $\mathbb{Y}$

- We now know how a problem instance of the JSSP looks like.
- What is a solution for the JSSP, for such an instance?
- A Gantt chart is a diagram which assigns each sub-job on each machine a start and end time.
- The solution space $\mathbb{Y}$ is the set of all possible feasible Gantt charts for one problem.
- Each of the m `int[]` lists in `schedule` holds n sub-jobs for each machine as three values jobID, start time, end time, i.e., has length $3n$.

Listing: A class `JSSPCandidateSolution` capable representing a Gantt chart.

```java
public class JSSPCandidateSolution {
  public int[][] schedule; // one row per machine
}
```

Output: Solution Space $\mathbb{Y}$

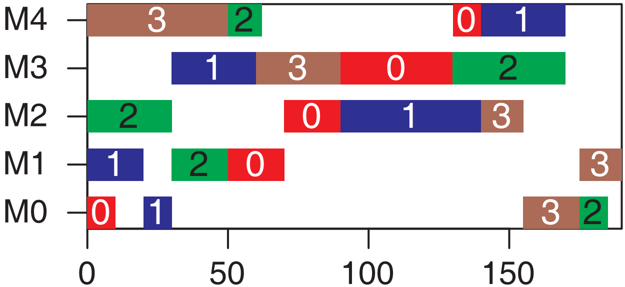
```
int[][] {
{ 0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
{ 1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
{ 2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
{ 1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
{ 3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
}
```

Output: Solution Space $\mathbb{Y}$

```
   int[][] {
M0 { 0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
M1 { 1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
M2 { 2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
M3 { 1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
M4 { 3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
   }
```

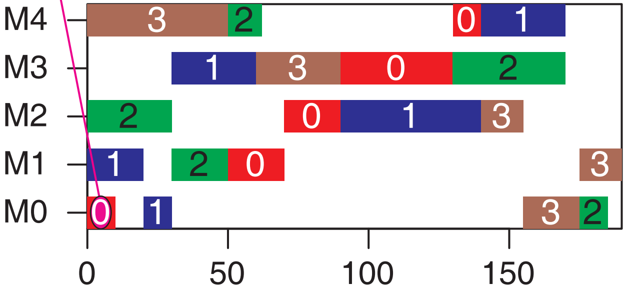
Output: Solution Space $\mathbb{Y}$

```
     int[][] {
M0 { 0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
M1 { 1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
M2 { 2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
M3 { 1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
M4 { 3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
     }
```



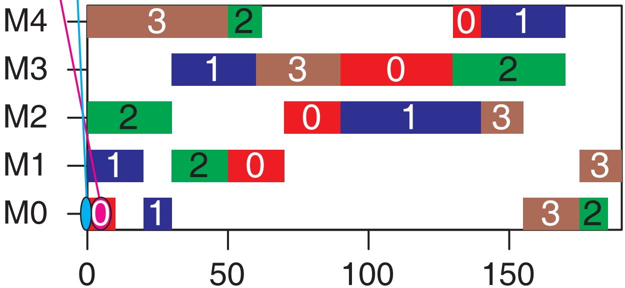
Output: Solution Space $\mathbb{Y}$

```
   int[][] {
M0 { 0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
M1 { 1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
M2 { 2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
M3 { 1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
M4 { 3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
   }
```



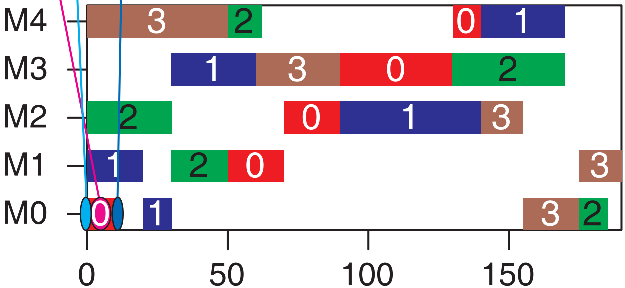
Output: Solution Space $\mathbb{Y}$

```
   int[][] {
M0 { 0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
M1 { 1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
M2 { 2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
M3 { 1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
M4 { 3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
   }
```



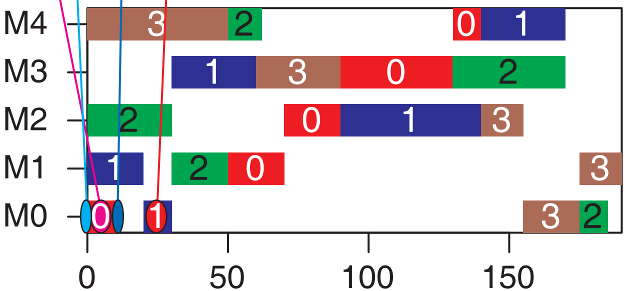
Output: Solution Space $\mathbb{Y}$

```
   int[][] {
M0 { 0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
M1 { 1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
M2 { 2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
M3 { 1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
M4 { 3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
   }
```



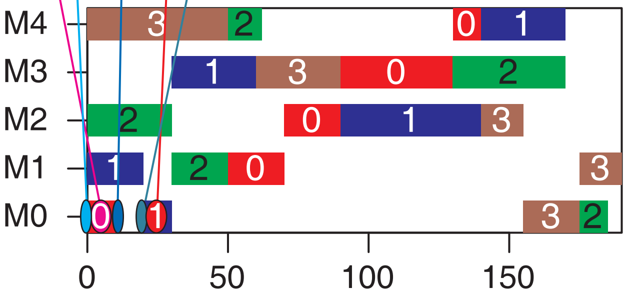
Output: Solution Space $\mathbb{Y}$

```
   int[][] {
M0 { 0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
M1 { 1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
M2 { 2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
M3 { 1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
M4 { 3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
   }
```



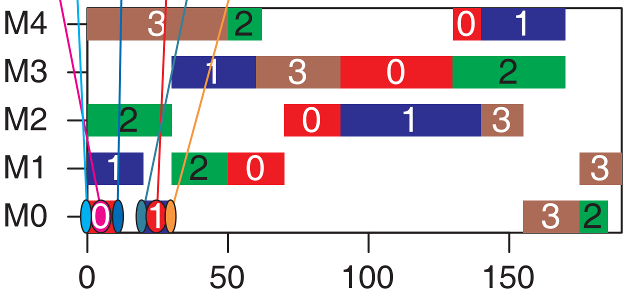
Output: Solution Space $\mathbb{Y}$

```
    int[][] {
M0 { 0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
M1 { 1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
M2 { 2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
M3 { 1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
M4 { 3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
    }
```



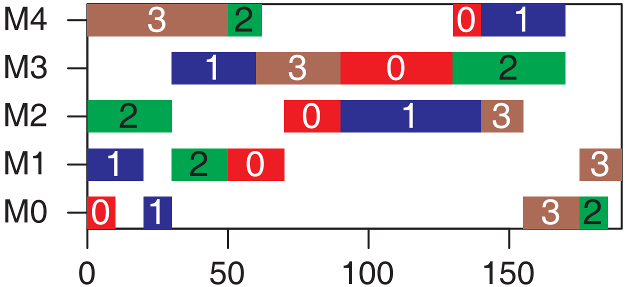
Output: Solution Space $\mathbb{Y}$

```
    int[][] {
M0 { 0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
M1 { 1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
M2 { 2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
M3 { 1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
M4 { 3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
    }
```



Output: Solution Space $\mathbb{Y}$

```
   int[][] {
M0 { 0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
M1 { 1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
M2 { 2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
M3 { 1, 30, 60, 3, 60, 90, 0,  90, 130, 2, 130, 170},
M4 { 3,  0, 50, 2, 50, 62, 0, 130, 140, 1, 140, 170}
   }
```



Output: Solution Space $\mathbb{Y}$

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    int[][] {
M0 { 0,  0, 10, 1, 20, 30, 3, 155, 175, 2, 175, 185},
M1 { 1,  0, 20, 2, 30, 50, 0,  50,  70, 3, 175, 190},
M2 { 2,  0, 30, 0, 70, 90, 1,  90, 140, 3, 140, 155},
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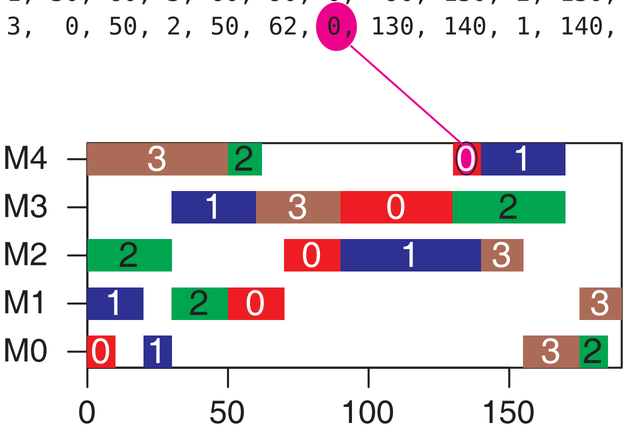

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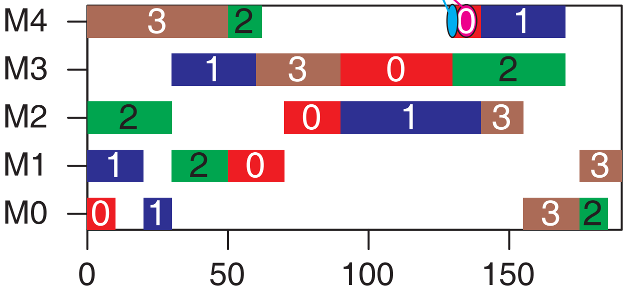

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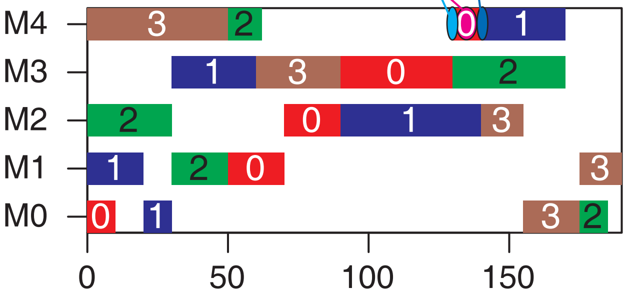

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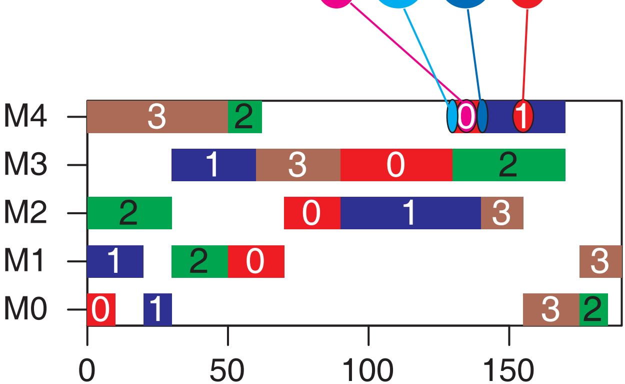

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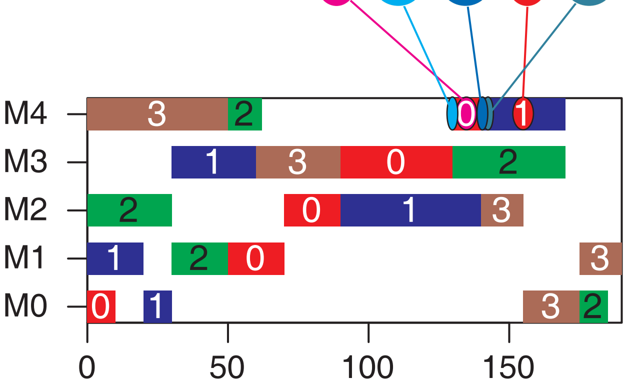

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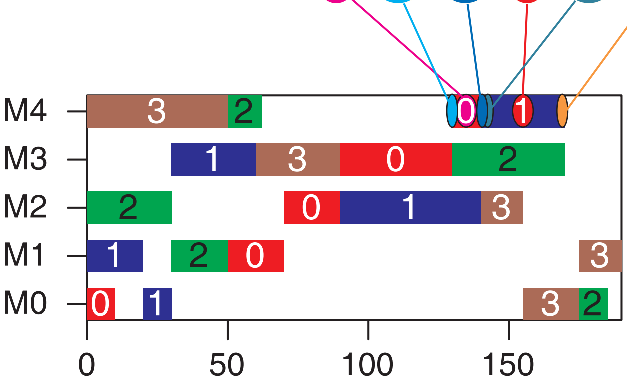
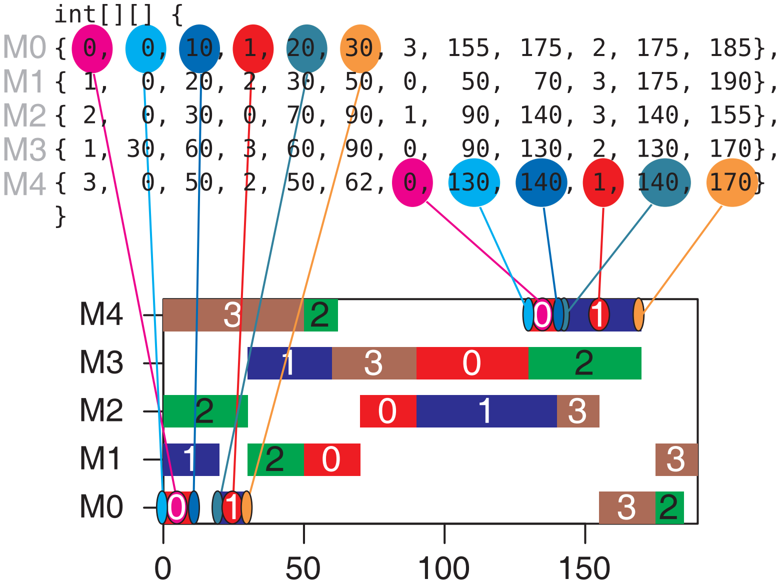


Output: Solution Space 𝕐



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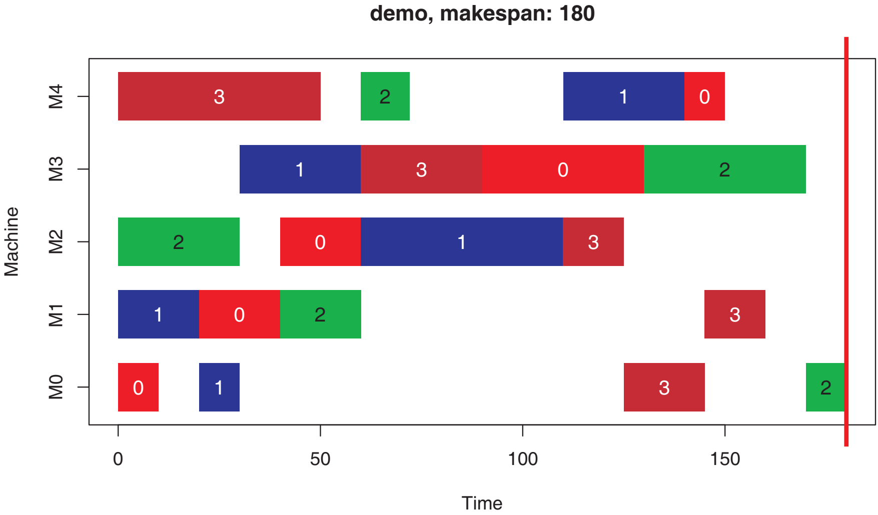

Solution Quality


la24, makespan: 1019

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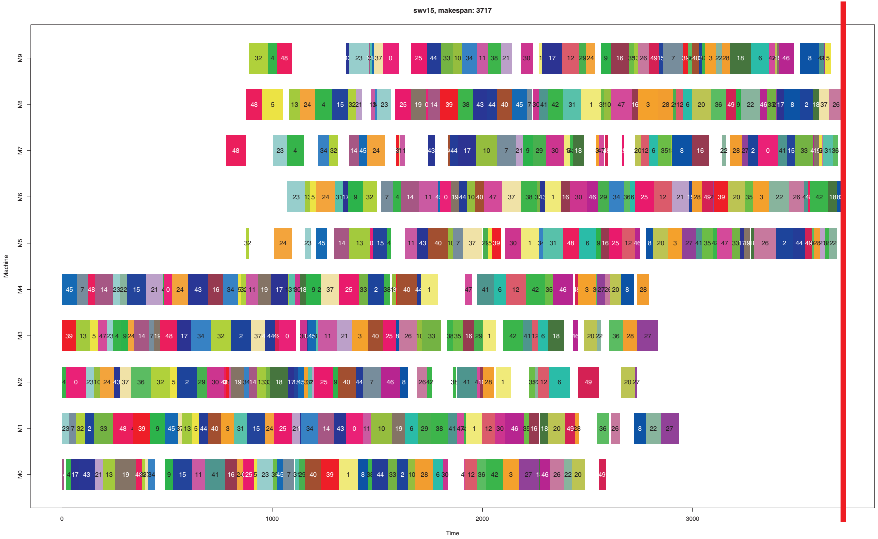

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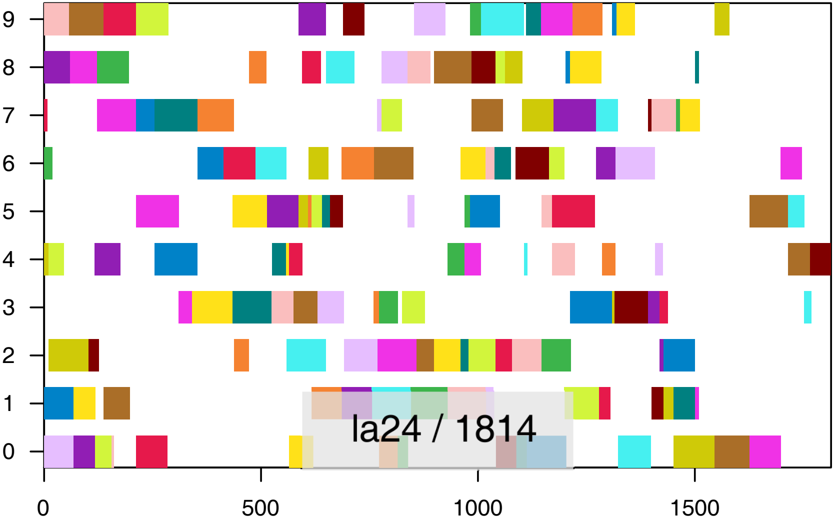
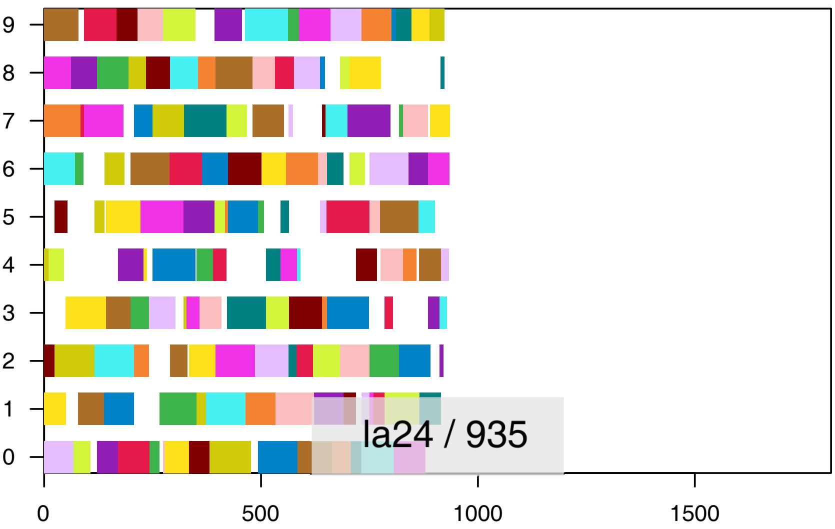

9
8
7
6
5
4
3
2
1
0
0
500
1000
1500
la24 / 1814

Solution Quality



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An Interface for Objective Functions in `Java`

Listing: An interface for objective functions.

```
public interface IObjectiveFunction<Y> {

  public abstract double evaluate(Y y);

}
```

The JSSP Objective Function in Java

Listing: The JSSP objective function.

```java
public final class JSSPMakespanObjectiveFunction
    implements IObjectiveFunction<JSSPCandidateSolution> {

  private final JSSPInstance m_instance;

  public final double evaluate(JSSPCandidateSolution y) {
    int makespan = 0;
// look at the schedule for each machine
    for (final int[] machine : y.schedule) {
// the end time of the last job on the machine is the last number
// in the array, as array machine consists of "flattened" tuples
// of the form ((job, start, end), (job, start, end), ...)
      final int end = machine[machine.length - 1];
      if (end > makespan) {
        makespan = end; // remember biggest end time
      }
    }
    return makespan;
  }
}
```

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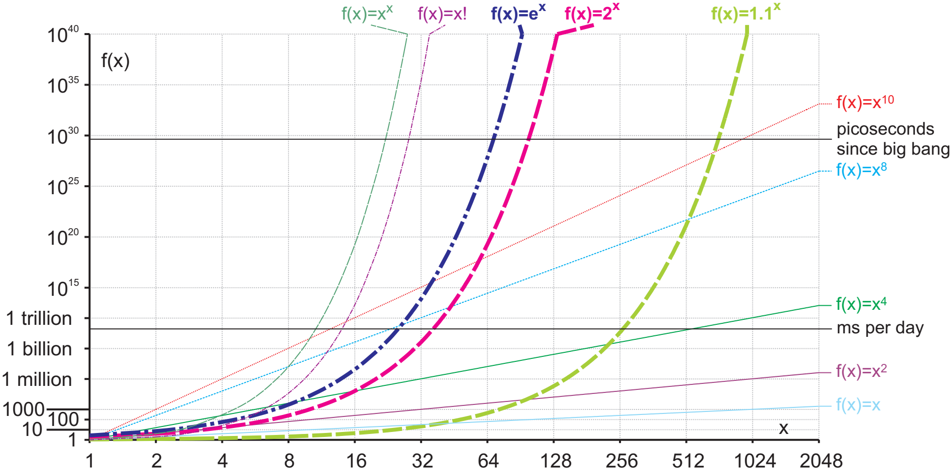

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- All what we can do is search somehow in $\mathbb{Y}$ and hope to get as close to $y^\star$ within reasonable time as possible.
- If we can find a solution with a slightly larger makespan than the best possible solution, but we can get it within a few minutes, that would also be nice.

Section Outline

1. Introduction
2. Smart Manufacturing Example Problem
3. Solution Space and Objective Function
4. **From Solution Space to Search Space**
5. Number of Solutions and Termination
6. Summary

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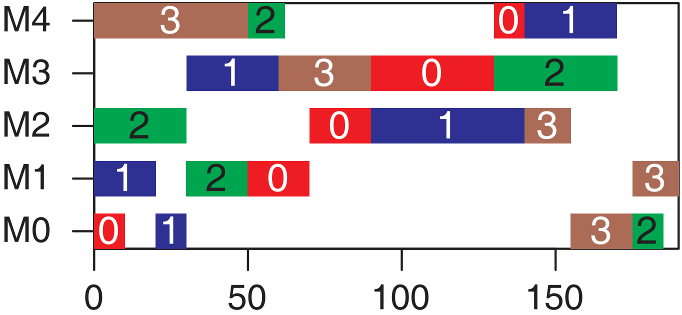

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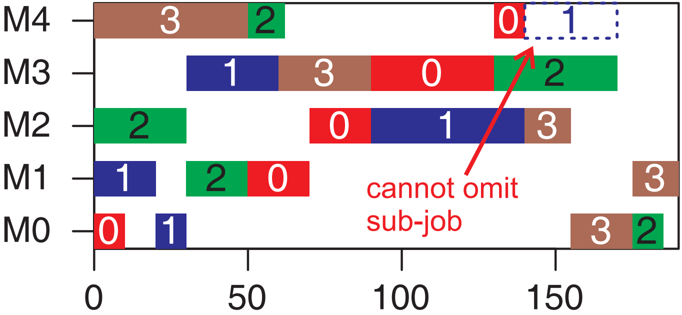

cannot omit
sub-job

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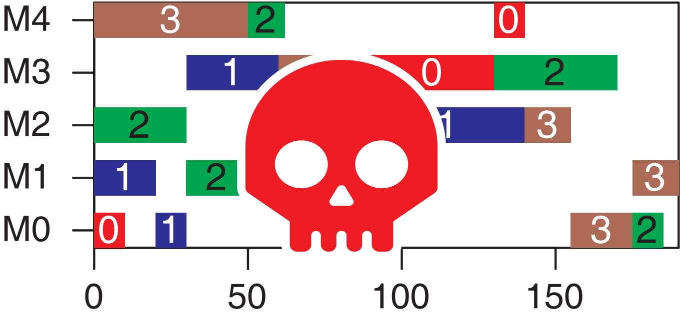

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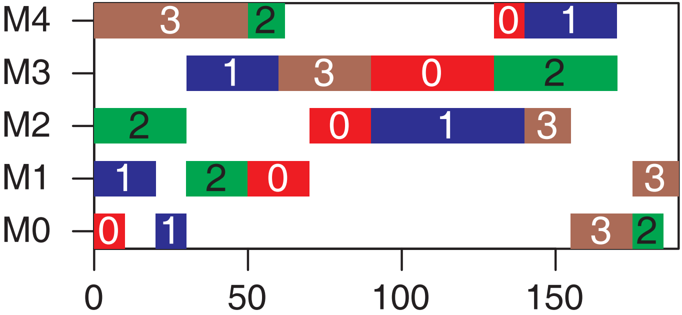

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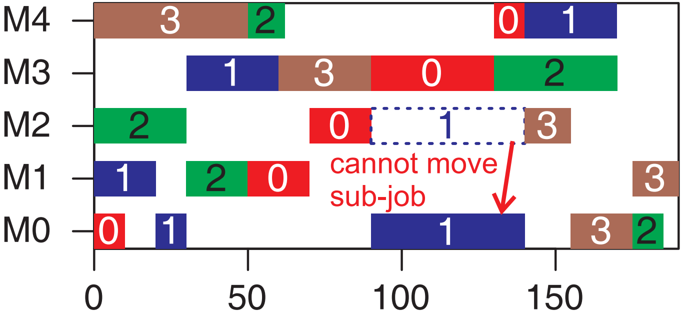

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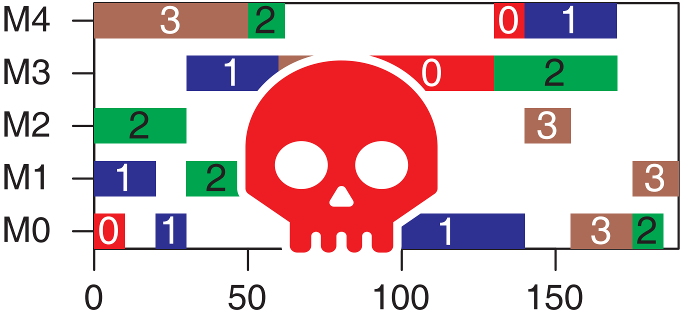

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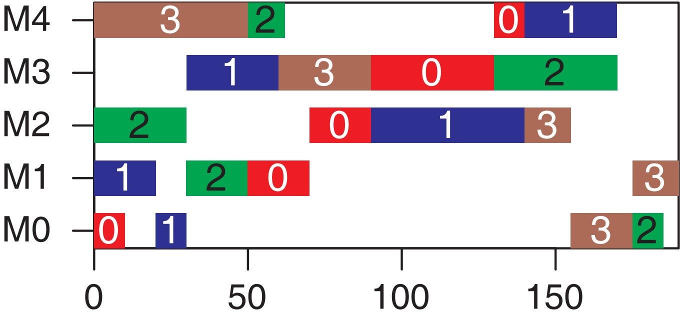

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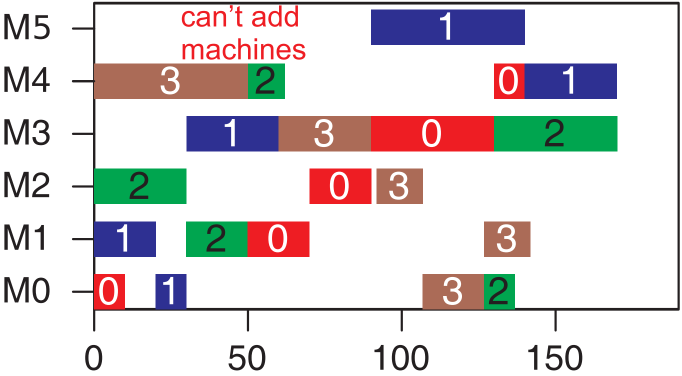

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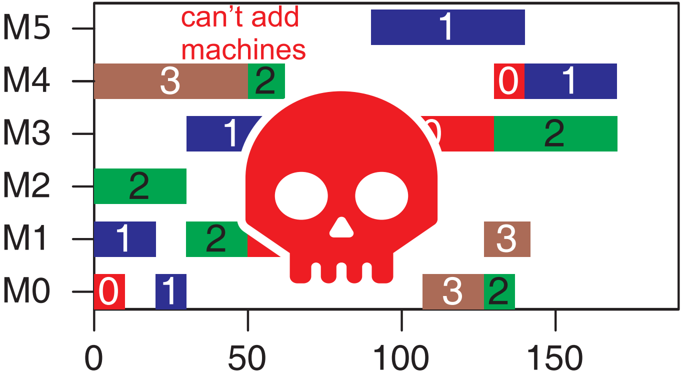

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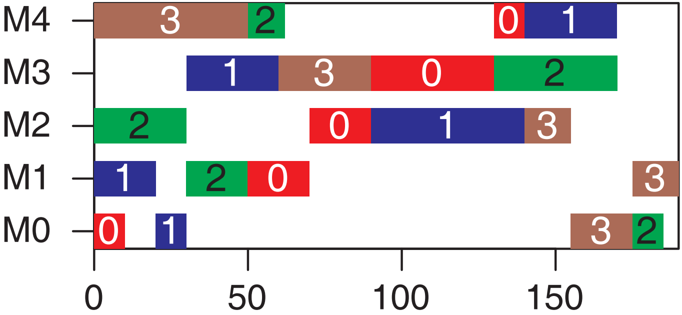

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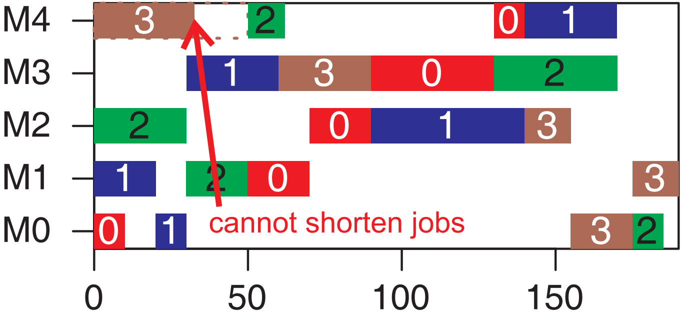

cannot shorten jobs

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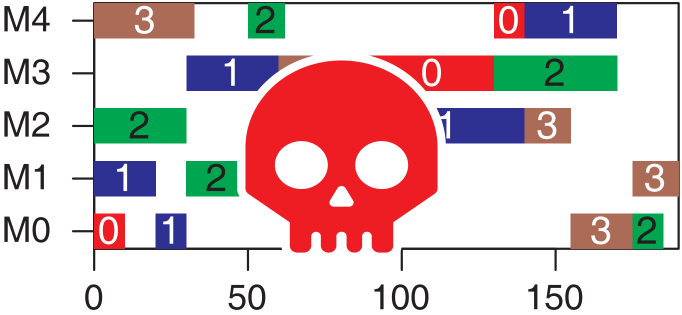

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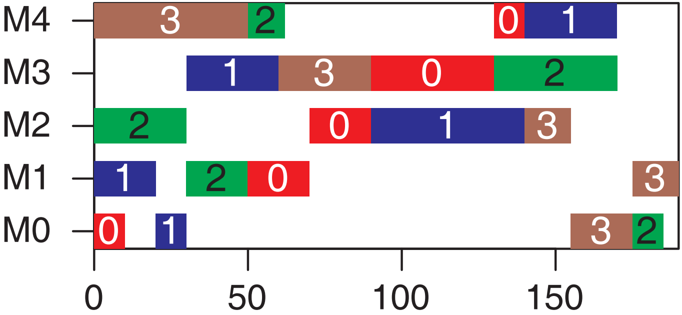
M4
M3
M2
M1
M0
3
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1
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3
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50
100
150

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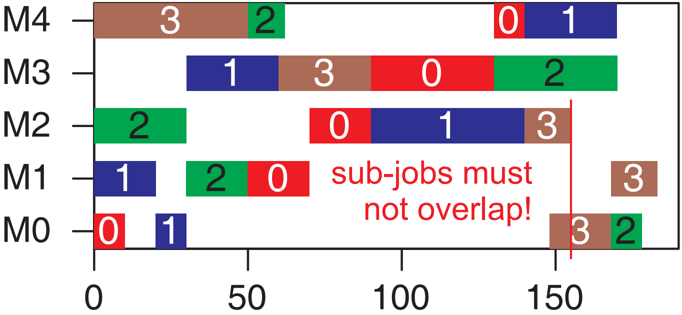

sub-jobs must not overlap!

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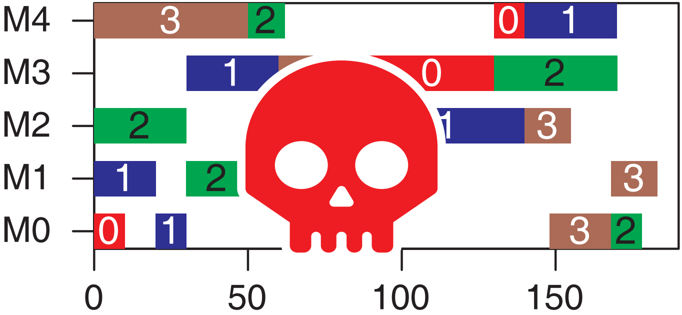

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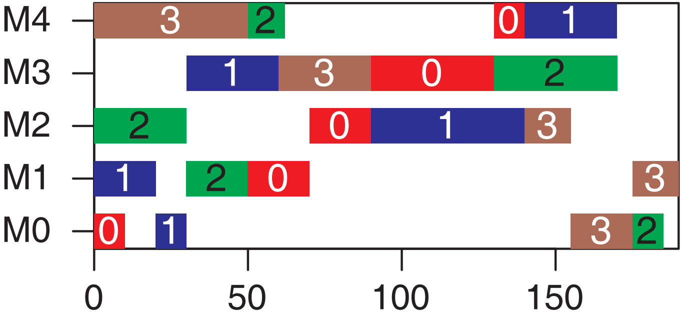

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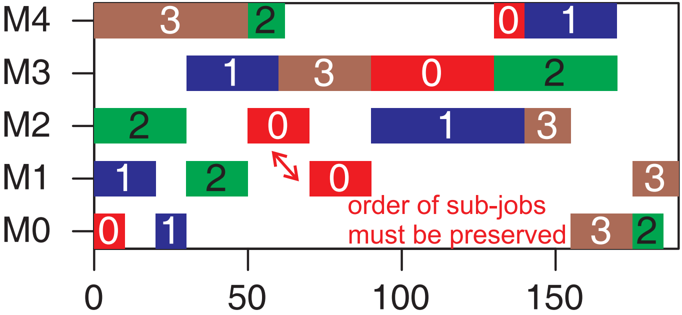

order of sub-jobs must be preserved

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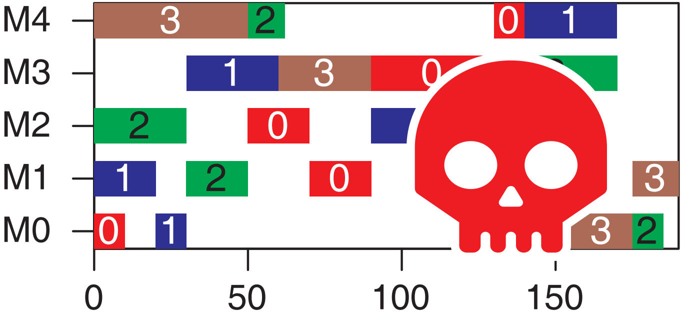

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- Only a Gantt chart obeying all of these constraints is feasible, i.e., can be implemented in practice.

Searching in $\mathbb{Y}$

- So how do we search in the space of Gantt charts?

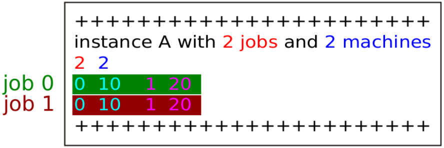
Searching in $\mathbb{Y}$

- So how do we search in the space of Gantt charts?
- We need to create Gantt charts that fulfill all the constraints.

Searching in $\mathbb{Y}$

- So how do we search in the space of Gantt charts?
- We need to create Gantt charts that fulfill all the constraints.
- For different *instances*, different solutions are feasible!

Searching in $\mathbb{Y}$



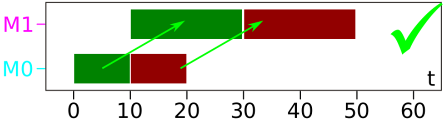
```
+++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10    1  20
0  10    1  20
+++++++++++++++++++++++++
```

Searching in $\mathbb{Y}$

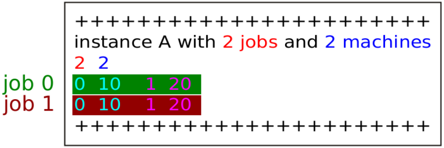
```
+++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10    1  20
0  10    1  20
+++++++++++++++++++++++
```

job 0
job 1

M0: Job 0, Job 1; M1: Job 0, Job 1

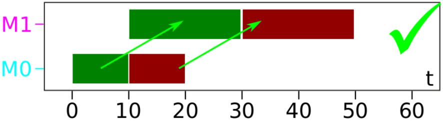


Searching in $\mathbb{Y}$

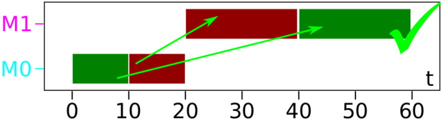


```
+++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2 2
0 10   1 20
0 10   1 20
+++++++++++++++++++++++++
```

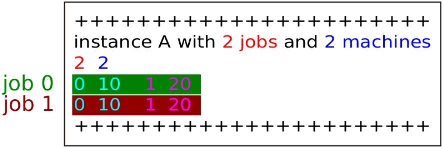
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0

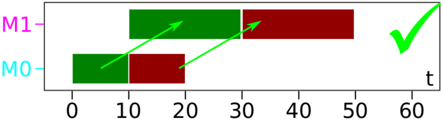


Searching in $\mathbb{Y}$

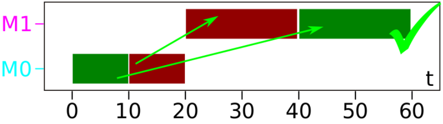


```
+++++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10   1  20
0  10   1  20
+++++++++++++++++++++++++
```

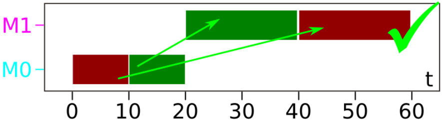
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



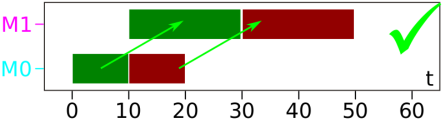
M0: Job 1, Job 0; M1: Job 0, Job 1

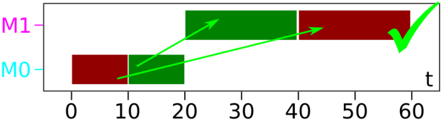


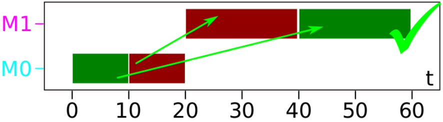
Searching in $\mathbb{Y}$

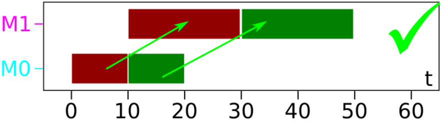
job 0
job 1

```
+++++++++++++++++++++++
instance A with 2 jobs and 2 machines
2  2
0  10    1  20
0  10    1  20
+++++++++++++++++++++++
```

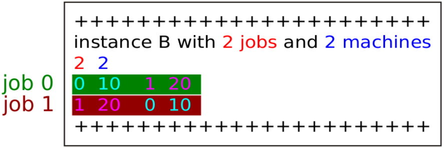

M0: Job 0, Job 1; M1: Job 0, Job 1


M0: Job 1, Job 0; M1: Job 0, Job 1


M0: Job 0, Job 1; M1: Job 1, Job 0


M0: Job 1, Job 0; M1: Job 1, Job 0

Searching in 𝕐



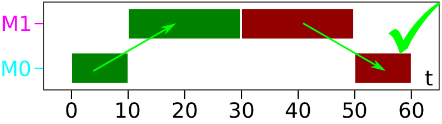
```
+++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
+++++++++++++++++++++++
```

Searching in $\mathbb{Y}$

job 0
job 1

```
+++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
+++++++++++++++++++++++
```

M0: Job 0, Job 1; M1: Job 0, Job 1

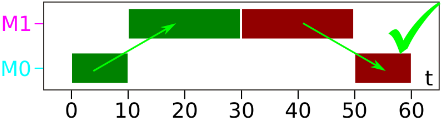


Searching in $\mathbb{Y}$

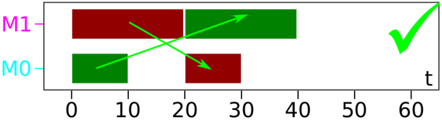
job 0
job 1

```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
++++++++++++++++++++++++
```

M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



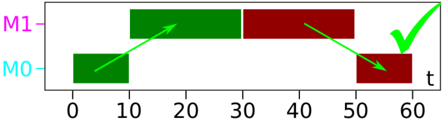
Searching in $\mathbb{Y}$

```
++++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
++++++++++++++++++++++++++
```

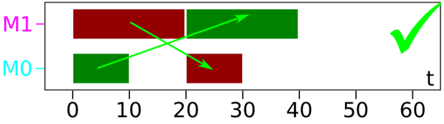
job 0
job 1

Machine 0 should begin by doing job 1.

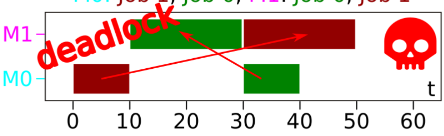
M0: Job 0, Job 1; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



M0: Job 1, Job 0; M1: Job 0, Job 1



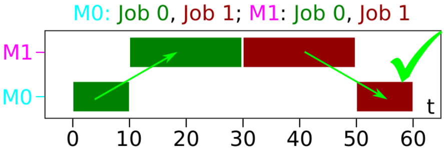
Searching in $\mathbb{Y}$

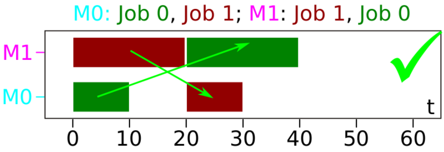
```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2 2
0 10   1 20
1 20   0 10
+++++++++++++++++++++++++
```

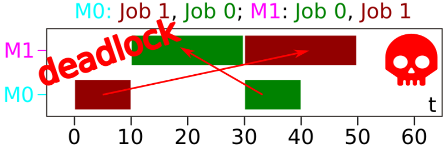
job 0
job 1

Machine 0 should begin by doing job 1.

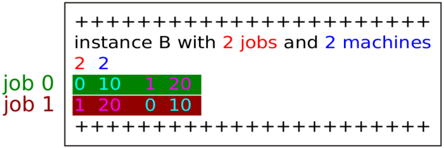
Job 1 can only start on machine 0 after it has been finished on machine 1.


M0: Job 0, Job 1; M1: Job 0, Job 1


M0: Job 0, Job 1; M1: Job 1, Job 0

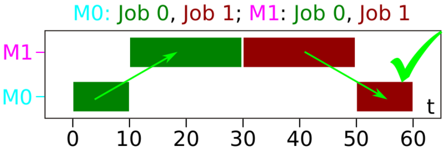

M0: Job 1, Job 0; M1: Job 0, Job 1

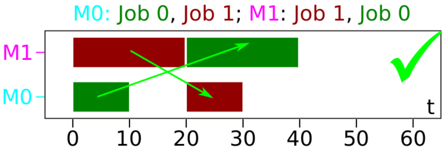
Searching in $\mathbb{Y}$

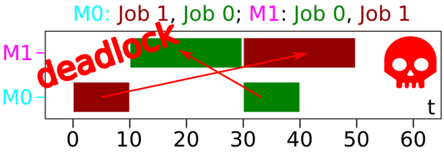


```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
+++++++++++++++++++++++++
```

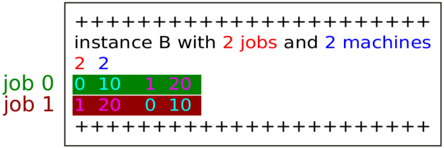
Machine 0 should begin by doing job 1. Job 1 can only start on machine 0 after it has been finished on machine 1. At machine 1, we should begin with job 0.





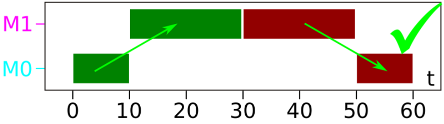


Searching in $\mathbb{Y}$

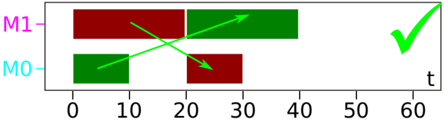


```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2 2
0 10   1 20
1 20   0 10
++++++++++++++++++++++++
```

M0: Job 0, Job 1; M1: Job 0, Job 1

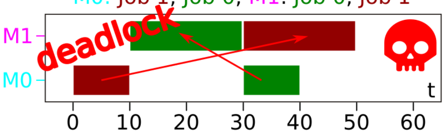


M0: Job 0, Job 1; M1: Job 1, Job 0

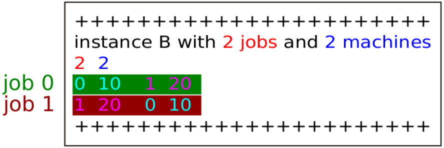


Job 1 can only start on machine 0 after it has been finished on machine 1. At machine 1, we should begin with job 0. Before job 0 can be put on machine 1, it must go through machine 0.

M0: Job 1, Job 0; M1: Job 0, Job 1



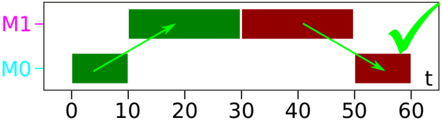
Searching in $\mathbb{Y}$



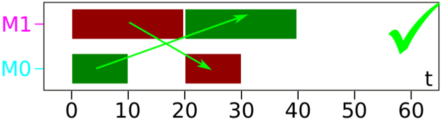
```
++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2 2
0 10  1 20
1 20  0 10
++++++++++++++++++++++++
```

So job 1 cannot go to machine 0 until it has passed through machine 1, but in order to be executed on machine 1, job 0 needs to be finished there first.

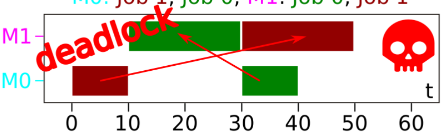
M0: Job 0, Job 1; M1: Job 0, Job 1



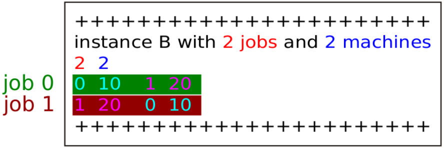
M0: Job 0, Job 1; M1: Job 1, Job 0



M0: Job 1, Job 0; M1: Job 0, Job 1

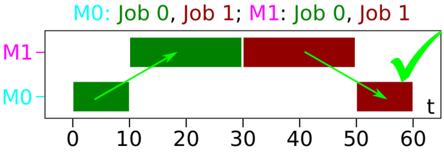


Searching in $\mathbb{Y}$

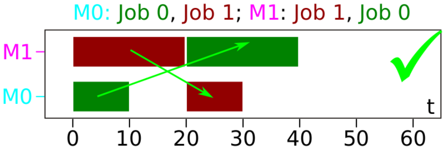


```
job 0
job 1
+++++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
+++++++++++++++++++++++++++
```

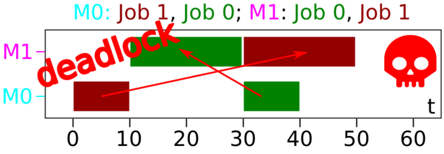
Job 0 cannot begin on machine 1 until it has been passed through machine 0, but it cannot be executed there, because job 1 needs to be finished there first.



M0: Job 0, Job 1; M1: Job 0, Job 1

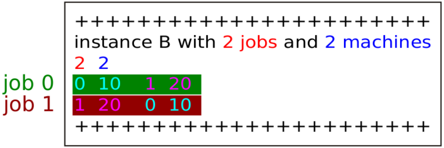


M0: Job 0, Job 1; M1: Job 1, Job 0



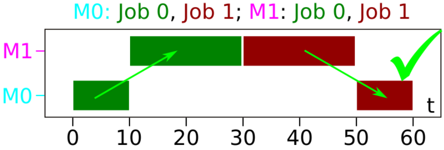
M0: Job 1, Job 0; M1: Job 0, Job 1

Searching in $\mathbb{Y}$

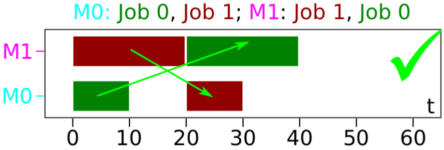


```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
+++++++++++++++++++++++++
```

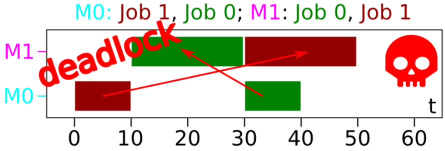
A cyclic blockage has appeared: no job can be executed on any machine if we follow this schedule.



M0: Job 0, Job 1; M1: Job 0, Job 1

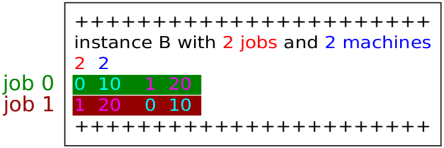


M0: Job 0, Job 1; M1: Job 1, Job 0



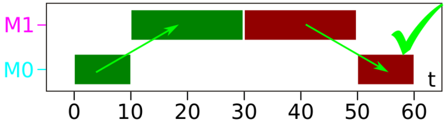
M0: Job 1, Job 0; M1: Job 0, Job 1

Searching in $\mathbb{Y}$

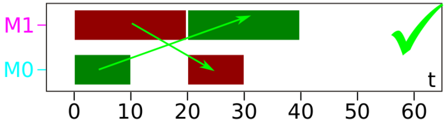


```
+++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10   1  20
1  20   0  10
+++++++++++++++++++++++++
```

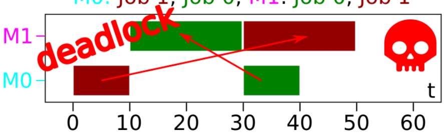
A cyclic blockage has appeared: no job can be executed on any machine if we follow this schedule. This is called a deadlock.



M0: Job 0, Job 1; M1: Job 0, Job 1

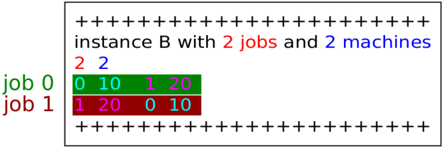


M0: Job 0, Job 1; M1: Job 1, Job 0



M0: Job 1, Job 0; M1: Job 0, Job 1

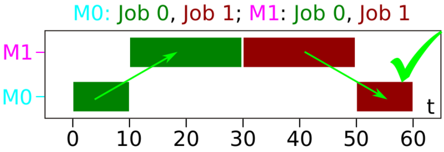
Searching in $\mathbb{Y}$

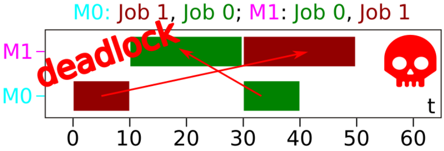


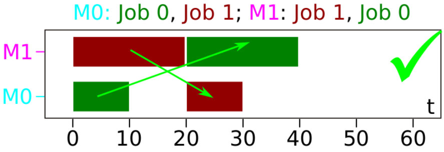
job 0
job 1

```
+++++++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
+++++++++++++++++++++++++++
```

This is called a deadlock. The schedule is infeasible, because it cannot be executed or written down without breaking the precedence constraint.





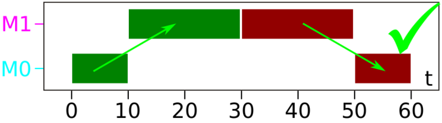


Searching in 𝕐

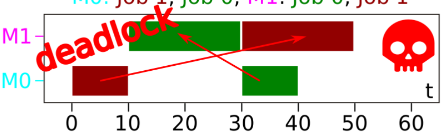
job 0
job 1

```
+++++++++++++++++++++++
instance B with 2 jobs and 2 machines
2  2
0  10    1  20
1  20    0  10
+++++++++++++++++++++++
```

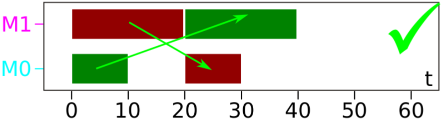
M0: Job 0, Job 1; M1: Job 0, Job 1



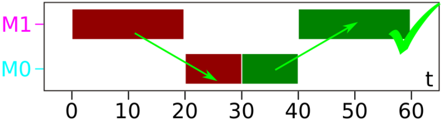
M0: Job 1, Job 0; M1: Job 0, Job 1



M0: Job 0, Job 1; M1: Job 1, Job 0



M0: Job 1, Job 0; M1: Job 1, Job 0



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- Solution: We develop a data structure $\mathbb{X}$ which we can handle easily and which can always be translated to feasible Gantt charts by a mapping $\gamma : \mathbb{X} \mapsto \mathbb{Y}$.

The Search Space $\mathbb{X}$

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- So how could a simple search space $\mathbb{X}$ for the JSSP look like?
- Let us revisit the demo problem instance.

```
++++++++++++++++++++++++++++++
A simple demo
4 5
0 10 1 20 2 20 3 40 4 10
1 20 0 10 3 30 2 50 4 30
2 30 1 20 4 12 3 40 0 10
4 50 3 30 2 15 0 20 1 15
++++++++++++++++++++++++++++++
```

number of jobs

number of machines

job 0

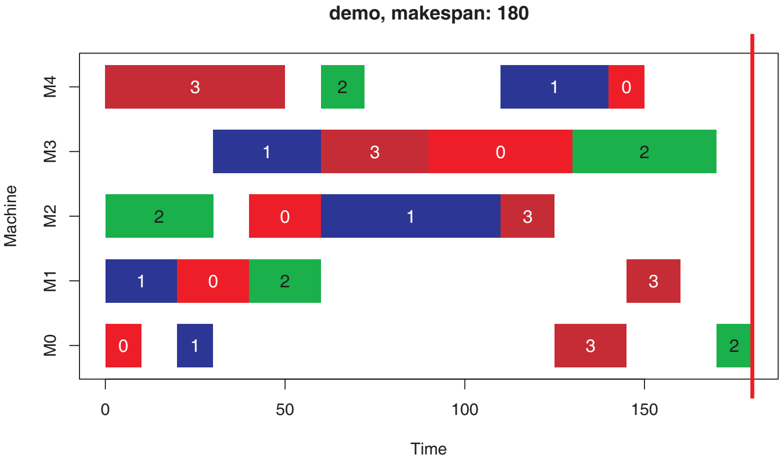
job 1

job 2

job 3

This is information that we have, which does not need to be stored in the elements x.

The Search Space $\mathbb{X}$



The instance data and the data from one point x should encode such a Gantt chart.

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- This way, we will always have the sub-jobs in the right order.

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$$x = \begin{array}{l}(3,\ 0,\ 2,\ 1,\ 0,\\ \ 1,\ 0,\ 1,\ 2,\ 3,\\ \ 2,\ 1,\ 1,\ 2,\ 3,\\ \ 0,\ 2,\ 0,\ 3,\ 3)\end{array}$$

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job 1 1 20 0 10 3 30 2 50 4 30
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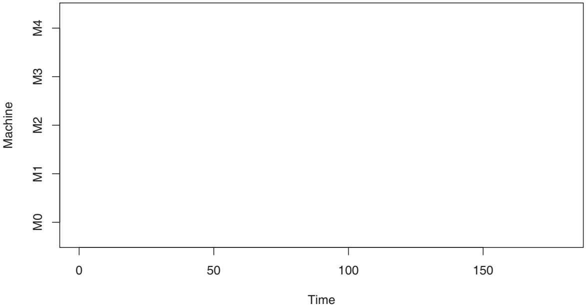
      +++++++++++++++++++++++++++++
```



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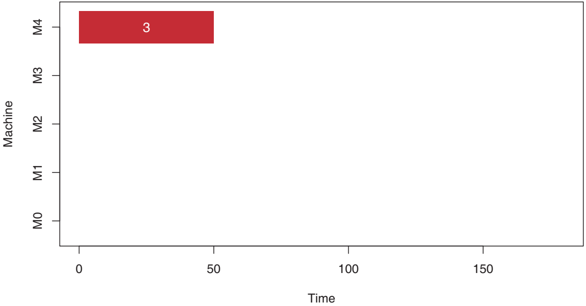

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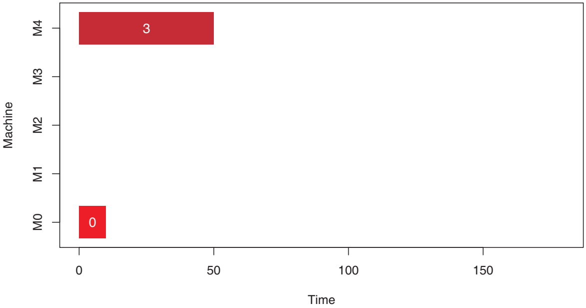

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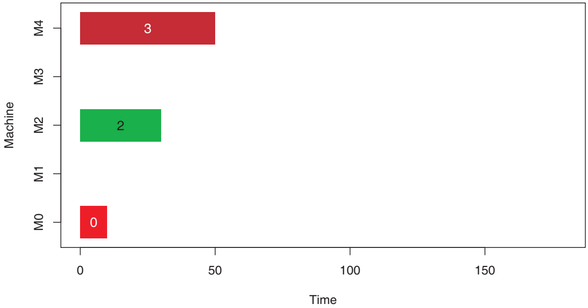

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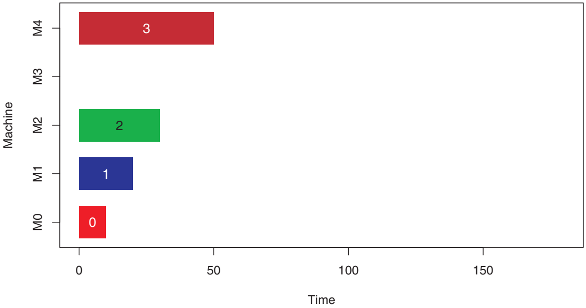

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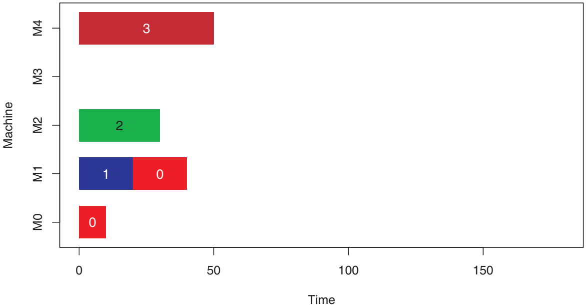

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$x =$

```
(3, 0, 2, 1, 0,
 1, 0, 1, 2, 3,
 2, 1, 1, 2, 3,
 0, 2, 0, 3, 3)
```

```
      +++++++++++++++++++++++++++++
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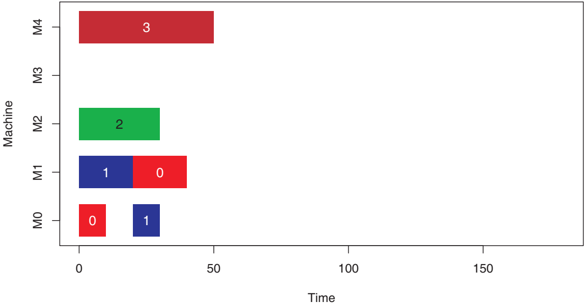

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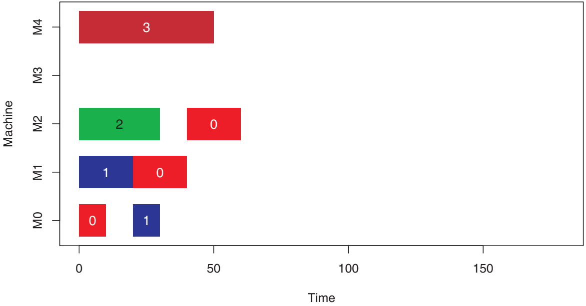

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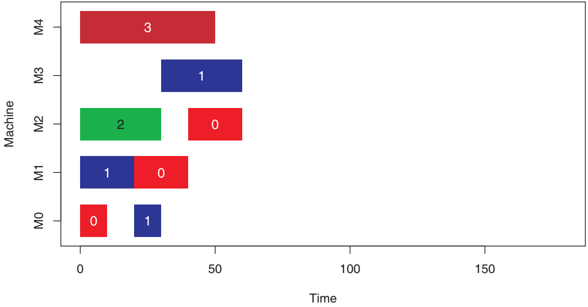

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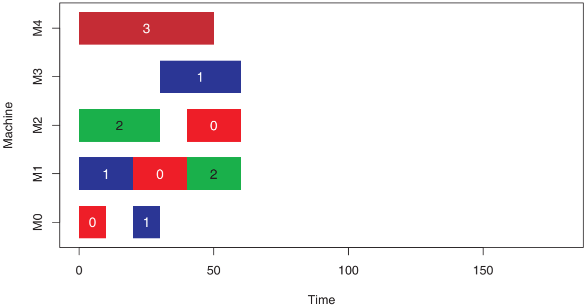

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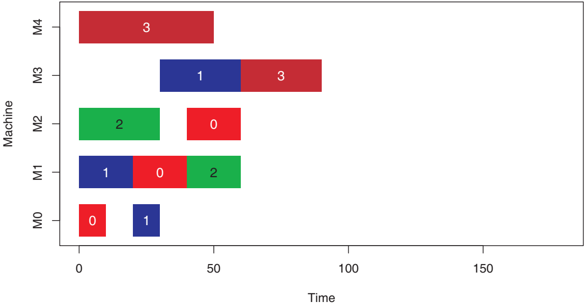

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```
      +++++++++++++++++++++++++++++
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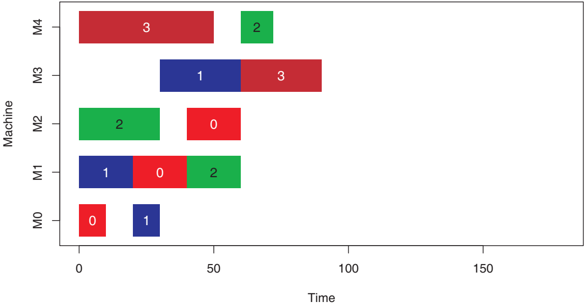

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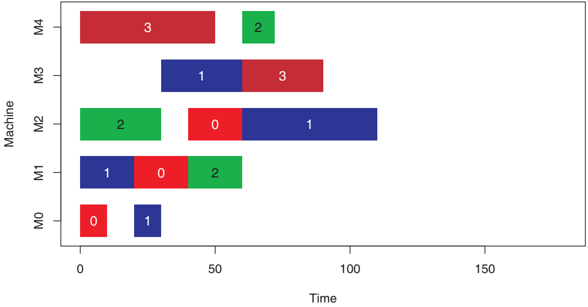

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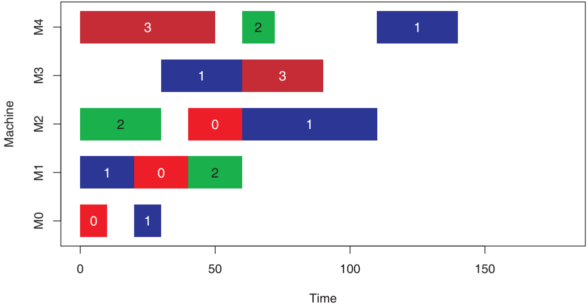

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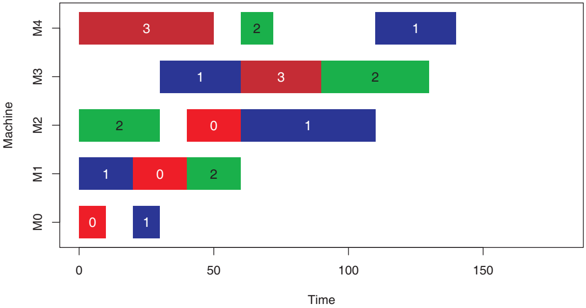

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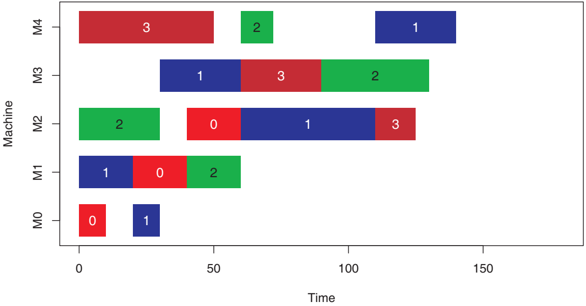

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       A simple demo
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job 0  0 10 1 20 2 20 3 40 4 10
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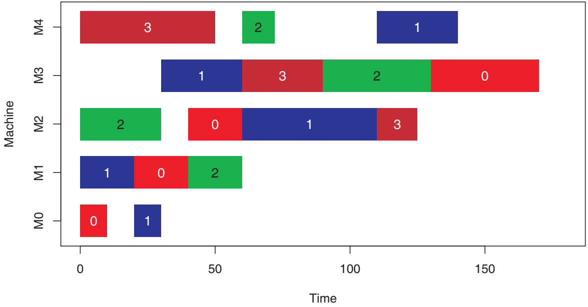

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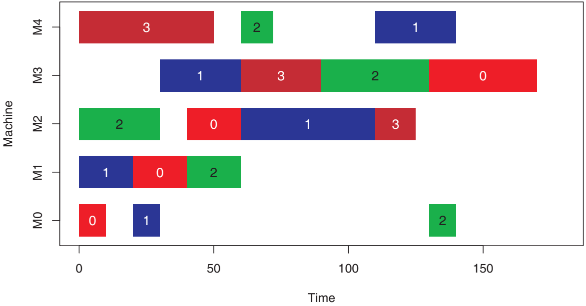

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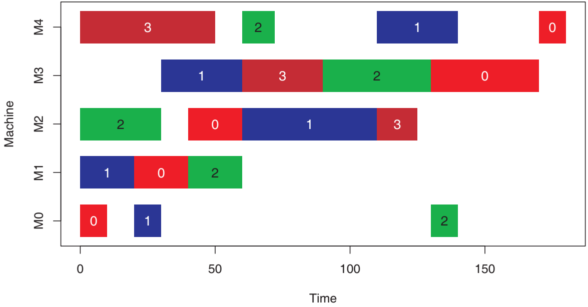

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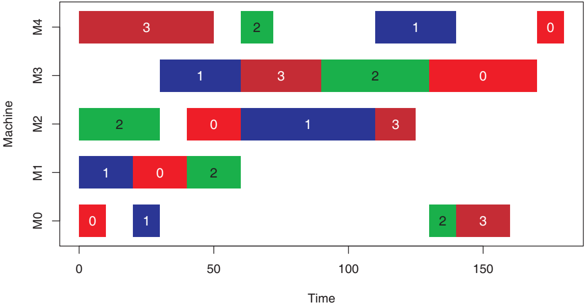
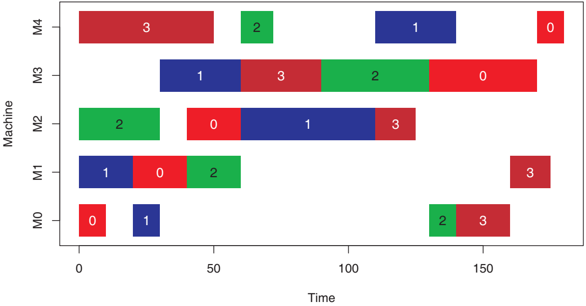


Demo Example for the Search Space

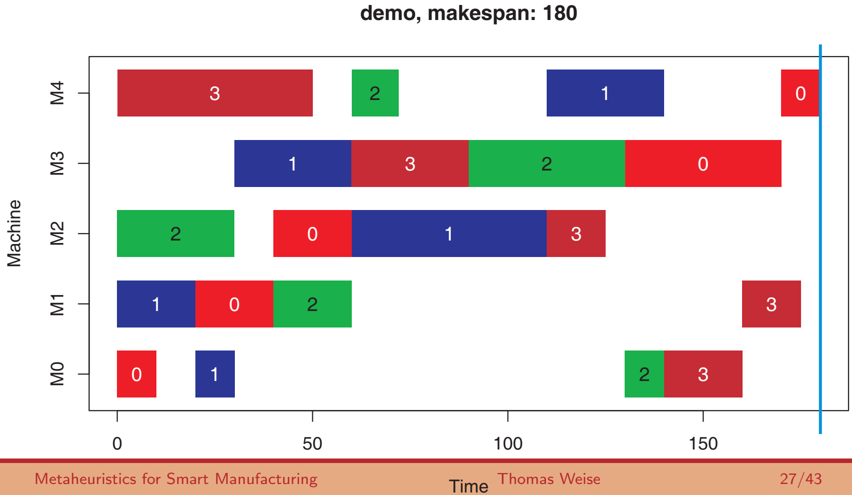
$$x = \begin{array}{l}(3,\ 0,\ 2,\ 1,\ 0, \\ \ 1,\ 0,\ 1,\ 2,\ 3, \\ \ 2,\ 1,\ 1,\ 2,\ 3, \\ \ 0,\ 2,\ 0,\ 3,\ 3)\end{array}$$

```
      +++++++++++++++++++++++++++++
      A simple demo
      4 5
job 0 0 10 1 20 2 20 3 40 4 10
job 1 1 20 0 10 3 30 2 50 4 30
job 2 2 30 1 20 4 12 3 40 0 10
job 3 4 50 3 30 2 15 0 20 1 15
      +++++++++++++++++++++++++++++
```



Demo Example for the Search Space

$$x = \begin{matrix}(3, & 0, & 2, & 1, & 0, & 1, & 0, & 1, & 2, & 3, \\ 2, & 1, & 1, & 2, & 3, & 0, & 2, & 0, & 3, & 3)\end{matrix}$$



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- We call this the representation.
- If necessary, we could also easily add more constraints, such as job-order specific machine setup times, or job/machine specific transport times – they would all go into the mapping γ.

An Interface for Representation Mappings in `Java`

Listing: An interface for representation mappings.

```java
public interface IRepresentationMapping<X, Y> {

  public abstract void map(X x, Y y);
}
```

The JSSP Representation Mapping in Java

Listing: The JSSP representation mapping.

```
public final class JSSPRepresentationMapping implements
    IRepresentationMapping<int[], JSSPCandidateSolution> {

  public void map(int[] x, JSSPCandidateSolution y) {
    int[] machineState = this.m_machineState; int[] machineTime = this.m_machineTime;
    int[] jobState = this.m_jobState;         int[] jobTime = this.m_jobTime;
    Arrays.fill(machineState, 0);             Arrays.fill(jobState, 0);
    Arrays.fill(machineTime, 0);              Arrays.fill(jobTime, 0);

    for (final int nextJob : x) {
      int[] jobSteps = this.m_jobs[nextJob];
      int   jobStep  = (jobState[nextJob]++) << 1;
      int   machine  = jobSteps[jobStep];

      int   start      = Math.max(machineTime[machine], jobTime[nextJob]);
      int   end        = start + jobSteps[jobStep + 1];
      jobTime[nextJob] = machineTime[machine] = end;

      int[] schedule = y.schedule[machine];
      schedule[machineState[machine]++] = nextJob;
      schedule[machineState[machine]++] = start;
      schedule[machineState[machine]++] = end;
    }
  }
}
```

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4. From Solution Space to Search Space
5. **Number of Solutions and Termination**
6. Summary

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- If there are 2 machines, this gives us $(n!) * (n!) = (n!)^2$ choices.
- For m machines, we are at $(n!)^m$ possible solutions.
- But some may be wrong, i.e., contain deadlocks!

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name	n	m	$\min(\#\text{feasible})$	$\lvert\mathbb{Y}\rvert$
	2	2	3	4

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	2	3	4	8

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name	n	m	$\min(\#\text{feasible})$	$\lvert\mathbb{Y}\rvert$
	2	2	3	4
	2	3	4	8
	2	4	5	16

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name	n	m	$\min(\#\text{feasible})$	$\lvert\mathbb{Y}\rvert$
	2	2	3	4
	2	3	4	8
	2	4	5	16
	2	5	6	32

Number of Solutions: Size of $\mathbb{Y}$

name	n	m	$\min(\#\text{feasible})$	$\vert\mathbb{Y}\vert$
	2	2	3	4
	2	3	4	8
	2	4	5	16
	2	5	6	32
	3	2	22	36
	3	3	63	216
	3	4	147	1'296
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	4	2	244	576
	4	3	1'630	13'824
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	3	2	36	90
	3	3	216	1'680
	3	4	1'296	34'650
	3	5	7'776	756'756
	4	2	576	2'520
	4	3	13'824	369'600
	4	4	331'776	63'063'000
	5	2	14'400	113'400
	5	3	1'728'000	168'168'000
	5	4	207'360'000	305'540'235'000
	5	5	24'883'200'000	623'360'743'125'120
`demo`	4	5	7'962'624	11'732'745'024
`la24`	15	10	$\approx 1.462{*}10^{121}$	$\approx 2.293{*}10^{164}$
`abz7`	20	15	$\approx 6.193{*}10^{275}$	$\approx 1.432{*}10^{372}$
`yn4`	20	20	$\approx 5.278{*}10^{367}$	$\approx 1.213{*}10^{501}$
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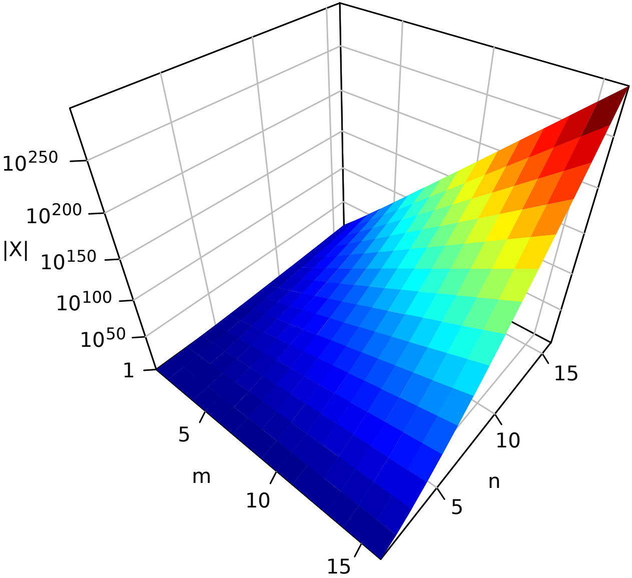

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- How many points are in our representations of the solution space?
- Both $\mathbb{X}$ and $\mathbb{Y}$ are very big for any relevant problem size.
- $\mathbb{X}$ is bigger, we pay with size for the simplicity and the avoidance of infeasible solutions.

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- This is called the *termination criterion*.

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- We assume that a human operator receives the job information, enters them into a computer (as JSSP instance), and then goes to drink a coffee.

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- If we have this, we can directly use any of the algorithms in the rest of the lecture (almost) as-is.

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- Let us now try to solve the JSSP using metaheuristics that search inside $\mathbb{X}$ (and thus can find solutions in $\mathbb{Y}$ within 3 minutes).

谢谢

Thank you

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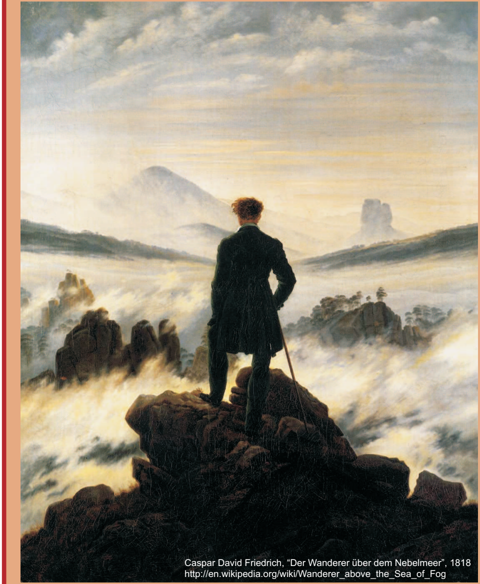

Caspar David Friedrich, “Der Wanderer über dem Nebelmeer”, 1818
http://en.wikipedia.org/wiki/Wanderer_above_the_Sea_of_Fog

Bibliography

Bibliography I

1. Thomas Weise. *An Introduction to Optimization Algorithms*. Institute of Applied Optimization (IAO), Faculty of Computer Science and Technology, Hefei University, Hefei, Anhui, China, 2019-06-25 edition, 2018–2019. URL `http://thomasweise.github.io/aitoa/`. see also [2].
2. Thomas Weise. *Global Optimization Algorithms – Theory and Application*. it-weise.de (self-published), Germany, 2009. URL `http://www.it-weise.de/projects/book.pdf`.
3. Fred Glover and Gary A. Kochenberger, editors. *Handbook of Metaheuristics*, volume 57 of *International Series in Operations Research & Management Science (ISOR)*. Springer Netherlands, Dordrecht, Netherlands, 2003. ISBN 0-306-48056-5. doi: 10.1007/b101874.
4. Zbigniew Michalewicz and David B. Fogel. *How to Solve It: Modern Heuristics*. Berlin/Heidelberg: Springer-Verlag, 2nd edition, 2004. ISBN 3-540-22494-7, 978-3-540-22494-5, and 978-3-642-06134-9. URL `http://books.google.de/books?id=RJbV_-JlIUQC`.
5. Ronald Lewis Graham, Eugene Leighton Lawler, Jan Karel Lenstra, and Alexander Hendrik George Rinnooy Kan. Optimization and approximation in deterministic sequencing and scheduling: A survey. *Annals of Discrete Mathematics*, 5: 287–326, 1979. doi: 10.1016/S0167-5060(08)70356-X.
6. Eugene Leighton Lawler, Jan Karel Lenstra, Alexander Hendrik George Rinnooy Kan, and David B. Shmoys. Sequencing and scheduling: Algorithms and complexity. In Stephen C. Graves, Alexander Hendrik George Rinnooy Kan, and Paul H. Zipkin, editors, *Handbook of Operations Research and Management Science*, volume IV: Production Planning and Inventory, chapter 9, pages 445–522. North-Holland Scientific Publishers Ltd., Amsterdam, The Netherlands, 1993. doi: 10.1016/S0927-0507(05)80189-6.
7. Eugene Leighton Lawler. Recent results in the theory of machine scheduling. In AAchim Bachem, Bernhard Korte, and Martin Grötschel, editors, *Math Programming: The State of the Art*, chapter 8, pages 202–234. Springer-Verlag, Bonn/New York, 1982. ISBN 978-3-642-68876-8. doi: 10.1007/978-3-642-68874-4_9.
8. Éric D. Taillard. Benchmarks for basic scheduling problems. *European Journal of Operational Research (EJOR)*, 64: 278–285, January 1993. doi: 10.1016/0377-2217(93)90182-M.
9. Jacek Błażewicz, Wolfgang Domschke, and Erwin Pesch. The job shop scheduling problem: Conventional and new solution techniques. *European Journal of Operational Research (EJOR)*, 93:1–33, August 1996. doi: 10.1016/0377-2217(95)00362-2. URL `http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.159.1650&type=pdf`.

Bibliography II

10. Richard M. Karp. Reducibility among combinatorial problems. In Raymond E. Miller and James W. Thatcher, editors, *Complexity of Computer Computations. The IBM Research Symposia Series.*, pages 85–103. Springer, Boston, MA, USA, 1972. ISBN 978-1-4684-2003-6. doi: 10.1007/978-1-4684-2001-2_9.
11. Stephen Arthur Cook. The complexity of theorem-proving procedures. In *Proceedings of the Third Annual ACM Symposium on Theory of Computing (STOC'71), May 3–5, 1971, Shaker Heights, OH, USA*, pages 151–158, New York, NY, USA, 1971. ACM. doi: 10.1145/800157.805047.
12. John Edward Beasley. Or-library: Distributing test problems by electronic mail. *The Journal of the Operational Research Society (JORS)*, 41:1069–1072, November 1990. doi: 10.1057/jors.1990.166.
13. Jelke Jeroen van Hoorn. Job shop instances and solutions, 2015. URL `http://jobshop.jjvh.nl`.
14. Jelke Jeroen van Hoorn. The current state of bounds on benchmark instances of the job-shop scheduling problem. *Journal of Scheduling*, 21:127–128, feb 2018. doi: 10.1007/s10951-017-0547-8.
15. Joseph Adams, Egon Balas, and Daniel Zawack. The shifting bottleneck procedure for job shop scheduling. *Management Science*, 34:391–401, 1988. doi: 10.1287/mnsc.34.3.391.
16. Stephen R. Lawrence. *Resource Constrained Project Scheduling: An Experimental Investigation of Heuristic Scheduling Techniques (Supplement)*. PhD thesis, Graduate School of Industrial Administration (GSIA), Carnegie-Mellon University, Pittsburgh, Pennsylvania, USA, 1984.
17. Robert H. Storer, S. David Wu, and Renzo Vaccari. New search spaces for sequencing problems with application to job shop scheduling. *Management Science*, 38:1495–1509, 1992. doi: 10.1287/mnsc.38.10.1495.
18. Takeshi Yamada and Ryohei Nakano. A genetic algorithm applicable to large-scale job-shop instances. In Reinhard Männer and Bernard Manderick, editors, *Proceedings of Parallel Problem Solving from Nature 2 (PPSN II), September 28–30, 1992, Brussels, Belgium*, pages 281–290, Amsterdam, The Netherlands, 1992. Elsevier.
19. James M. Wilson. Gantt charts: A centenary appreciation. *European Journal of Operational Research (EJOR)*, 149: 430–437, September 2003. doi: 10.1016/S0377-2217(02)00769-5. URL `http://www-public.imtbs-tsp.eu/~gibson/Teaching/Teaching-ReadingMaterial/Wilson03.pdf`.
20. Robert Klein. *Scheduling of Resource-Constrained Projects*, volume 10 of *Operations Research/Computer Science Interfaces Series*. Springer US, New York, NY, USA, 2000. ISBN 978-0-7923-8637-7. doi: 10.1007/978-1-4615-4629-0.
21. Christian Bierwirth. A generalized permutation approach to job shop scheduling with genetic algorithms. *Operations-Research-Spektrum (OR Spectrum)*, 17:87–92, June 1995. doi: 10.1007/BF01719250. URL `http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.52.7392&type=pdf`.

Bibliography III

22. Christian Bierwirth, Dirk C. Mattfeld, and Herbert Kopfer. On permutation representations for scheduling problems. In Hans-Michael Voigt, Werner Ebeling, Ingo Rechenberg, and Hans-Paul Schwefel, editors, *Proceedings of the 4th International Conference on Parallel Problem Solving from Nature (PPSN IV), September 22–24, 1996, Berlin, Germany*, volume 1141/1996 of *Lecture Notes in Computer Science (LNCS)*, pages 310–318, Berlin, Germany, 1996. Springer-Verlag GmbH. ISBN 3-540-61723-X. doi: 10.1007/3-540-61723-X_995. URL `http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.51.8377&type=pdf`.
23. Bo Chen, Chris N. Potts, and Gerhard J. Woeginger. A review of machine scheduling: Complexity, algorithms and approximability. In Ding-Zhu Du and Panos M. Pardalos, editors, *Handbook of Combinatorial Optimization*, pages 1493–1641. Springer-Verlag US, Boston, MA, USA, 1998. ISBN 978-1-4613-7987-4. doi: 10.1007/978-1-4613-0303-9_25. also pages 21–169 in volume 3/3 by Kluwer Academic Publishers.
24. David Paul Williamson, Leslie A. Hall, J. A. Hoogeveen, Cor A. J. Hurkens, Jan Karel Lenstra, Sergey Vasil'evich Sevast'janov, and David B. Shmoys. Short shop schedules. *Operations Research*, 45(2):288–294, March–April 1997. doi: 10.1287/opre.45.2.288.
25. Klaus Jansen, Monaldo Mastrolilli, and Roberto Solis-Oba. Approximation schemes for job shop scheduling problems with controllable processing times. *European Journal of Operational Research (EJOR)*, 167(2):297–319, December 2005. doi: 10.1016/j.ejor.2004.03.025. URL `http://people.idsia.ch/~monaldo/papers/EJOR-varJsp-05.pdf`.
26. Monaldo Mastrolilli and Ola Svensson. Hardness of approximating flow and job shop scheduling problems. *Journal of the ACM (JACM)*, 58(5):20:1–20:32, October 2011. doi: 10.1145/2027216.2027218. URL `http://theory.epfl.ch/osven/Ola%20Svensson_publications/JACM11.pdf`.