Metaheuristic Optimization

16. Constraint Handling

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Outline

website

Section Outline

Constraints

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Definition (Feasibility)

In an optimization problem, $q \geq 0$ inequality constraints g and $r \geq 0$ equality constraints h may be imposed additionally to the objective functions. Then, a candidate solution x is *feasible*, if and only if it fulfils all constraints:

$$\text{isFeasible}(x) \Leftrightarrow \begin{array}{rcll} g_i(x) & \geq & 0 & \forall i \in 1 \ldots q \wedge \\ h_j(x) & = & 0 & \forall j \in 1 \ldots r \end{array} \tag{1}$$

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✘ Requires knowledge about "what makes a candidate solution feasible"

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  - The transition from one feasible individual to another one may be unlikely, the objective landscape becomes rugged with large neutral planes at the worst possible fitness levels
  - The information gained from sampling infeasible individuals is lost!

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  1. If $h > 0$ or $h < 0$, there should be a penalty:

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$$\nu(p) = \nu'(p) + \sum_{i=1}^{q} z * [g_i(p.x)]^{-1} \tag{3}$$

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- Good and a nice method for single-objective optimization, but a bit harder to understand in multi-objective scenarios

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✘ Too many objectives may make the problem very hard to solve (many-objective optimization optimization [3, 12–19])

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  3. fulfills none of the goals, i.e.,

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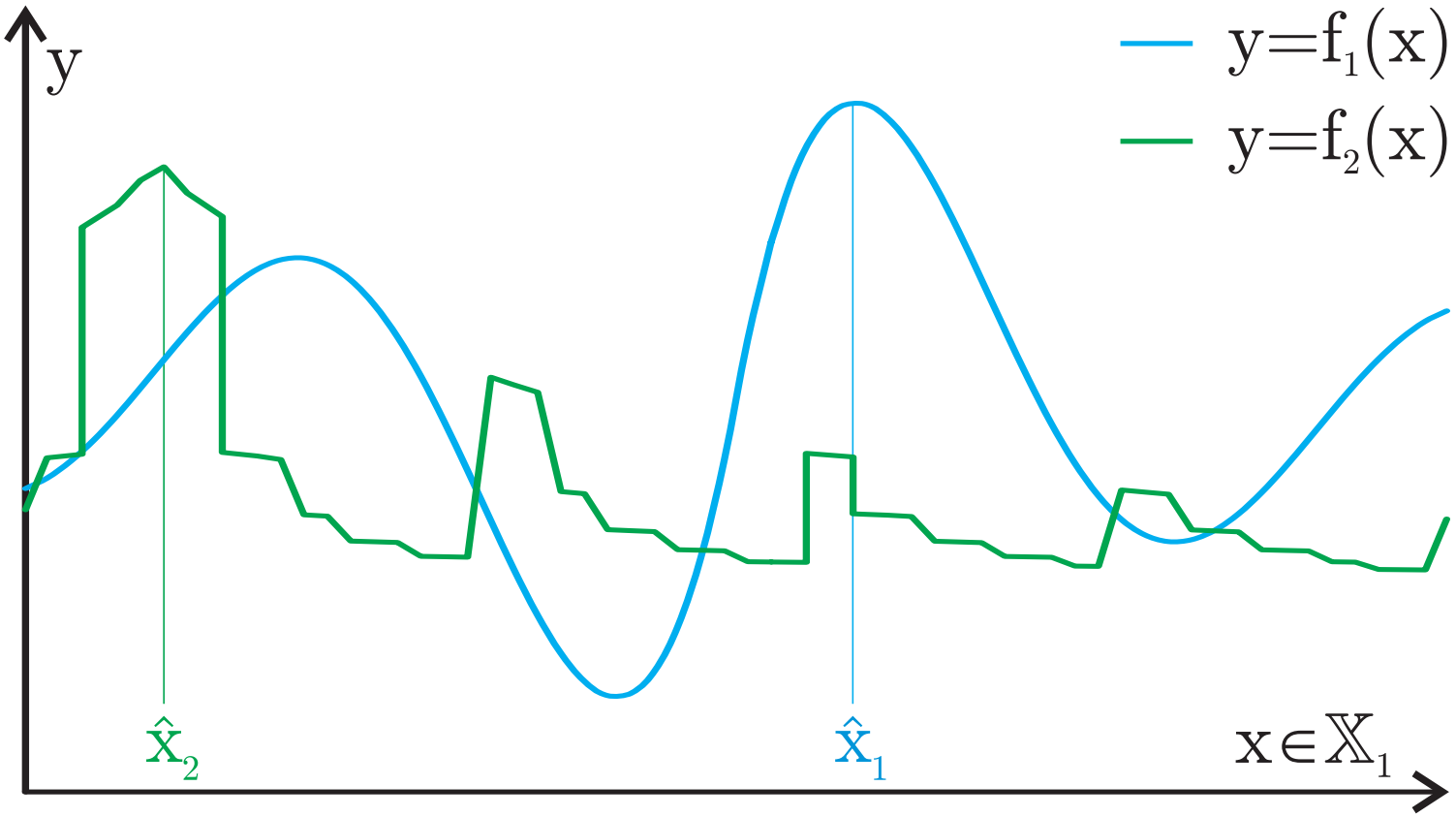
- Pareto ranking is performed based on this comparison method

Example A: Two 1d-Functions

- Two 1-dimensional functions subject to maximization:
  $\vec{f} = \{f_1, f_2\}, f_i : \mathbb{R} \mapsto \mathbb{R} \, \forall i \in \{1, 2\}$



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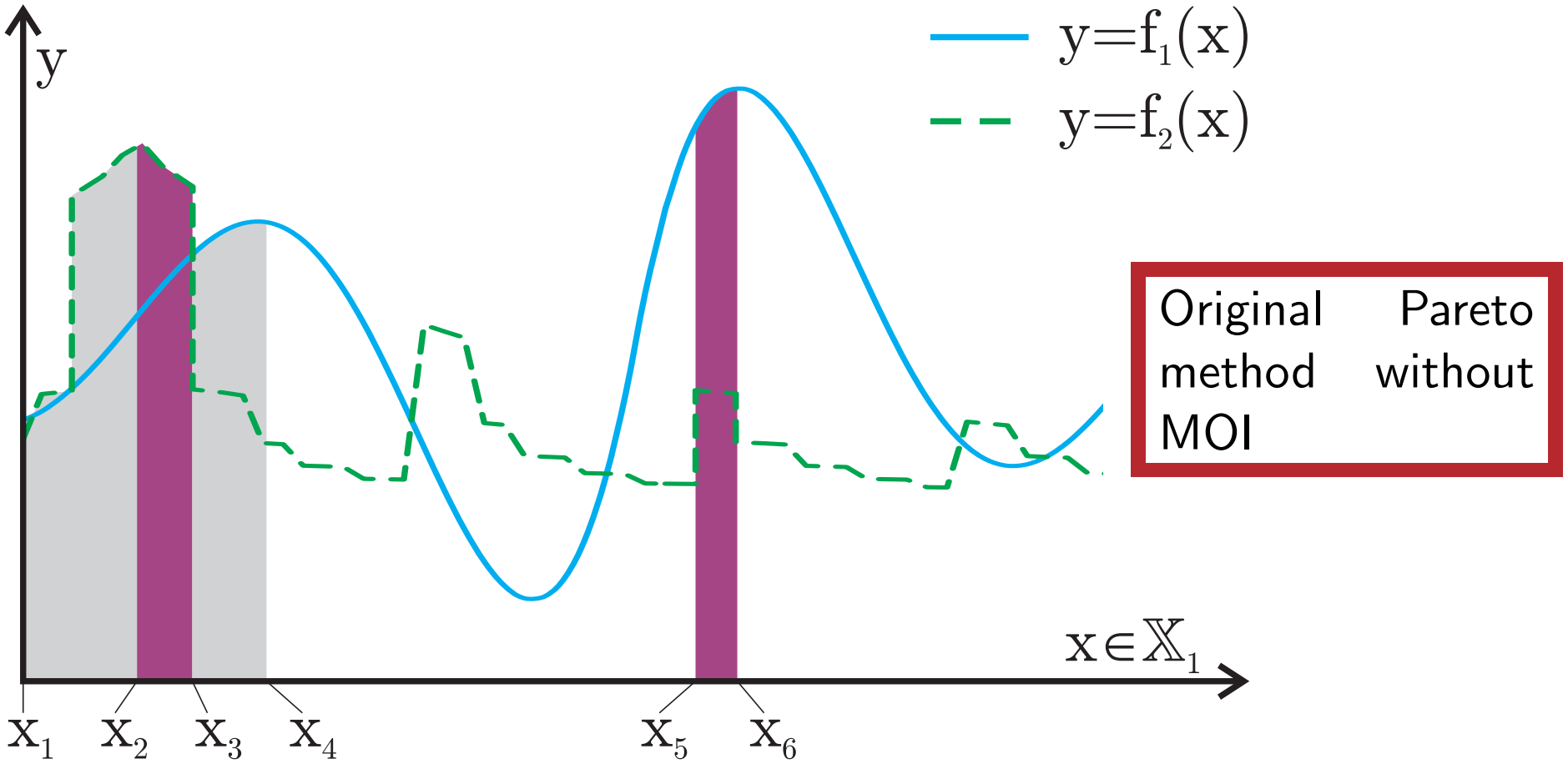

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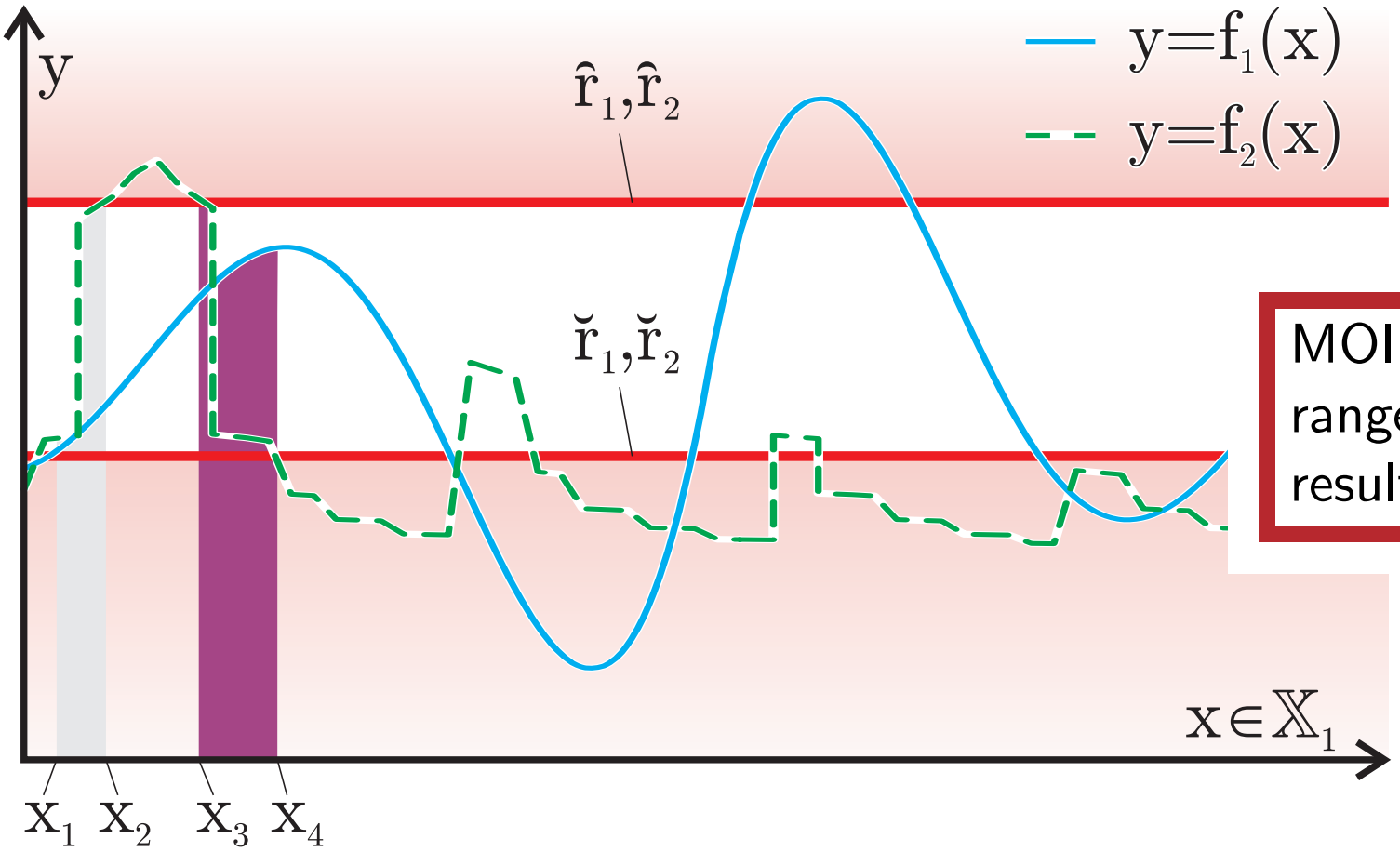

MOI with goal ranges $[\underline{r}i, \overline{r}i]$ – results change

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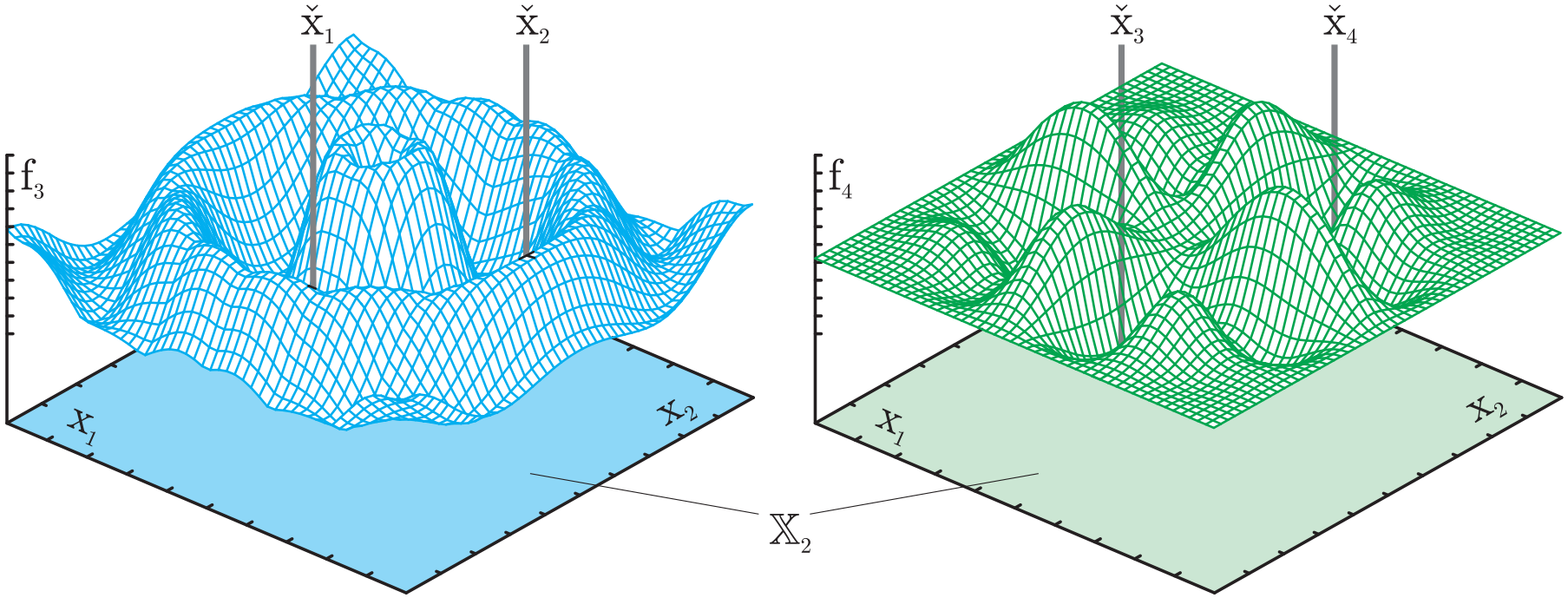

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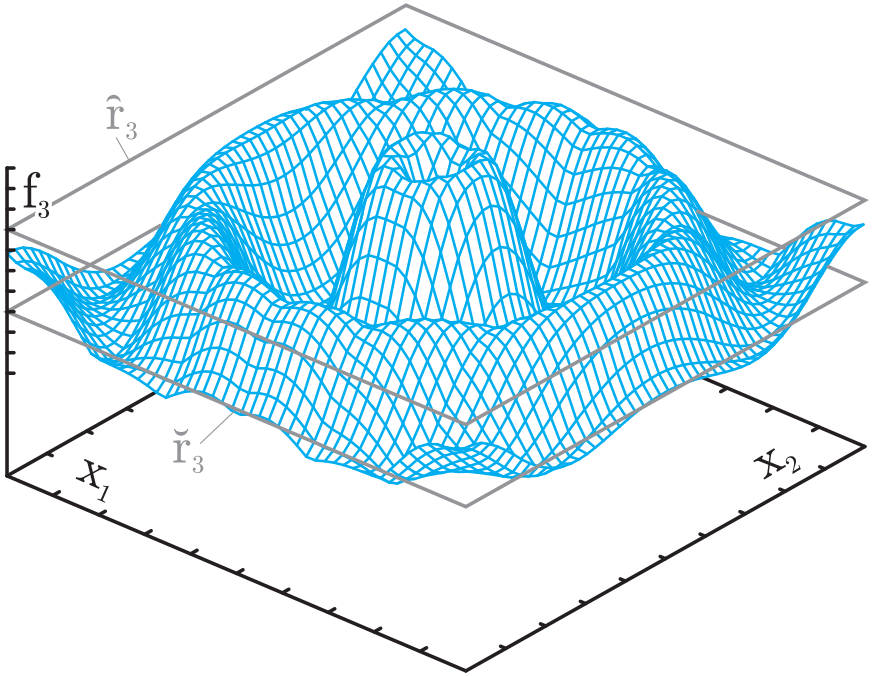

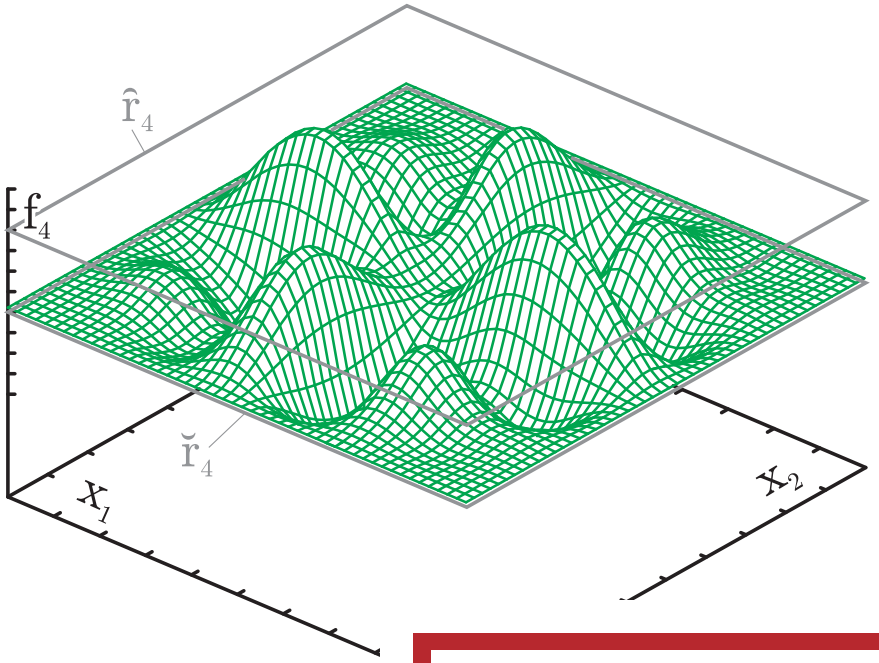

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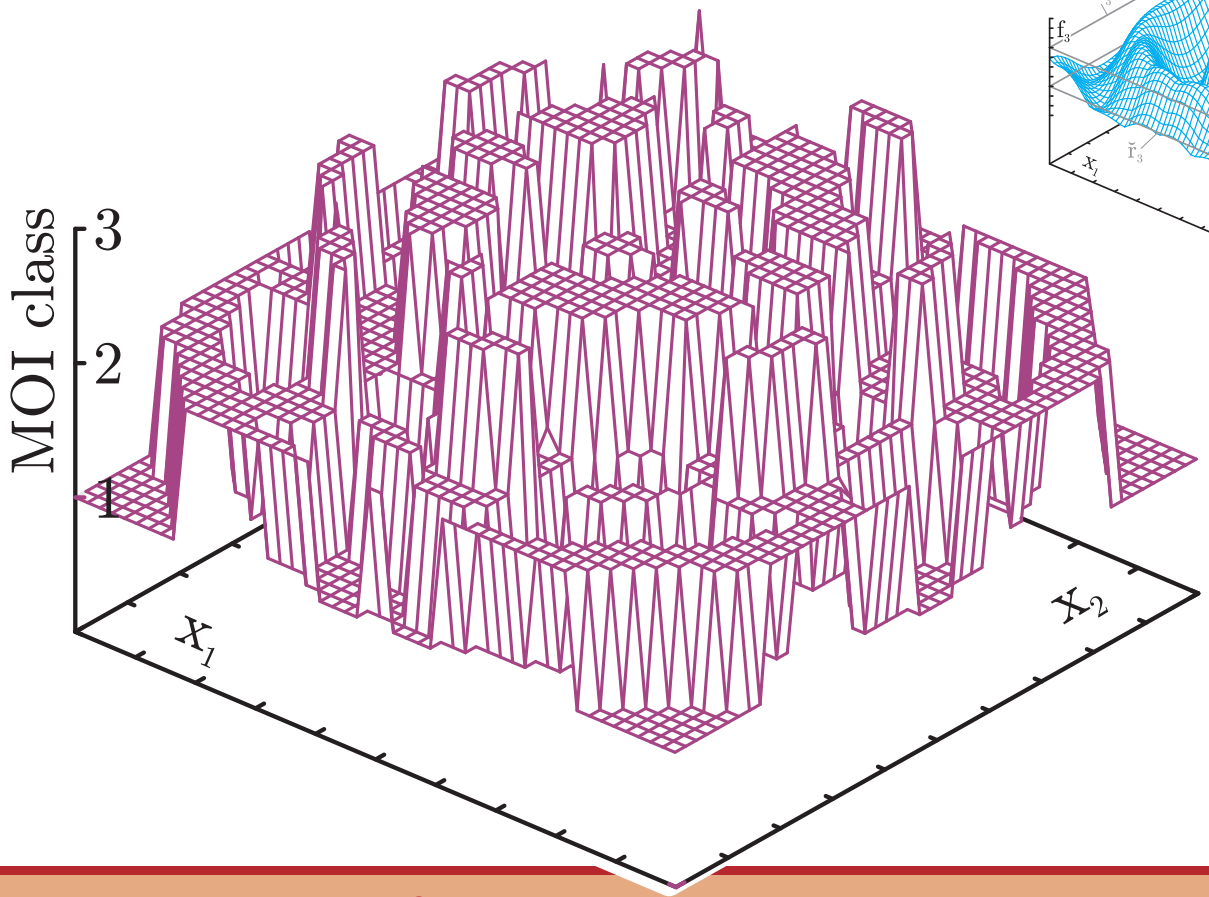

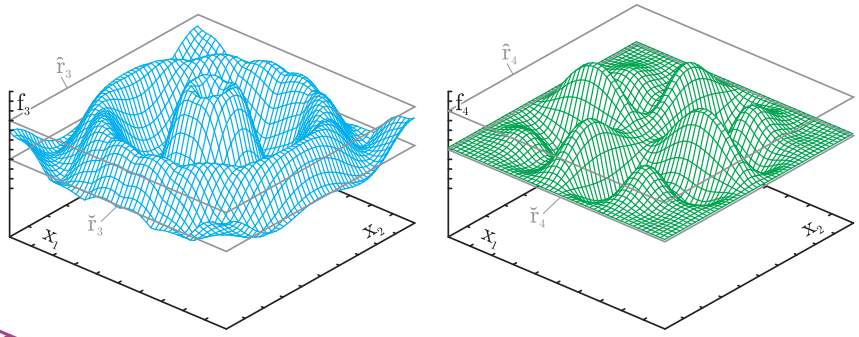

The three different classes

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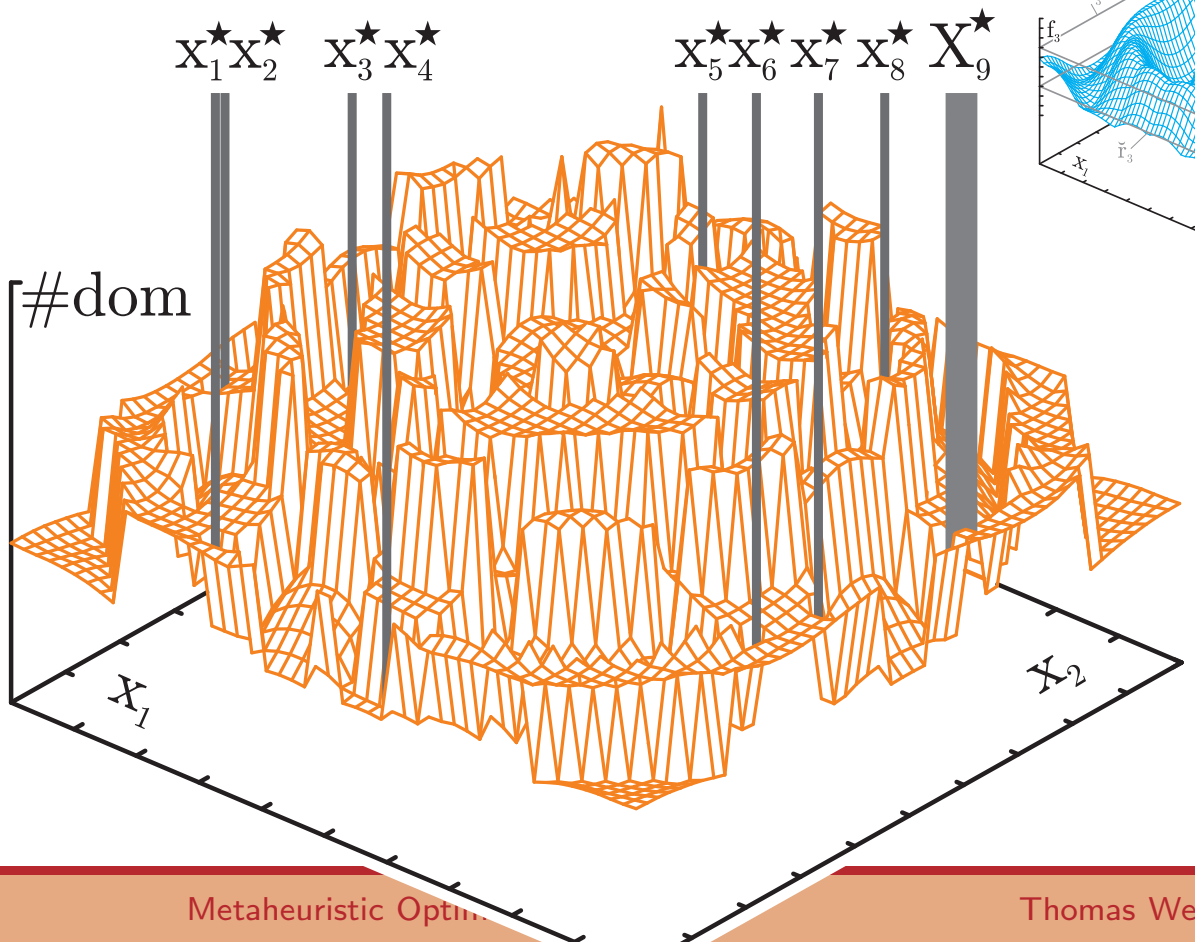

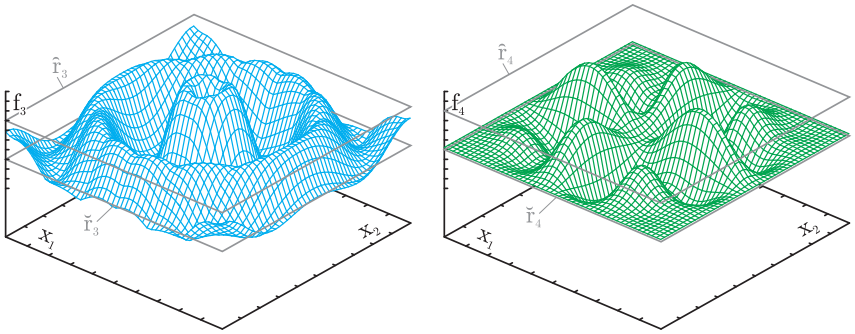

The MOI-domination ranks and optima

$$\#\mathbf{dom}(\mathbf{x}, \mathbf{X}) = |\{\mathbf{x}' : \ (x' \in X) \wedge (x' \dashv_c x)\}|$$

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- ✘ Constraints must be formulated as objective function ranges

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✔ Constraints can be of arbitrary nature

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- They are different from objectives: Objectives put "always" optimization pressure, whereas constraints only put pressure as long as they are not satisfied.

谢谢

Thank you

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Caspar David Friedrich, “Der Wanderer über dem Nebelmeer”, 1818
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