Benchmark Set Reduction for Cheap Empirical Algorithmic Studies

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Outline

- Goal
- Method
- Traveling Thief Problem
- Computational Results
- Conclusion and Future Research

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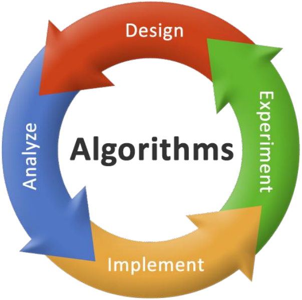
Goal | Motivation[2]

Algorithm Design[1] is an iterative process in a loop of

- Implement $\rightsquigarrow$ Experiment $\rightsquigarrow$ Modify $\rightsquigarrow$ Experiment . . .
- Recurring experimentation can be a burden for whom with limited computational resources



1 Skiena, S.S. (2008). *Algorithm Design Manual*. Springer

2 image source: https://www.coursera.org/courses?query=datastructuresandalgorithms

Goal[3]

Reducing a given benchmark set so that the experimental evaluation cost for the algorithmic studies can be significantly degraded

- specifically for large benchmark sets

[3] image source: https://www.univention.com/blog-en/2018/11/systematic-approach-to-evaluate-software-for-your-business/

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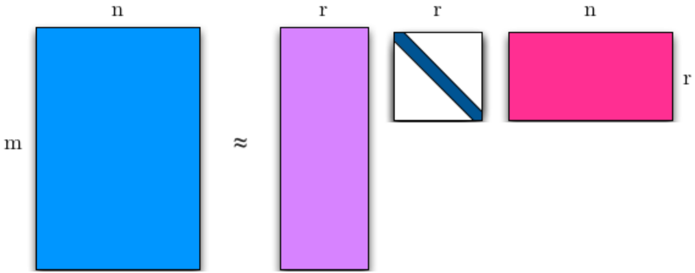
Method | Inspiration – ALORS[5][6]

An **Algorithm Selection** / **Recommender** system, operates through mapping instances' features to instances' latent (hidden) features

- Use matrix factorization to extract latent features – Singular Value Decomposition (SVD)[4] is used



4 Strang, G., 1980. Linear Algebra and its Applications. Academic Press, New York

5 Mısır, M. and Sebag, M., 2017. ALORS: An algorithm recommender system. Artificial Intelligence, 244, pp.291-314

6 image source: https://staging.njtrainingacademy.com/2019/01/11/singular-value-decomposition-svd/

Method

$\mathcal{M}_{n \times m}$ is a rank matrix of n instances and m algorithms

Input
Performance matrix $\mathcal{M}_{n \times m}$
Matrix rank for dimensionality reduction r
Latent feature extraction
Apply SVD to $\mathcal{M}$ to extract U_r and V_r
Instance clustering
Find best k for k-means(U_r) w.r.t. Silhouette score
Compute clusters C_k
Instance subset selection
Return $I_s = \cup \left\{ \texttt{select}(C_j, \left\lceil size(C_j) / \min_{i=1 \ldots k}(size(C_i)) \right\rceil) \right\}$ for $j = 1 \ldots k$

Method | Feature Extraction (Step 1)

SVD is used to decompose $\mathcal{M}$:

$$\mathcal{M} = U\Sigma V^t$$

- U is a matrix representing the rows of $\mathcal{M}$, i.e. instances
- V is a matrix representing the columns of $\mathcal{M}$, i.e. algorithms
- Σ is a diagonal matrix of **sorted** singular values, denotes importance

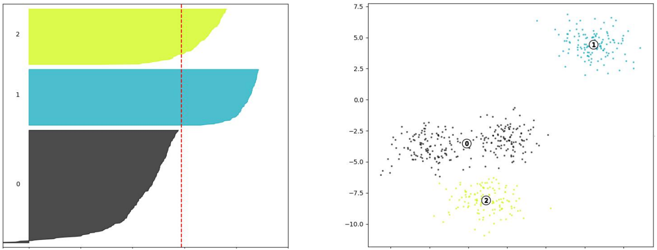
The dimensions of the resulting matrices can be reduced to r by using the first r dimensions

$$\mathcal{M} \approx U_r \Sigma_r V_r^t$$

Method | Instance Clustering (Step 2)[7]

Explore different instance types through clustering

- k-means is used to cluster the instances based on the extracted latent instance features
- Silhouette score, i.e. mean Silhouette coefficient, is employed to evaluate cluster quality, for $k = \{2, \ldots, 100\}$
- Next, binary search is performed between $k = 101$ and $n/2$



[7] figure source: https://scikit-learn.org/stable/auto_examples/cluster/plot_kmeans_silhouette_analysis.html

Method | Instance Subset Selection (Step 3)

The closest instances to the centroid of each cluster are selected, taking the cluster sizes into account

$$\left\lceil size(C_j) / \min_{i=1 \ldots k} (size(C_i)) \right\rceil$$

where $size(C_j)$ is the number of instances in the cluster C_j

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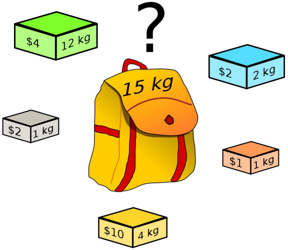
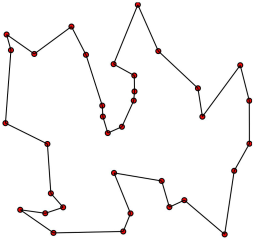
Traveling Thief Problem (TTP)[11]

An NP-hard problem concerned with two other, well-known optimization problems, namely[8]

- Traveling Salesman Problem (TSP)[9]
- Knapsack Problem (KP)[10]





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8 image source: https://en.wikipedia.org/wiki/Knapsack_problem – /Travelling_salesman_problem

9 M. M. Flood, "The traveling-salesman problem," Operations research, vol. 4, no. 1, pp. 61–75, 1956

10 H. M. Salkin and C. A. De Kluyver, "The knapsack problem: a survey," Naval Research Logistics Quarterly, vol. 22, no. 1, pp. 127–144, 1975

11 S. Polyakovskiy, M. R. Bonyadi, M. Wagner, Z. Michalewicz, and F. Neumann, "A comprehensive benchmark set and heuristics for the traveling thief problem," in Proceedings of the Annual Conference on Genetic and Evolutionary Computation (GECCO), 2014, pp. 477–484

TTP[12]

- A set of cities: $N = \{1, \dots n\}$
- A set of items: $M = \{1, \dots m\}$
- The distance between the city i and city j: d_{ij}
- The city i (except the starting city) has a set of items: $M_i = \{1, \dots, m_i\}$, $M = \bigcup_{i \in N} M_i$
- The item k from the city i has profit p_{ik} and weight w_{ik}
- W is the knapsack capacity
- R is the renting rate (cost) for the knapsack, per time unit
- v_{max} and v_{min} denote **max** and **min** speed of the thief, affected by the total weight of the collected items

[12] Wagner, M., Lindauer, M., Mısır, M., Nallaperuma, S. and Hutter, F., 2018. A case study of algorithm selection for the traveling thief problem. Journal of Heuristics, 24(3), pp.295-320

TTP

The **goal** is to specify a tour maximizing the total profit

- The tour consists of all the cities exactly once, starting from the first city and returning back there

The **objective function**[13] for a tour $\Pi = (x_1, \ldots, x_n)$, $x_i \in N$ and a packing plan $P = (y_{21}, \ldots, y_{nm_i})$:

$$Z(\Pi, P) = \sum_{i=1}^{n} \sum_{k=1}^{m_i} p_{ik} y_{ik} - R\left(\left(\sum_{i=1}^{n-1} \frac{d_{x_i x_{i+1}}}{\upsilon_{max} - \nu W_{x_i}}\right) + \frac{d_{x_n x_1}}{\upsilon_{max} - \nu W_{x_n}}\right)$$

where

- $y_{ik} \in \{0, 1\}$ shows whether the item k is picked from the city i
- W_i is the total item weight when the thief leaves the city i
- $\nu = \frac{\upsilon_{max} - \upsilon_{min}}{W}$ is a constant

[13] Within the knapsack's rent term, the first part is the traveling cost between cities while the second part refers to the cost of going back to the starting city

TTP | Instances

9720 TTP instances, from the literature

- Based on TSPLIB[14]
- Considering 3 KP variations, i.e. uncorrelated, uncorrelated with similar weights and bounded strongly correlated
- Different number of per city items
- Distinct renting rates, R

[14] http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/

TTP | Algorithms

21 candidate TTP algorithms, from the literature

- Simple Heuristic, Random Local Search (RLS), (1+1)-Evolutionary Algorithm (EA)
- Density-based Heuristic (DH)
- Memetic Algorithm with the Two-stage Local Search (MATLS)
- S1, S2, S3, S4, S5, C1, C2, C3, C4, C5, C6
- CoSolver with 2-OPT and Simulated Annealing (CS2SA)
- Variants of MAX-MIN Ant System (MMAS): MMASls3 (M3), MMASls4 (M4), MMASls3boost (M3B), MMASls4boost (M4B)

Outline

Computational Results | Performance

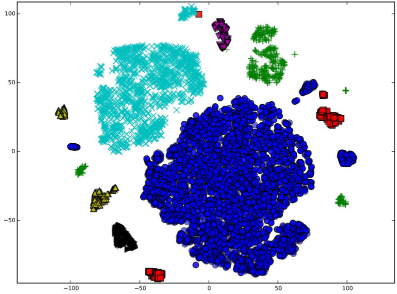
Spearman's rank coefficient test evaluates the marginal algorithm contribution to any algorithm (portfolio) subset, for Oracle

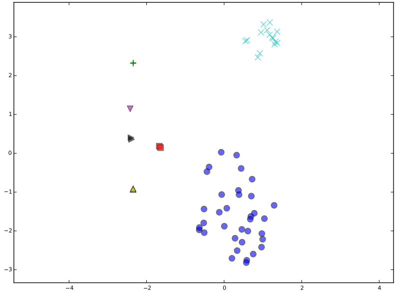
- ranking is preserved in most cases, i.e. ρ-values of > 0.9
- Subset-k5-1 achieves with its 62 instances $\rho = 0.974$, which is the best score among the smallest subsets
- Overall, Subset-k5-20 achieves with its 1240 instances $\rho = 0.991$

Scenario	ρ
Subset-k5-1	0.974
Subset-k5-5	0.986
Subset-k5-10	0.988
Subset-k5-20	0.991
Subset-k5-30	0.988
Subset-k6-1	0.805
Subset-k6-5	0.813
Subset-k6-10	0.788
Subset-k6-20	0.804
Subset-k6-30	0.808
Subset-k7-1	0.957
Subset-k7-5	0.970
Subset-k7-10	0.979
Subset-k7-20	0.986
Subset-k7-30	0.986
Subset-k8-1	0.971
Subset-k8-5	0.981
Subset-k8-10	0.981
Subset-k8-20	0.983
Subset-k8-30	0.979

Computational Results | Instance Set Reduction

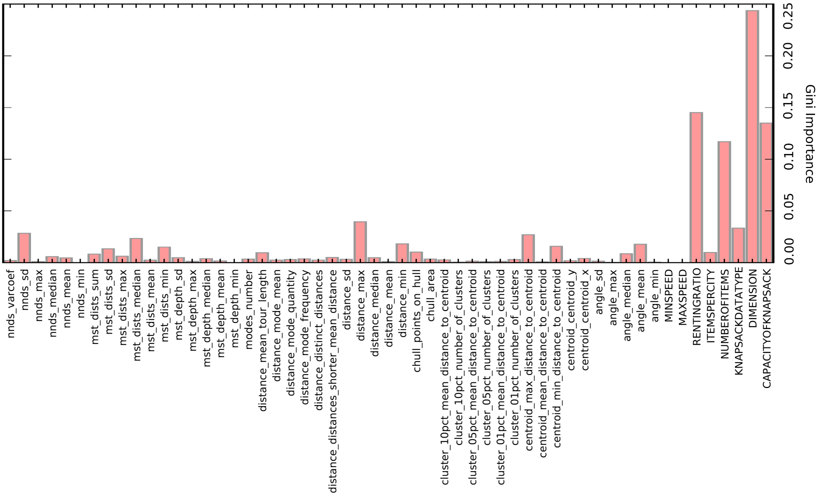
62 selected TTP instances as a representative benchmark set for the complete 9720 instances





Computational Results | Feature Importance

For 55 TTP features, from the literature



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Conclusion and Future Research

The proposed method is able to come up with representative instance sets, with less than %1 of the complete instance set

Follow-up research:

- repeating the analysis on other problems
- offering a new clustering approach for determining the number of clusters cheaper
- benefiting from Matrix Completion (MC)[15][16] to expand the applicability of the method
- recommending instance subsets not as a representative set of the large one but small yet a fair benchmark set

[15] M. Mısır. Data sampling through collaborative filtering for algorithm selection, in the 16th IEEE CEC, 2017, pp. 2494–2501

[16] M. Mısır. Active matrix completion for algorithm selection, in LOD. LNCS, Springer, 2019, pp. 321–334