Metaheuristic Optimization

18. Ant Colony Optimization

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Outline

1. Introduction
2. Ant Colony Optimization

website

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  5. Many combinatorial problems can be considered as finding the shortest path on a graph. Example: Traveling Salesman Problem
- Dorigo et al. [5] have the idea to use a simulation of the way ants form a path in order to solve optimization problems which can be represented as graphs – Ant Colony Optimization (ACO)

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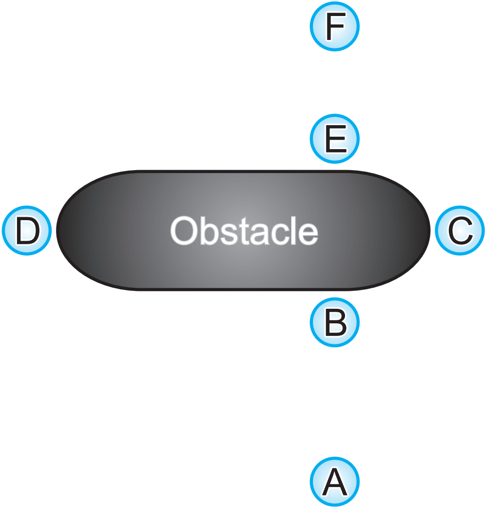
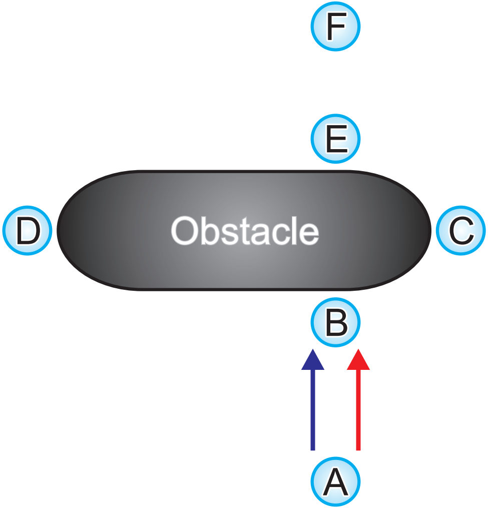
- ant nest (A) separated from food source (F) by obstacle



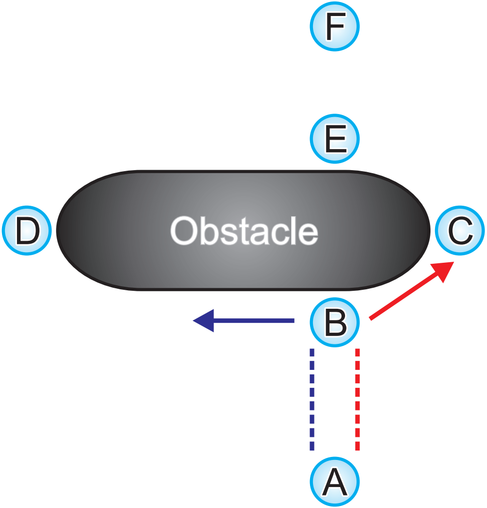
Ant Path Finding

- two ants (red and blue) leave the nest at the same time

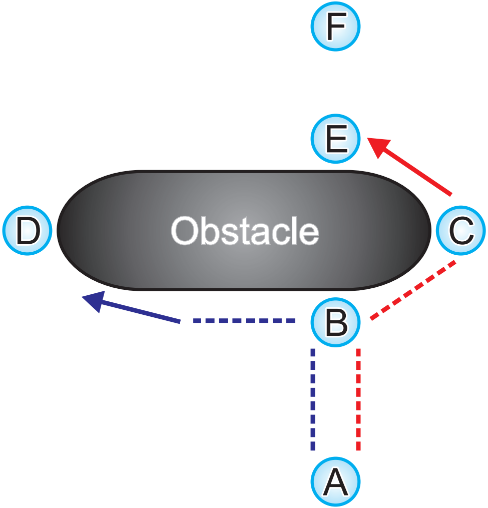


Ant Path Finding

- at the crossroad, one turns left and the other one right

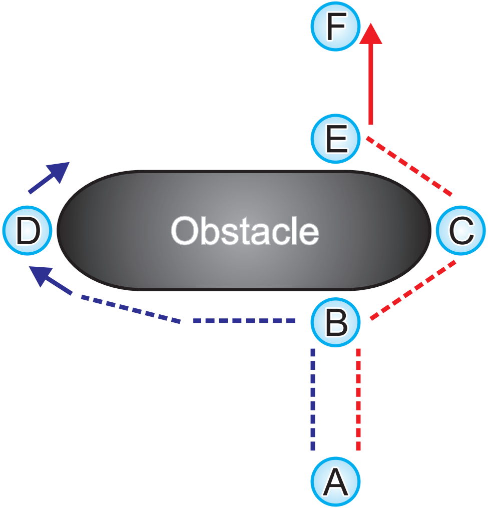


- when moving, ants leave pheromone behind (dotted lines)

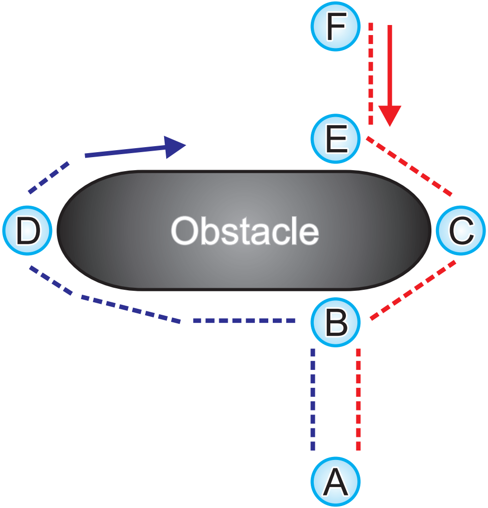


Ant Path Finding

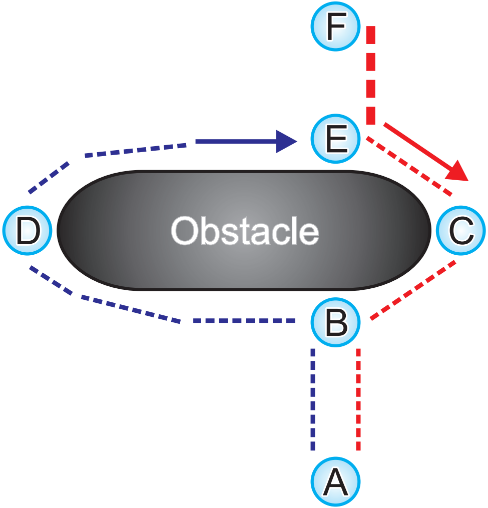
- the one with the shorter path arrives at the food source first



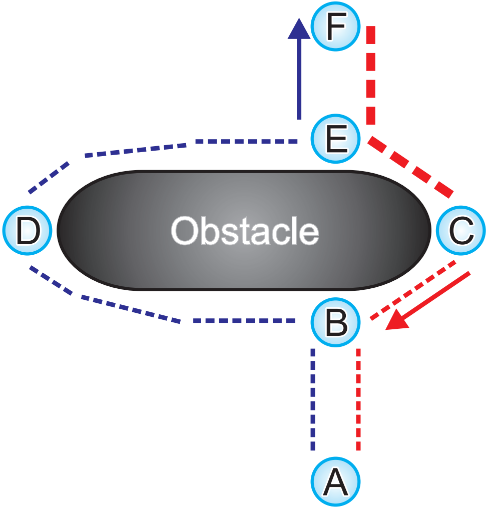
- when it turns back, it finds pheromone on one path and follows it



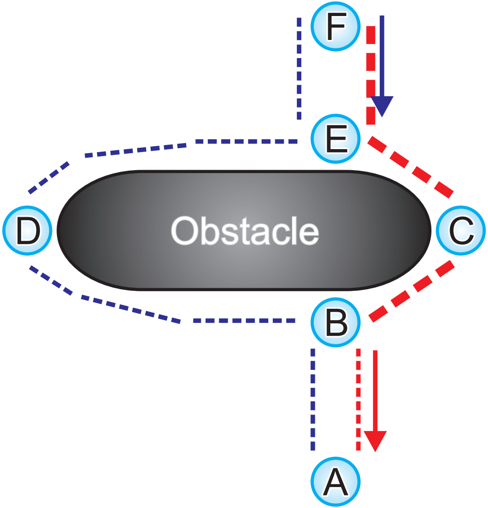
- by doing so, it leaves even more pheromone on the path



- now the second ant arrives at the food source



- when it turns back, there is pheromone on both paths – but more on the red one



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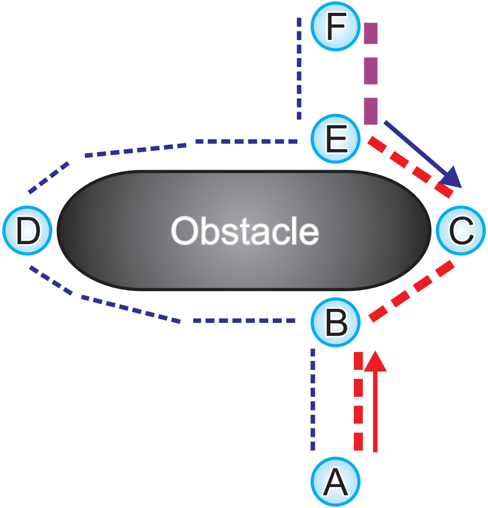

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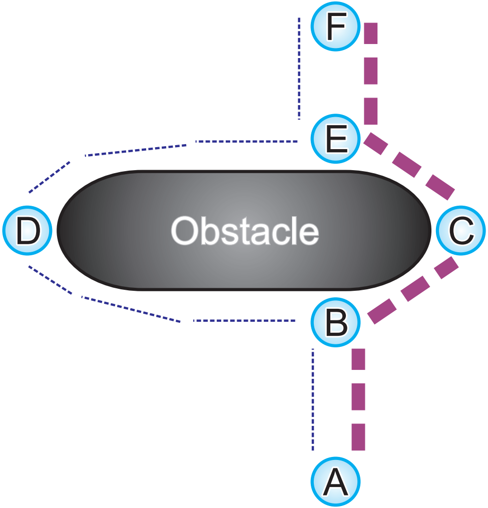
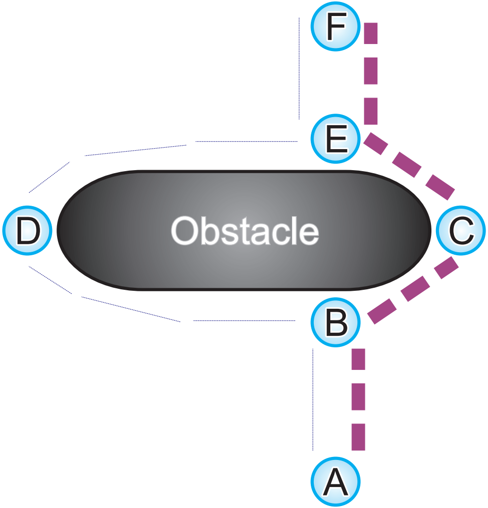
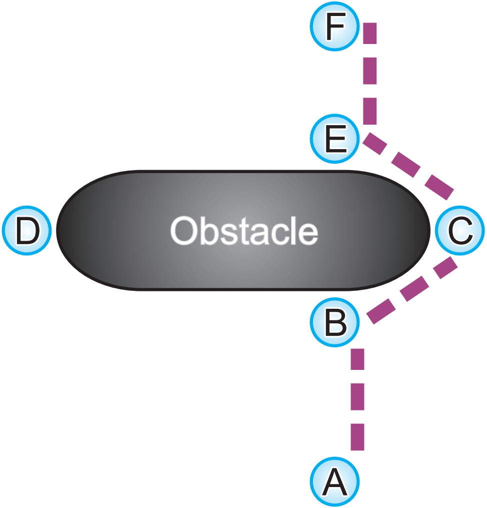


- while the one on the blue path evaporates



- until only the short path has pheromone. . .



ACO on a Graph

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    - based on the objective value of the resulting paths, τ is updated

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$p_{i,j}$	probability of an ant to go to j if at location i	$\tau_{i,j}$	amount of pheromone on the edge connecting i and j
α, β	weight parameters	$\eta_{i,j}$	visibility of node j from i: inversely proportional to distance between j and i

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- Objective Function: Minimize $f(x) = \sum_{i=0}^{4} dist(x[i], x[i+1]) + dist(x[4], x[0])$

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  - after all ants have completed their tour, pheromones are updated

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2. Calculate pheromone amount $\Delta\tau_{i,j}$ to be dispersed on the edge $\overline{ij}$ connecting city i with city j

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1. For each ant k of the ps ants:
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   3. Add the trip back to the starting node $\Longrightarrow$ We get tour x_k
2. Calculate pheromone amount $\Delta\tau_{i,j}$ to be dispersed on the edge $\overline{ij}$ connecting city i with city j as:

$$\sum_{k=1}^{ps} \begin{cases} \frac{1}{f(x_k)} & \text{if tour } x_k \text{ contains edge } \overline{ij} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

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   Alternative method: only the best ant contributes to $\Delta\tau$

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$$\tau_{i,j} = (1-\rho)\tau_{i,j} + \Delta\tau_{i,j} \tag{3}$$

where the evaporation coefficient $\rho \in [0,1]$ lets old pheromone disappear

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New Perspective: Path through Graph $\Leftrightarrow$ Permutation

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New Perspective: Path through Graph $\Leftrightarrow$ Permutation $\Longrightarrow$ ACO is good for permutation-based problems

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- High memory consumption $(\mathcal{O}\left(n^2\right))$ and update step of this matrix is also slow $(\mathcal{O}\left(n^2\right))$...

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  - and is suitable for dynamically changing problems [8]

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  - Swarm Intelligence
  - PSO: Copy flocking behavior
  - ACO: Copy ground-based movements / stigmergy

谢谢

Thank you

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Caspar David Friedrich, “Der Wanderer über dem Nebelmeer”, 1818
http://en.wikipedia.org/wiki/Wanderer_above_the_Sea_of_Fog

Bibliography

Bibliography I

1. Jean-Louis Deneubourg, Jacques M. Pasteels, and J. C. Verhaeghe. Probabilistic behaviour in ants: A strategy of errors? *Journal of Theoretical Biology*, 105(2):259–271, 1983. doi: 10.1016/S0022-5193(83)80007-1.
2. Jean-Louis Deneubourg
and Simon Goss. Collective patterns and decision-making. *Ethology, Ecology & Evolution*, 1(4):295–311, December 1989. URL
`http://www.ulb.ac.be/sciences/use/publications/JLD/53.pdfhttp://www.ulb.ac.be/sciences/use/publications/JLD/`
3. Simon Goss, R. Beckers, Jean-Louis Deneubourg, S. Aron, and Jacques M. Pasteels. How trail laying and trail following can solve foraging problems for ant colonies. In Roger N. Hughes, editor, *NATO Advanced Research Workshop on Behavioural Mechanisms of Food Selection*, volume 20 of *NATO Advanced Science Institutes (ASI) Series G. Ecological Sciences (NATO ASI)*, pages 661–678, Gregynog, Wales, UK, July 17–21, 1989. Berlin, Germany: Springer-Verlag GmbH.
4. T. R. Stickland, Chris M. N. Tofts, and Nigel R. Franks. A path choice algorithm for ants. *Naturwissenschaften – The Science of Nature*, 79(12):567–572, December 1992. doi: 10.1007/BF01131415.
5. Marco Dorigo, Vittorio Maniezzo, and Alberto Colorni. The ant system: Optimization by a colony of cooperating agents. *IEEE Transactions on Systems, Man, and Cybernetics – Part B: Cybernetics*, 26(1):29–41, February 1996. doi: 10.1109/3477.484436. URL `ftp://iridia.ulb.ac.be/pub/mdorigo/journals/IJ.10-SMC96.pdf`.
6. Mark Zlochin, Mauro Birattari, Nicolas Meuleau, and Marco Dorigo. Model-based search for combinatorial optimization: A critical survey. *Annals of Operations Research*, 132(1-4):373–395, November 2004. doi: 10.1023/B:ANOR.0000039526.52305.af.
7. Michael Guntsch and Martin Middendorf. A population based approach for aco. In Stefano Cagnoni, Jens Gottlieb, Emma Hart, Martin Middendorf, and Günther R. Raidl, editors, *Applications of Evolutionary Computing, Proceedings of EvoWorkshops 2002: EvoCOP, EvoIASP, EvoSTIM/EvoPLAN (EvoWorkshops'02)*, volume 2279 of *Lecture Notes in Computer Science (LNCS)*, pages 72–81, Kinsale, Ireland, April 2–4, 2002. Berlin, Germany: Springer-Verlag GmbH. doi: 10.1007/3-540-46004-7_8. URL `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.13.2514`.
8. Michael Guntsch and Martin Middendorf. Applying population based aco to dynamic optimization problems. In Marco Dorigo, Gianni A. Di Caro, and Michael Samples, editors, *From Ant Colonies to Artificial Ants – Proceedings of the Third International Workshop on Ant Colony Optimization (ANTS'02)*, volume 2463/2002 of *Lecture Notes in Computer Science (LNCS)*, pages 111–122, Brussels, Belgium, September 12–14, 2002. Berlin, Germany: Springer-Verlag GmbH. doi: 10.1007/3-540-45724-0_10. URL `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.12.6580`.