合肥学院
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Metaheuristic Optimization

17. Particle Swarm Optimization

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Outline

1. Introduction
2. Basic Phenomena
3. Particle Swarm Optimization

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- Self-Organization
- Swarm Intelligence (SI) methods make use of these phenomena for optimization [1–4]

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Definition (Emergence)

Emergence is the spontaneous assemblence of new properties or structures on a macro-level of a system as a result of the joint behavior of its elements on a micro-level. Emergent properties cannot be traced back to the properties that the elements of a system's micro-level exhibit in an isolated state in an obvious manner.

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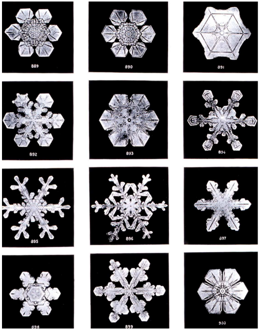

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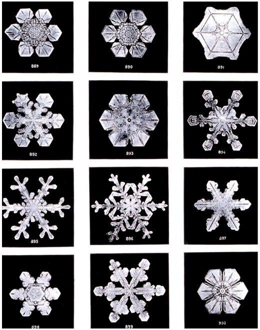

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single termites aggregate pieces of clay, forming a giant nest whose structure is not obviously related to the behavioral pat-terns of a single termite

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A swarm results from the interactions of the single birds without the need of any "lead bird" or controler outside of the swarm

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- Wilson [5] states about fish schools: *"In theory at least, individual members of the school can profit from the discoveries and previous experience of all other members of the school during the search for food. This advantage can become decisive, outweighing the disadvantages of competition for food items, whenever the resource is unpredictably distributed in patches."* [6]
  (school = swarm of fish)

Swarms and PSO

- Wilson [5] states about fish schools: *"In theory at least, individual members of the school can profit from the discoveries and previous experience of all other members of the school during the search for food. This advantage can become decisive, outweighing the disadvantages of competition for food items, whenever the resource is unpredictably distributed in patches."* [6]
  (school = swarm of fish)
- Particle Swarm Optimization (PSO) [6–14] was developed by Eberhart and Kennedy [6, 10, 11] in 1995 to make use of this phenomenon for optimization

- Copy the behavior with which swarms / schools / flocks in nature find food for solving optimization problems

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$$\begin{aligned} p.\vec{v}_i = p.\vec{v}_i + \; & [\{\text{randomly from } [0, c]\} * (\mathrm{best}(p).g_i - p.g_i)] + \\ & [\{\text{randomly from } [0, d]\} * (\mathrm{best}(\mathrm{N}(p)).g_i - p.g_i)] \end{aligned} \quad (1)$$

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- Parameters $c, d \in [0, 1]$ influence convergence speed
- Warning: Velocity may increase without bound ... update must bound velocity into a $[min, max]$ interval!

PSO Algorithm

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$p_{best} \longleftarrow \mathrm{PSO}(f, ps)$

begin
- pop $\longleftarrow$ create population of ps particles
- **while** $\neg shouldTerminate$ **do**
  - **for** $i \longleftarrow 0$ **up to** $ps - 1$ **do**
    - $\mathrm{pop}[i] \longleftarrow \mathrm{psoUpdate}(\mathrm{pop}[i], \mathrm{pop})$
- **return** $best(pop)$

PSO Individual Record

Listing: The PSO Individual Record

```
public class PSOIndividual<X> extends Individual<double[], X> {

  /** the velocity vector */
  public final double[] velocity;

  /** the best position seen by this individual */
  public final Individual<double[], X> best;
}
```

Listing: The PSO Algorithm

```java
public class PSO<X> extends OptimizationAlgorithm<double[], X> {
  public Individual<double[], X> solve(final IObjectiveFunction<X> f) {
    final PSOIndividual<X>[] swarm;
    PSOIndividual<X> cur;
    Individual<double[], X> best;
    double limitV;
    int i, j;

    swarm = new PSOIndividual[this.ps];
    best = new Individual<>();
    best.g = new double[this.rn.dim];

    limitV = 0.1 * (this.rn.max - this.rn.min);

    for (i = swarm.length; (--i) >= 0;) {
      swarm[i] = cur = new PSOIndividual<>(this.nullary.create(this.random));
      cur.x = this.gpm.gpm(cur.g);
      cur.v = f.compute(cur.x);
      copyIndividual(cur.best, cur);
      if (cur.v < best.v) {
        copyIndividual(best, cur);
      }

      if (this.termination.shouldTerminate()) {
        return best;
      }
    }

    for (;;) {

      for (i = swarm.length; (--i) >= 0;) {
        cur = swarm[i];
        for (j = this.rn.dim; (--j) >= 0;) {
          cur.velocity[j] = Math.min(limitV, Math.max(-limitV,
              cur.velocity[j] = ((this.random.nextDouble() * this.c) * (cur.best.g[j] - cur.g[j]))
                  + ((this.random.nextDouble() * this.d) * (best.g[j] - cur.g[j]))));
        }
      }

      for (i = swarm.length; (--i) >= 0;) {
        cur = swarm[i];
        for (j = this.rn.dim; (--j) >= 0;) {
          cur.g[j] = Math.max(this.rn.min, Math.min(this.rn.max, cur.g[j] + cur.velocity[j]));
          cur.x = this.gpm.gpm(cur.g);
          cur.v = f.compute(cur.x);
          if (cur.v < cur.best.v) {
            copyIndividual(cur.best, cur);
            if (cur.v < best.v) {
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- But: PSO is *not* rotationally invariant! [15]
  - It performs well on (axis-parallel) separable functions (potentially better than CMA-ES)
  - But much worse if the same functions are rotated or the problems are non-separable (epistatic) [15]

谢谢
Thank you

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Caspar David Friedrich, “Der Wanderer über dem Nebelmeer”, 1818
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