Blending Dynamic Programming with Monte Carlo Simulation for Bounding the Running Time of Evolutionary Algorithms

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Introduction

- Dynamic parameter settings can greatly improve the efficiency of evolutionary algorithms (EAs)
- Runtime lower bounds give a baseline, which is important for algorithm comparison and development
- Proving precise lower bounds for algorithms with dynamic parameter choices is challenging
- Previously, a dynamic programming approach was proposed to derive lower bounds for simple problems [Buzdalov, Doerr, PPSN 2020]
  - transition probabilities between different states can be expressed by mathematical expressions
  - applied to derive optimal mutation rates for OneMax problem
- We propose a method that combines dynamic programming with Monte Carlo sampling, which is applicable for a broader problem class

Considered Evolutionary Aglorithms

Data: n: problem size; $f : \{0,1\}^n \to \mathbb{R}$: function to maximize; λ: population size; $\mathcal{D}(p)$: a family of parameterized distributions over $[0..n]$

```
Sample parent x ∈ {0,1}^n uniformly at random;
for t ← 1, 2, ... do
    for i ∈ [1..λ] do
        Choose a distribution parameter p_i^t ;
        Sample k_i ~ D(p_i^t), the number of bits to flip;
        Create y_i by flipping k_i different bits in x chosen uniformly at random ;
    Select x ← arg max_{z ∈ {x, y_1, ..., y_λ}} f(z) breaking ties arbitrarily;
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for $t \leftarrow 1, 2, \ldots$ **do**
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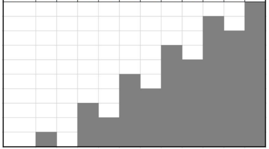
    Choose a distribution parameter p_i^t ;
    Sample $k_i \sim \mathcal{D}(p_i^t)$, the number of bits to flip;
    Create y_i by flipping k_i different bits in x chosen uniformly at random ;
  Select $x \leftarrow \arg\max_{z \in \{x, y_1, \ldots, y_\lambda\}} f(z)$ breaking ties arbitrarily;

Parameter control in $(1+\lambda)$ EA with mutation rate p:

- 2-rate: try $p/2$ and $2p$ on two halves of population
- Ab rule: multiply p by A or b based on success
- HQEA: multiply p by A or b according to Q-learning

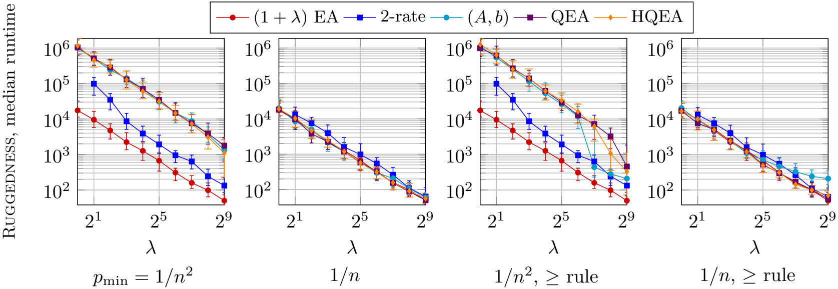
Ruggedness Problem and Benchmarking

i	0	1	2	3	4	5	6	7	8	9	10
$r_2(i)$	1	0	3	2	5	4	7	6	9	8	10



Optimum: $f(z) = n$.
Points at Hamming distance one from z have fitness $n - 2$,
those at distance two have fitness $n - 1$,
those at distance three have fitness $n - 4$,
those at distance four have fitness $n - 3$, and so on

Previous results for parameter control on Ruggedness:



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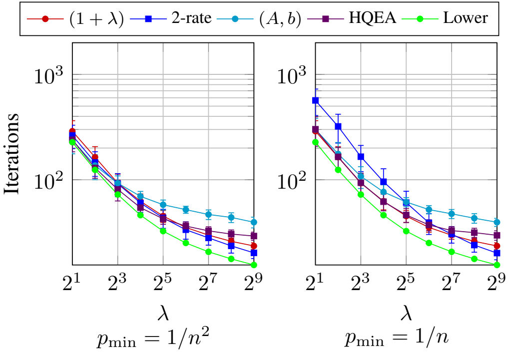
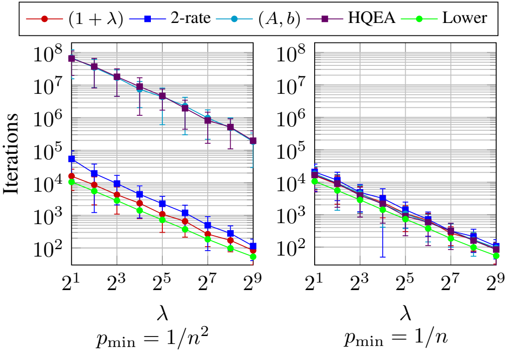
Requirement: the optimal choice of p depends on the fitness value exclusively

Lower Runtime Bounds for Parameter Control

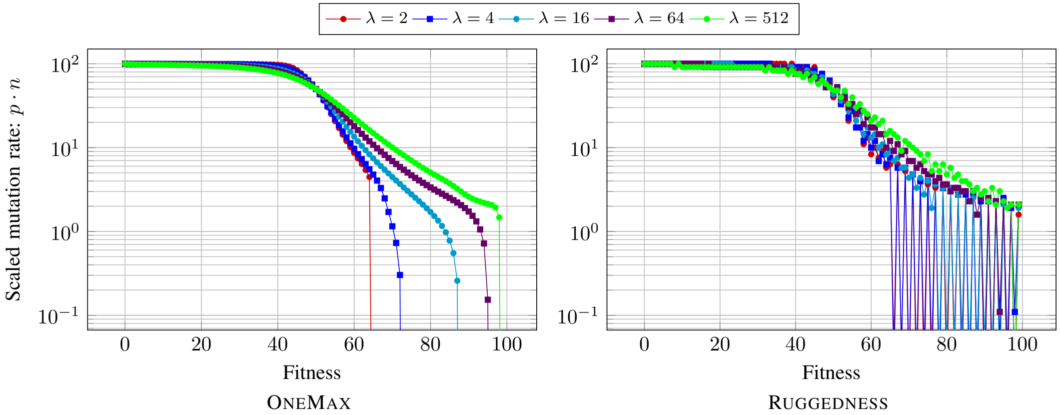
Iterations until the optimum of OneMax (left) and Ruggedness (right)





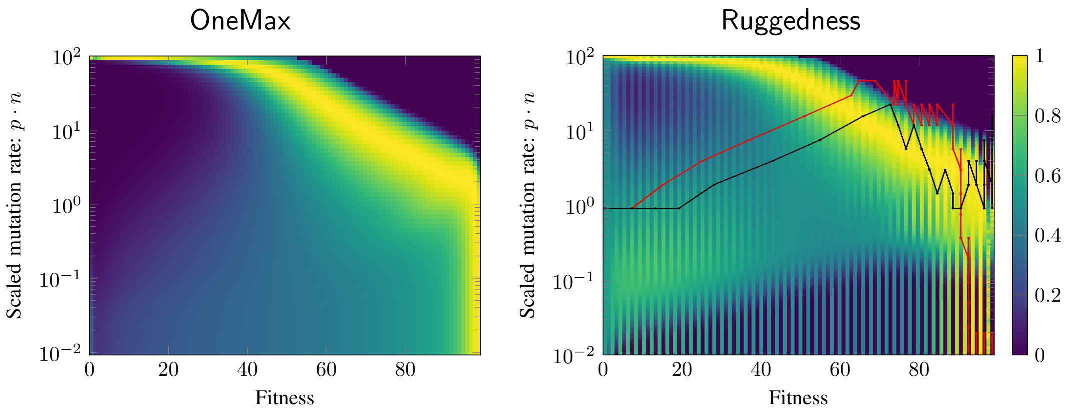
- New insight: on Ruggedness, only a constant-factor improvement is possible
- Why does (A,b) rule performs so much worse than 2-rate when using $p_{min} = 1/n^2$?

Optimal Mutation Rates



- Regular oscillations on Ruggedness with a period of 2
- It may be difficult to track precisely – is this a problem?

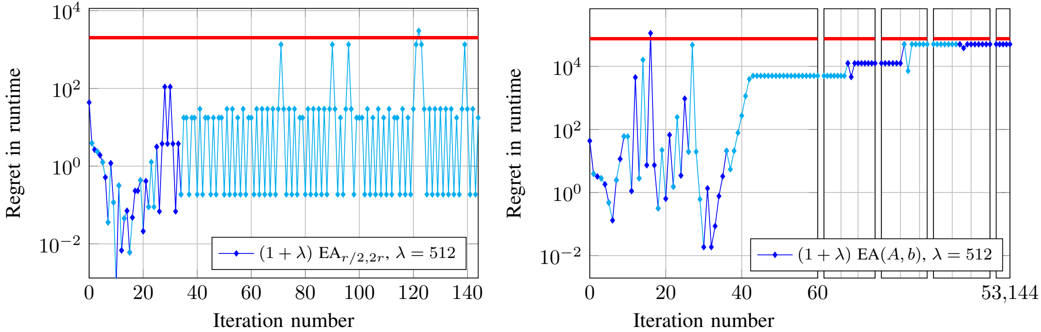
Parameter Efficiency Heatmaps



- Relative efficiency of the corr. p among all mutation rates for the corr. f
- The range of nearly equally good rates is wide enough
- On Ruggedness, for odd fitness values the best rates are higher
- (A, b) rule (red) gets stuck with too small rates near the optimum

Regret Plots

Regret $|T_{f,p} - T^*_f|$ for p chosen by 2-rate (left) and (A, b) rule (right)



- How much of the performance the method loses from acting suboptimally
- (A, b) rule spends most of its time with very large regrets

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Thank you!