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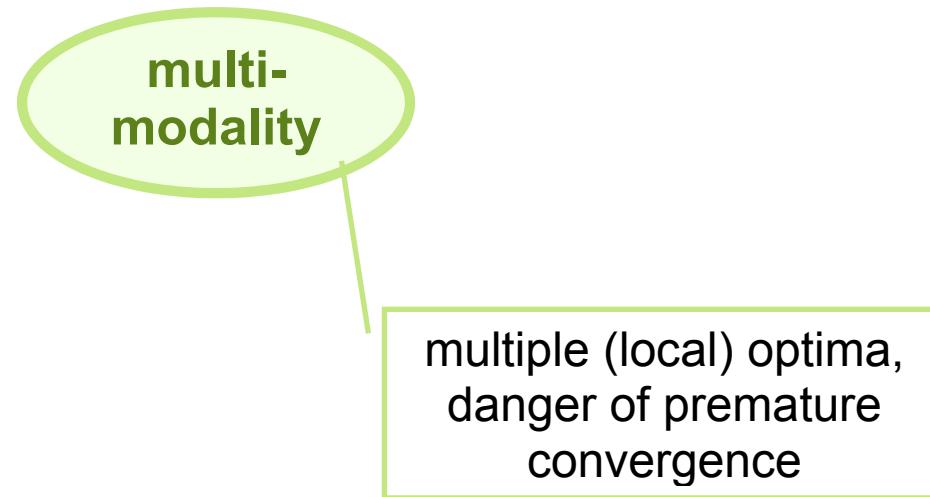
# A Tunable Model for Multi-Objective, Epistatic, Rugged, and Neutral Fitness Landscapes

GECCO 2008  
Genetic and Evolutionary Computation Conference  
Theory II Track, Roswell Room, 2008-07-16 11:30-11:55, Chair: Thomas Jansen  
Renaissance Atlanta Hotel Downtown, 590 West Peachtree Street NW, Atlanta, Georgia 30308, USA

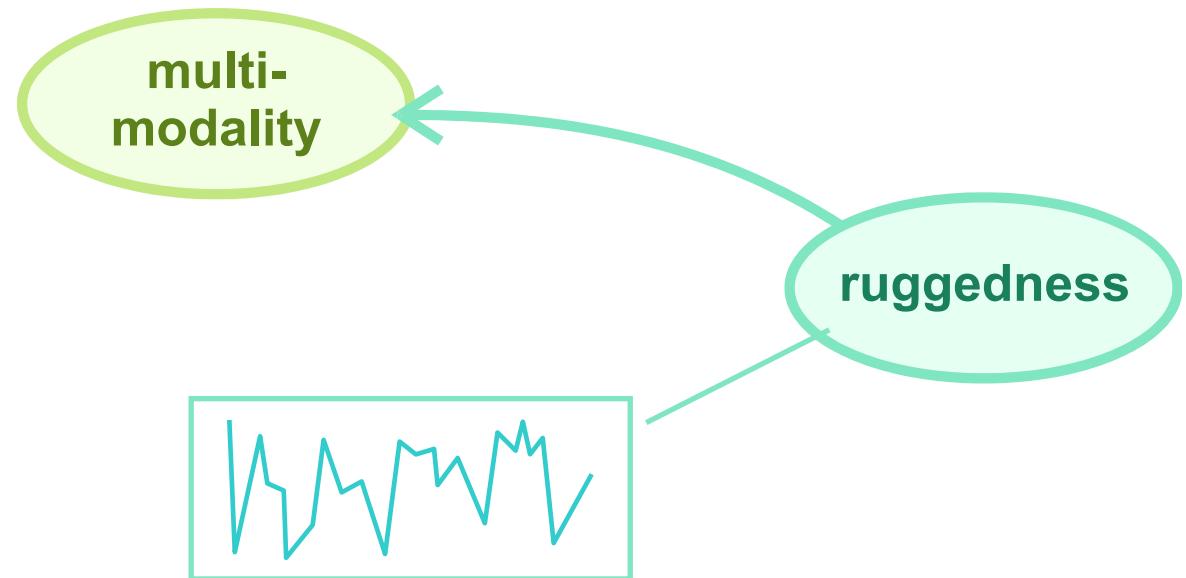
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- Model Problem
  - Neutrality
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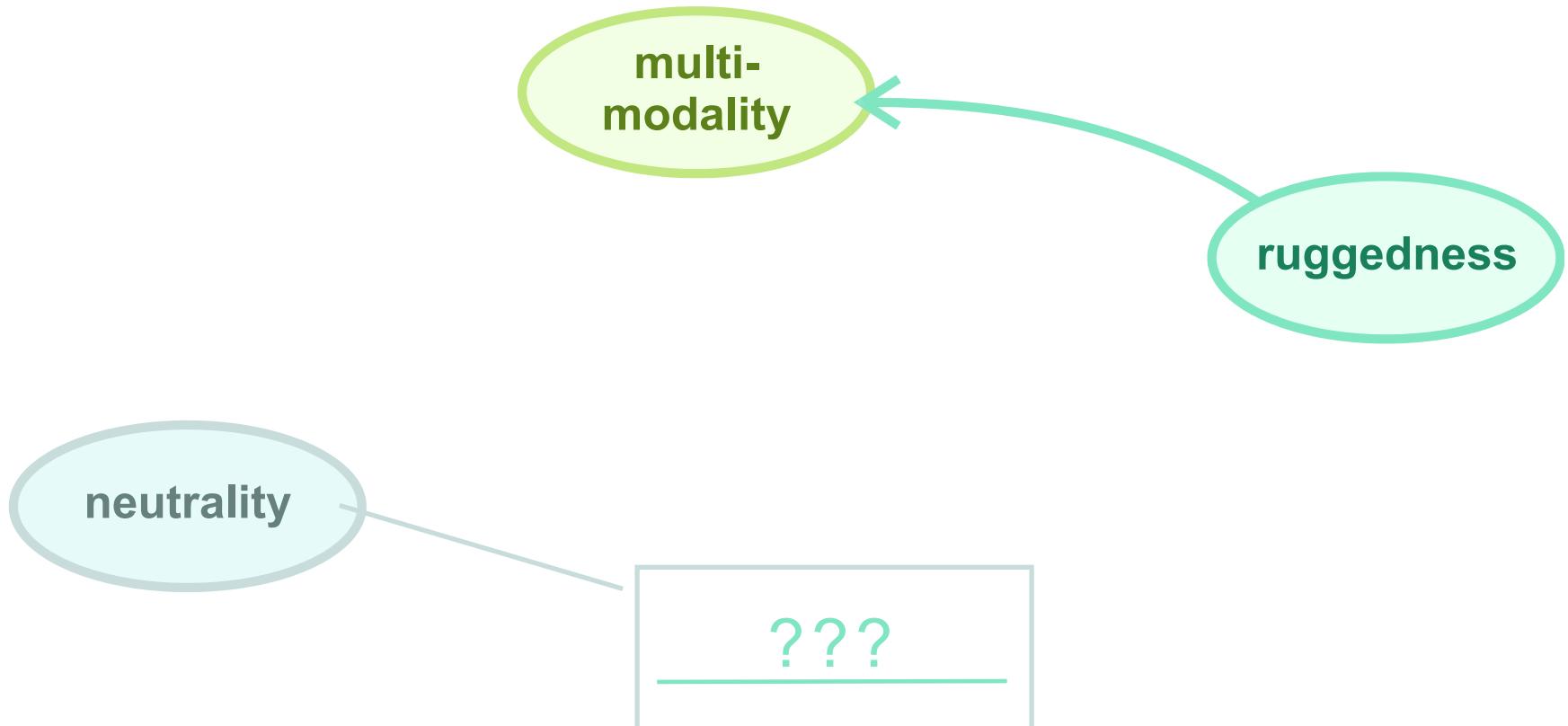
# Problems in Optimization



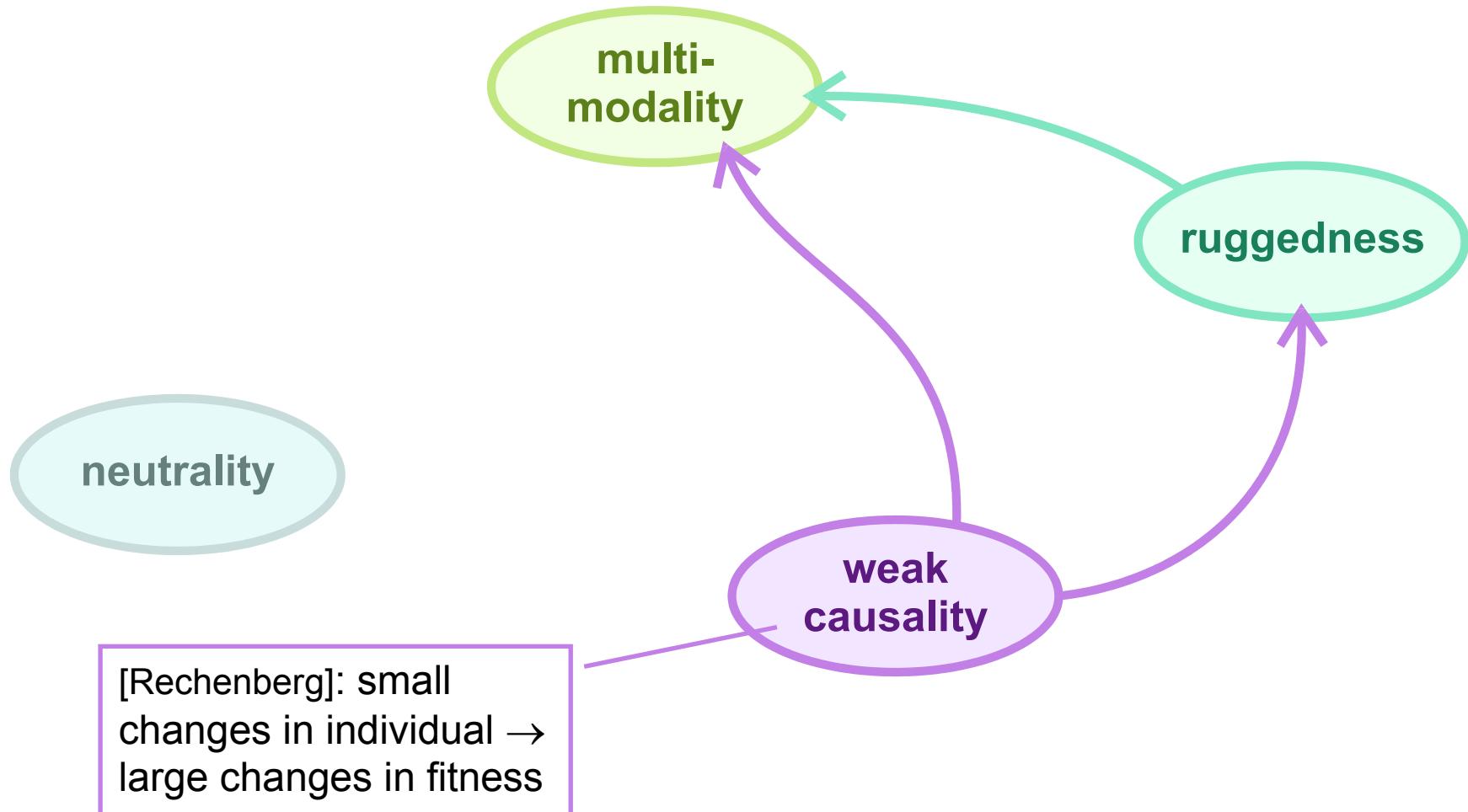
# Problems in Optimization



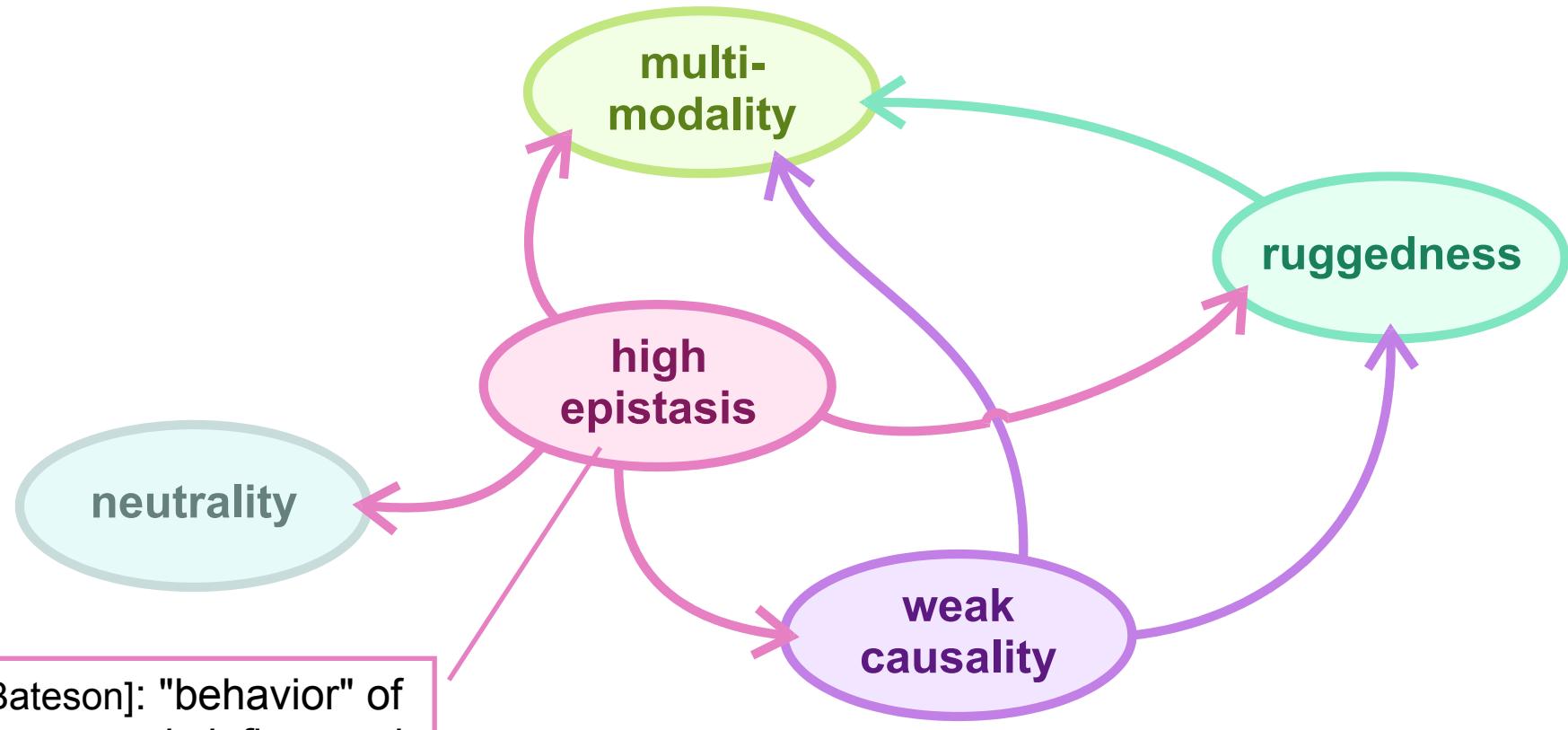
# Problems in Optimization



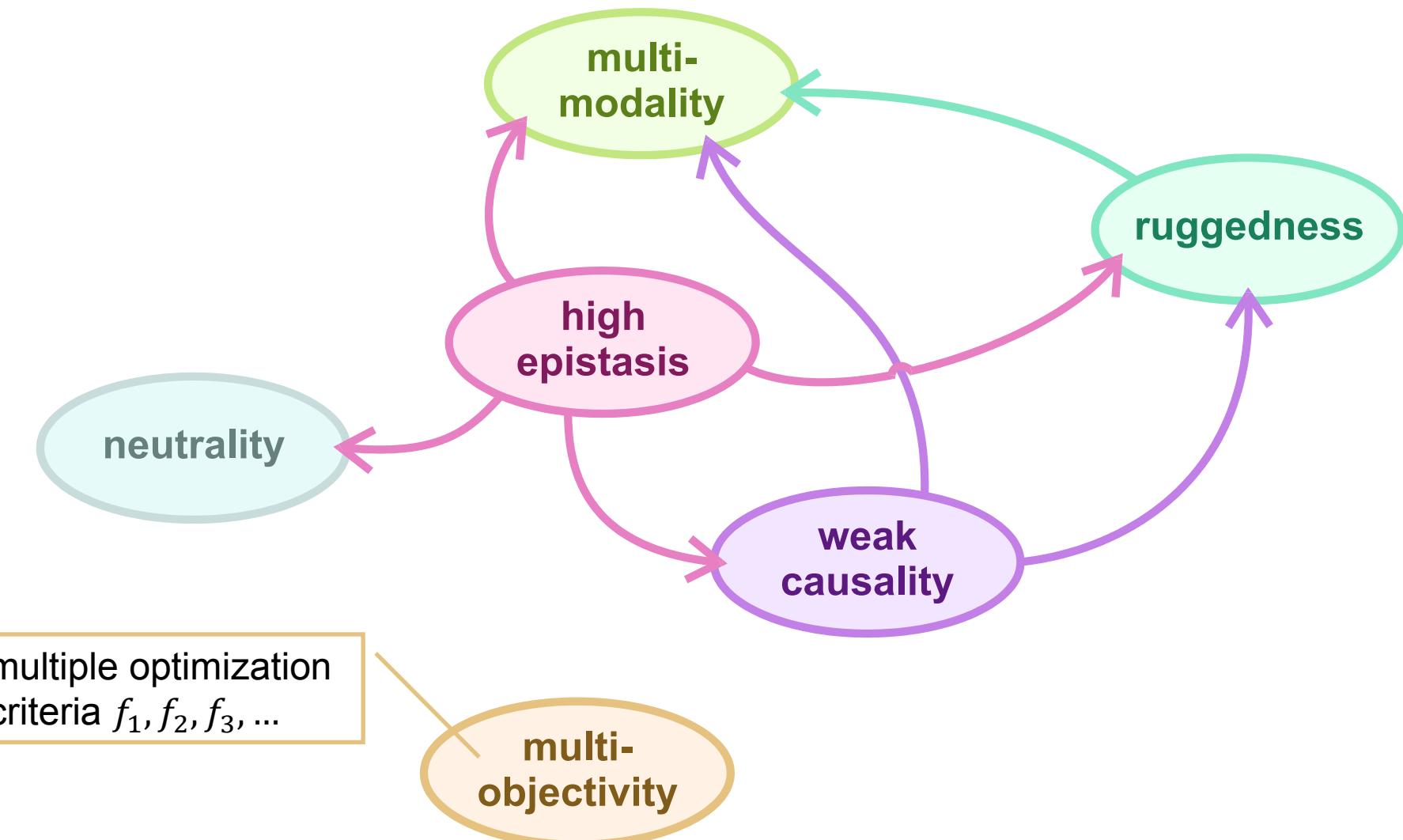
# Problems in Optimization



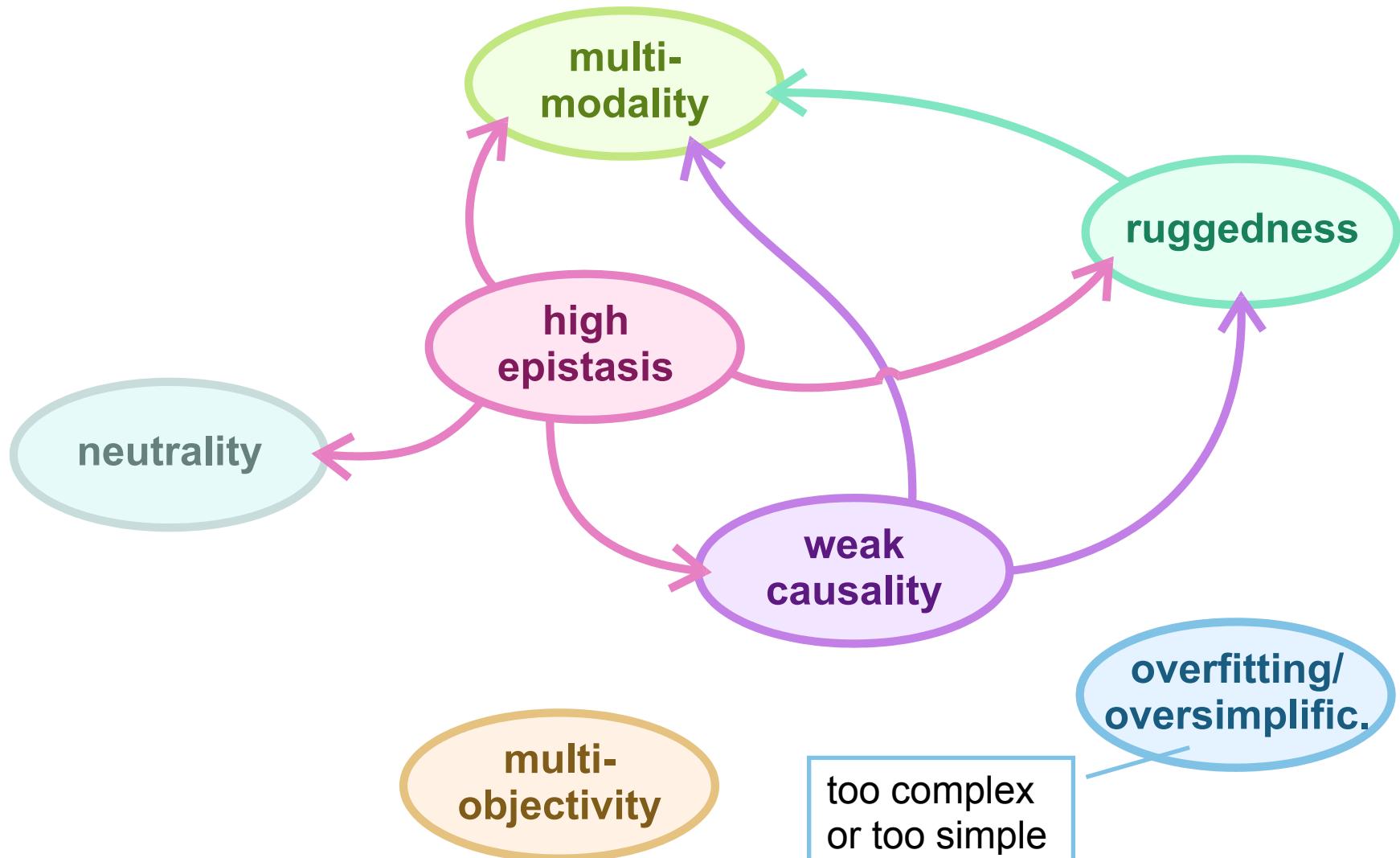
# Problems in Optimization



# Problems in Optimization



# Problems in Optimization



# Motivation

- optimization tasks exhibit these features to different degrees
- which settings of an EA perform best?
- experiments with "real" tasks often take very long
  - can test only very few settings
- want: model problem
  - fast
  - all features tunable

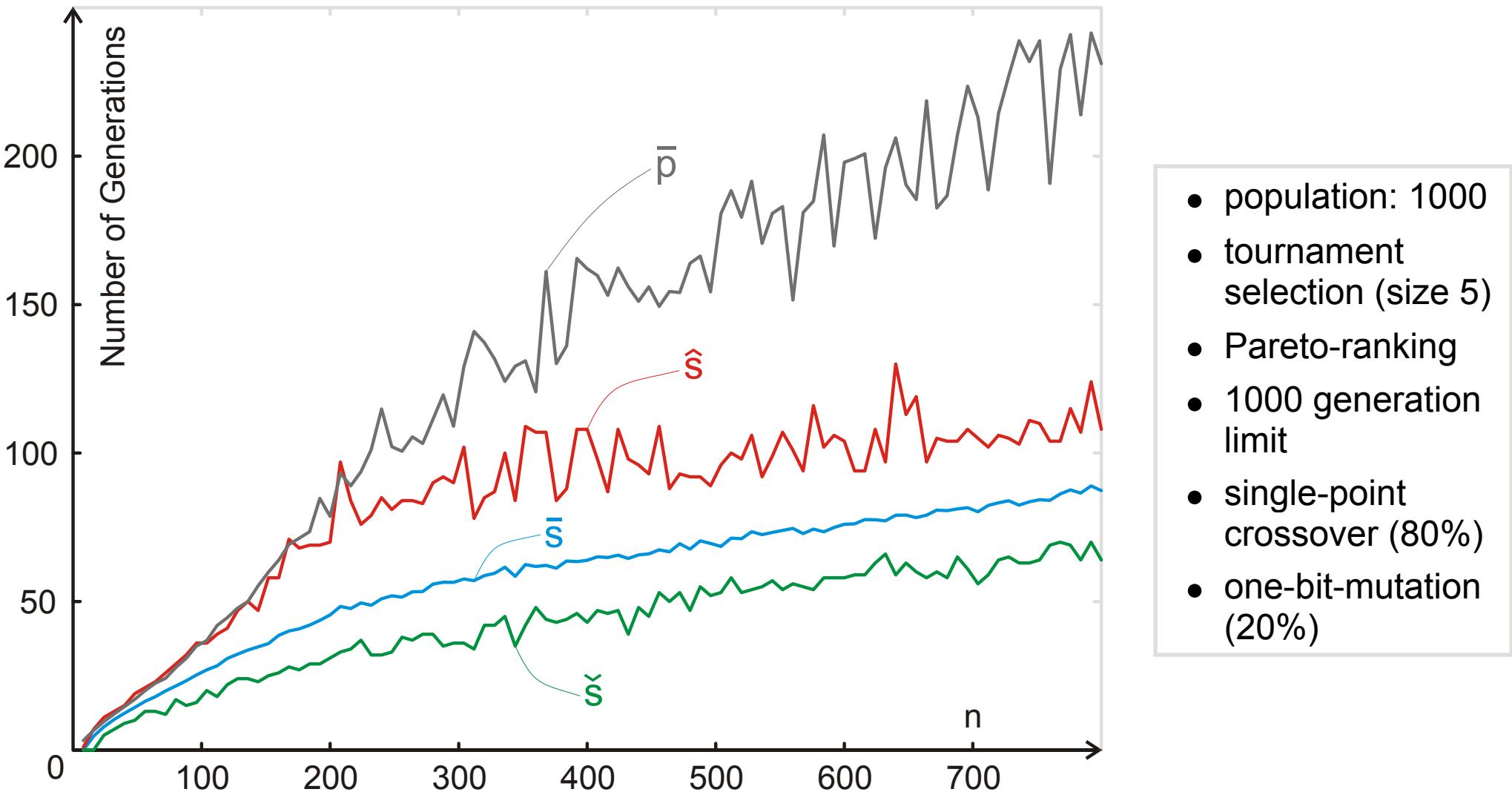
# Model Problem – Basics

- each feature introduced as tunable filter
- search space: variable length bit strings (or fixed length)
- basic problem: find "optimal" bit string  $x^*$  of length  $n$

$$x^* = 0101010101 \dots 01$$

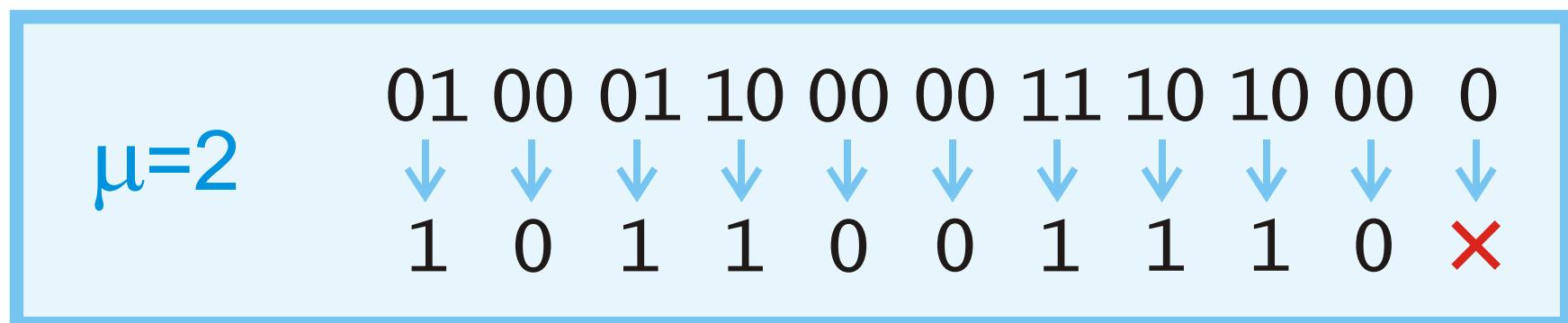
- objective functions
  - Hamming distance (first  $n$  bits) to  $x^*$
  - length

# Model Problem – Basics

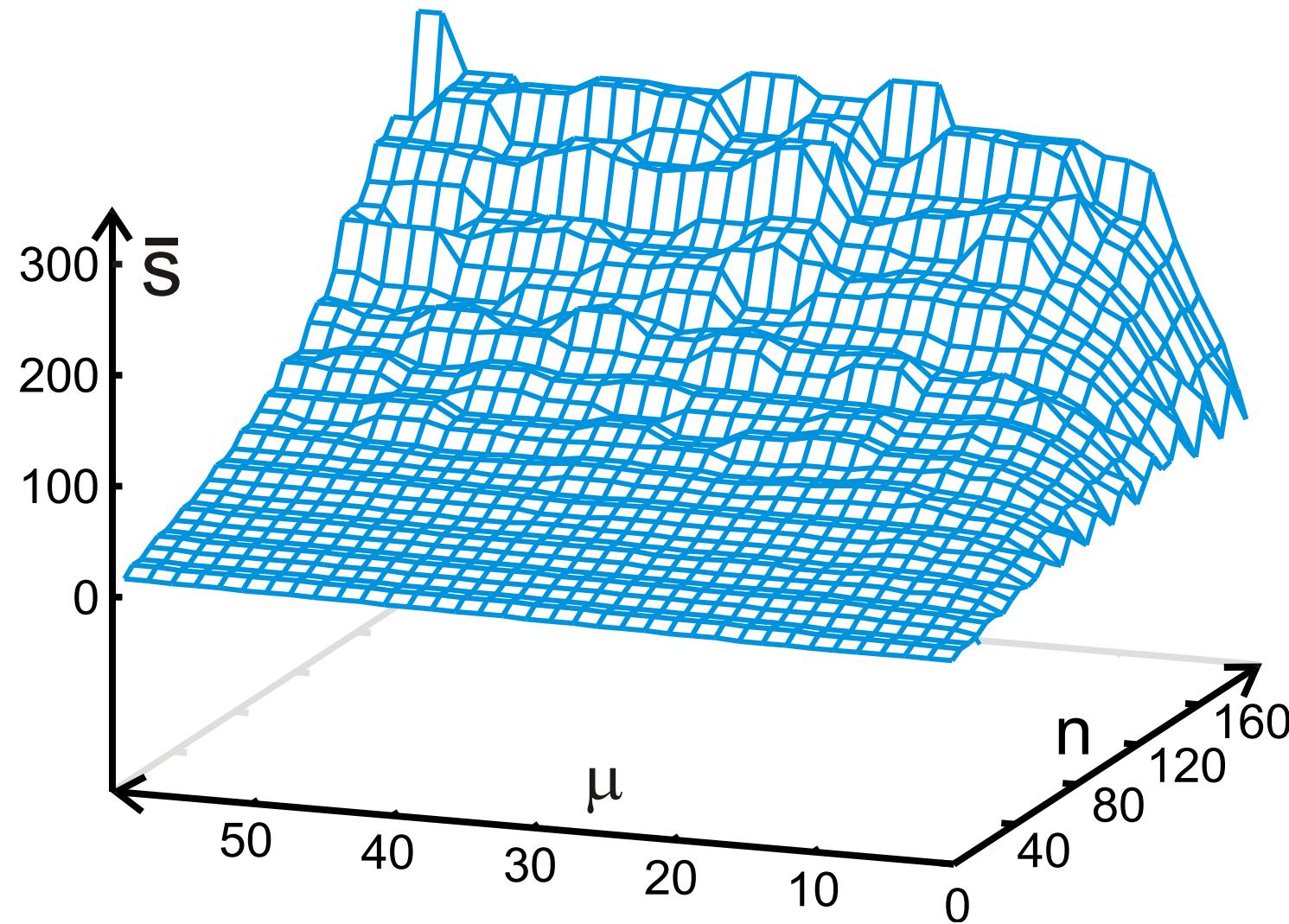


# Model Problem – Neutrality

- generate neutrality via redundancy
- combine blocks of  $\mu$  bits to one bit via majority
- use 1 in draw situations
- genotype length  $l$  no multiple of  $\mu \rightarrow$  ignore last  $l \bmod \mu$  bits



# Model Problem – Neutrality



# Model Problem – Epistasis

- bijective mapping  $e_\eta$  of blocks  $z$  of the fixed length  $\eta$
- modify the Hamming distance of the blocks
- $h(z_1, z_2) = 1 \Rightarrow h(e_\eta(z_1), e_\eta(z_2)) \geq \eta - 1 \quad \forall z_1, z_2 \in \{0,1\}^\eta$

$z$	$e_4(z)$	$z$	$e_4(z)$	$z$	$e_4(z)$
0000	0000	1111	1110	0011	0110
0001	1101	0111	0001	0101	1010
0010	1011	1011	1001	0110	1100
0100	0111	1101	0101	1001	0010
1000	1111	1110	0011	1010	0100

$h=1$

$\text{h} \leq 1$

# Model Problem – Epistasis

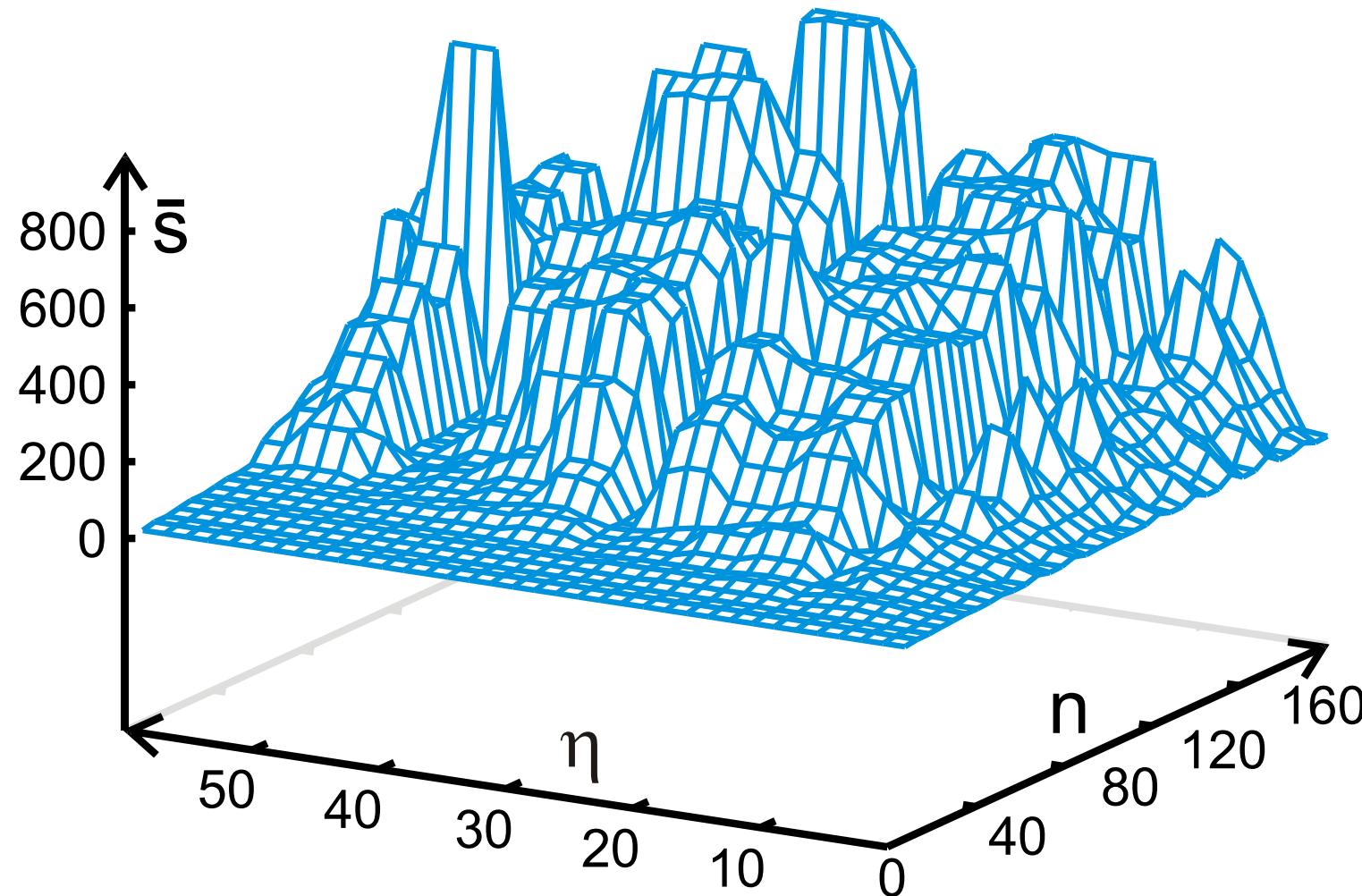
$$\bullet \quad e_\eta(z)_{[k]} = \begin{cases} \text{xor } z_{[i]} & \forall z: 0 \leq z < 2^{\eta-1} \\ \forall i: 0 \leq i < \eta, \\ i \neq (\textcolor{red}{k-1}) \% \eta \\ \frac{e_\eta(z - 2^{\eta-1})_{[k]}}{e_\eta(z - 2^{\eta-1})_{[k]}} & \text{otherwise} \end{cases}$$

 $\eta=4$ 

1011	0011	10
$e_4 \downarrow$	$e_4 \downarrow$	$e_2 \downarrow$
1001	0110	11

insufficient bits,  
at the end, use  
 $\eta=2$  instead of  
 $\eta=4$

# Model Problem – Epistasis



# Model Problem – Multi-Objectivity

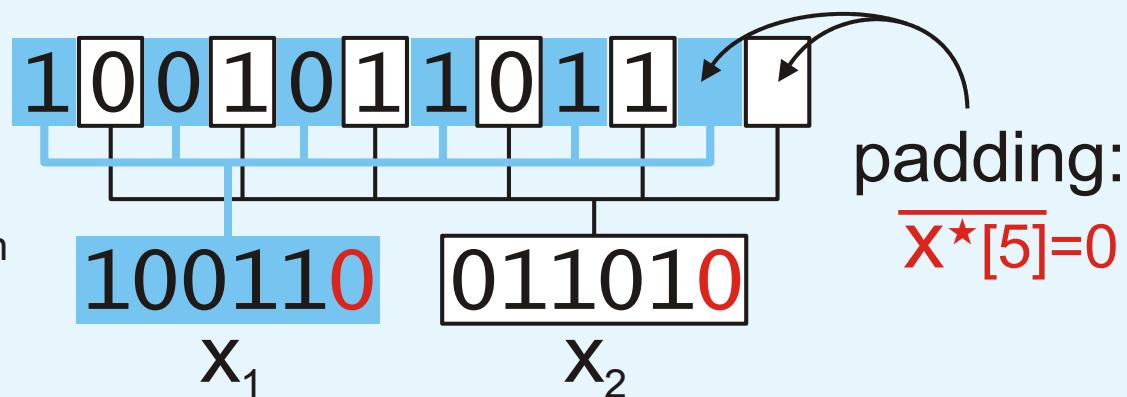
- using the functional objective function  $m$  times

$$x_i = (g_{i-1}, g_{m+i-1}, g_{2m+i-1}, \dots, g_{(n-1)m+i-1})$$

- conflicts will ensue if epistasis is turned on
- non-functional objective: length of string (in the VL case)

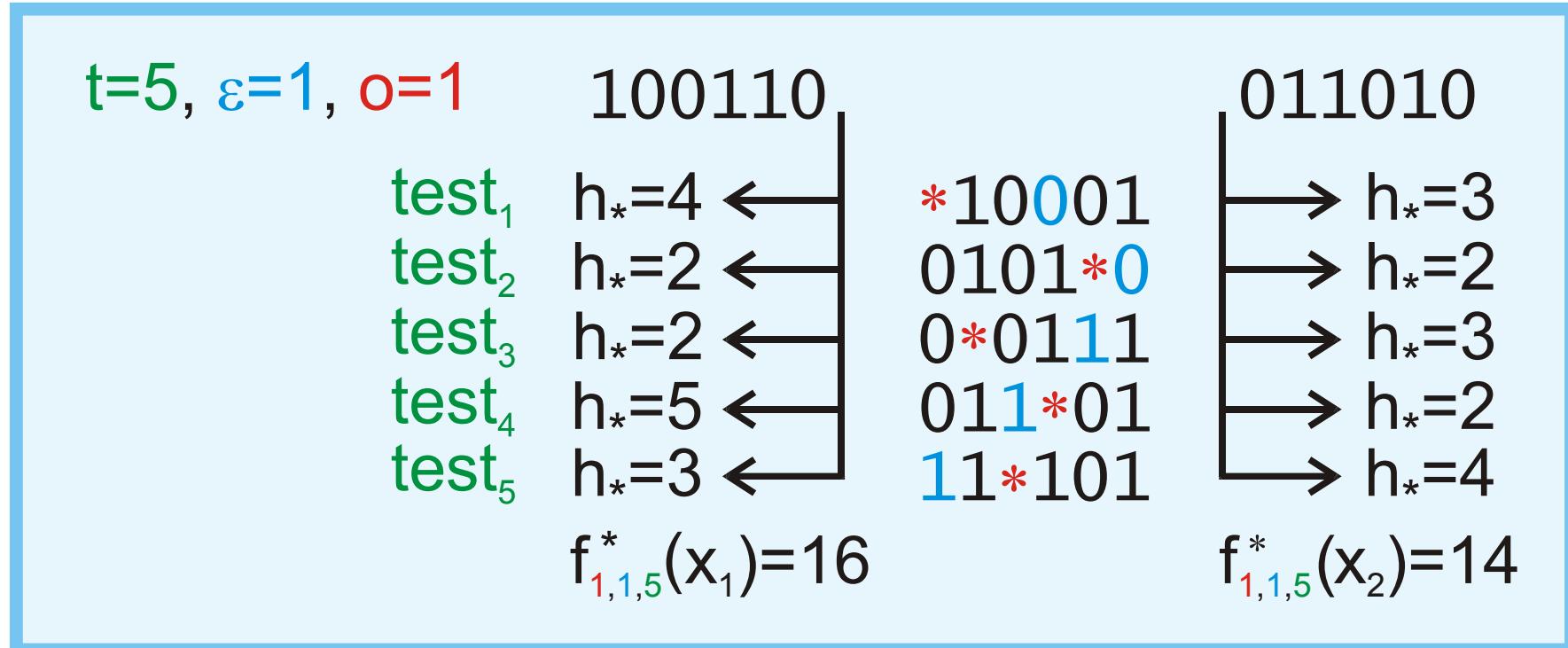
$m=2, n=6$

$(x_1, x_2) \in \{0,1\}^n$



# Model Problem – Overfitting/~simplification

- instead of direct comparison with  $x^* = (0101 \dots)$  use modified Hamming  $h_*$  distance to  $t$  test cases



# Model Problem – Ruggedness

- introduce ruggedness via weak causality
- objective values: 0 is best,  $q \approx (n - o)t$  is worst

$$0 \leftarrow 1 \leftarrow 2 \leftarrow 3 \leftarrow \dots \leftarrow q-2 \leftarrow q-1 \leftarrow q$$

# Model Problem – Ruggedness

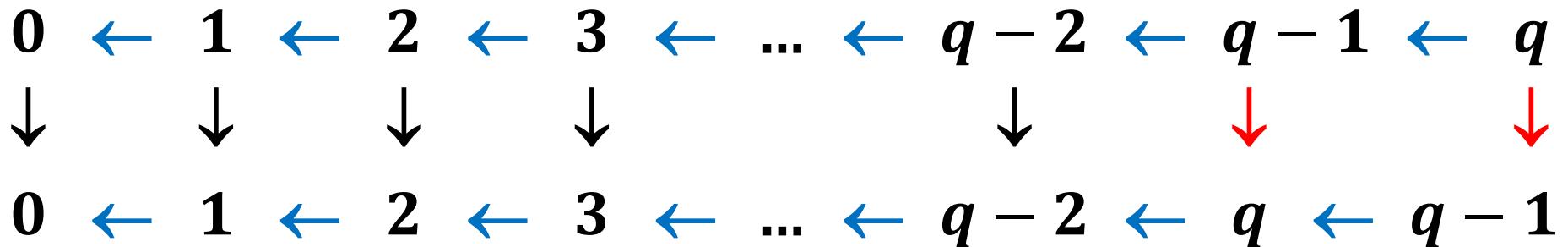
- introduce ruggedness via weak causality
- objective values: 0 is best,  $q \approx (n - o)t$  is worst

$$\begin{array}{ccccccccccccccccc} 0 & \leftarrow & 1 & \leftarrow & 2 & \leftarrow & 3 & \leftarrow & \dots & \leftarrow & q-2 & \leftarrow & q-1 & \leftarrow & q \\ & & 1 & + & 1 & + & 1 & + & 1 & + & 1 & + & 1 & + & 1 & + & 1 \end{array}$$

- $\gamma = 0 \Rightarrow \Delta(r_\gamma) = q$

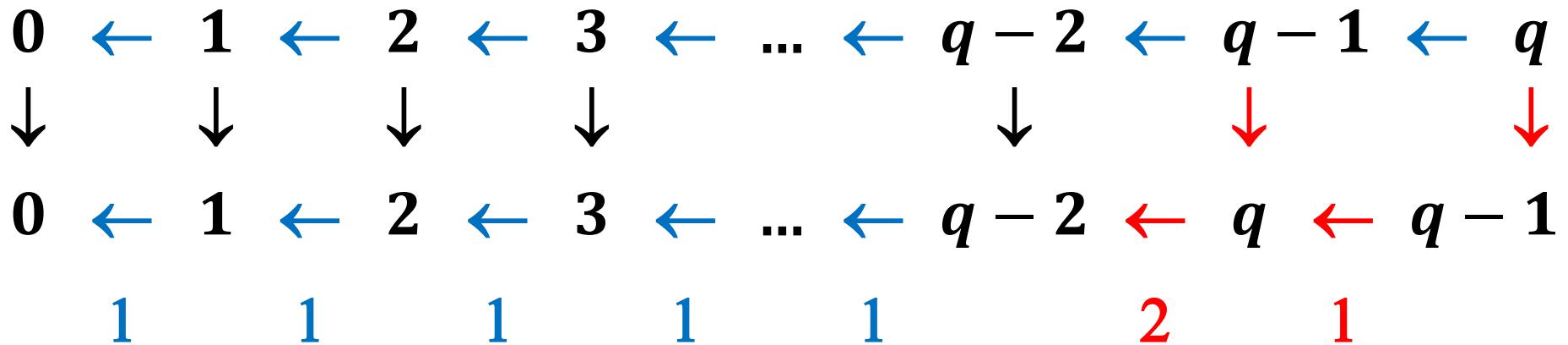
# Model Problem – Ruggedness

- introduce ruggedness via weak causality
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# Model Problem – Ruggedness

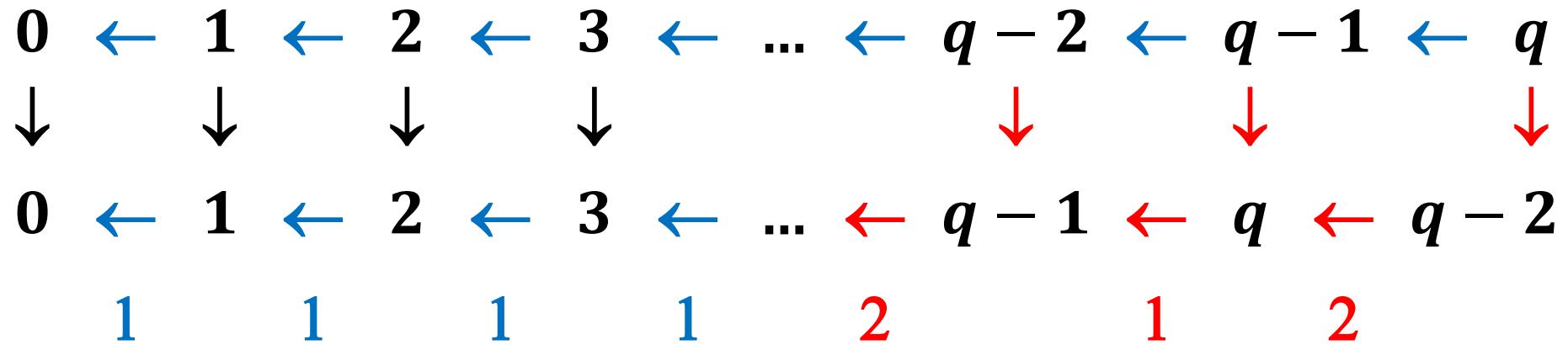
- introduce ruggedness via weak causality
- objective values: 0 is best,  $q \approx (n - o)t$  is worst



- $\gamma = 1 \Rightarrow \Delta(r_\gamma) = q + 1$

# Model Problem – Ruggedness

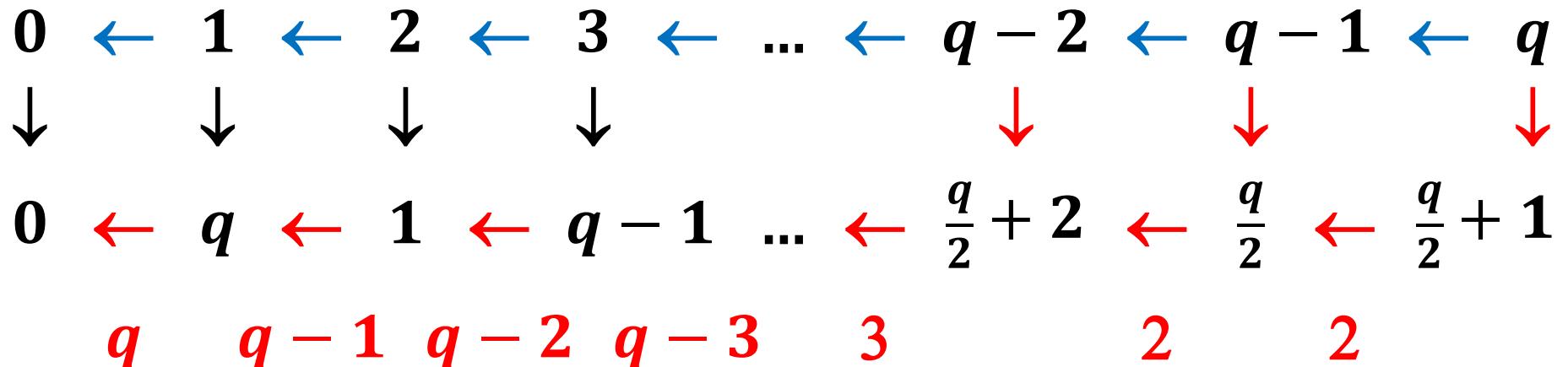
- introduce ruggedness via weak causality
  - objective values: 0 is best,  $q \approx (n - o)t$  is worst



- $\gamma = 2 \Rightarrow \Delta(r_\gamma) = q + 2$

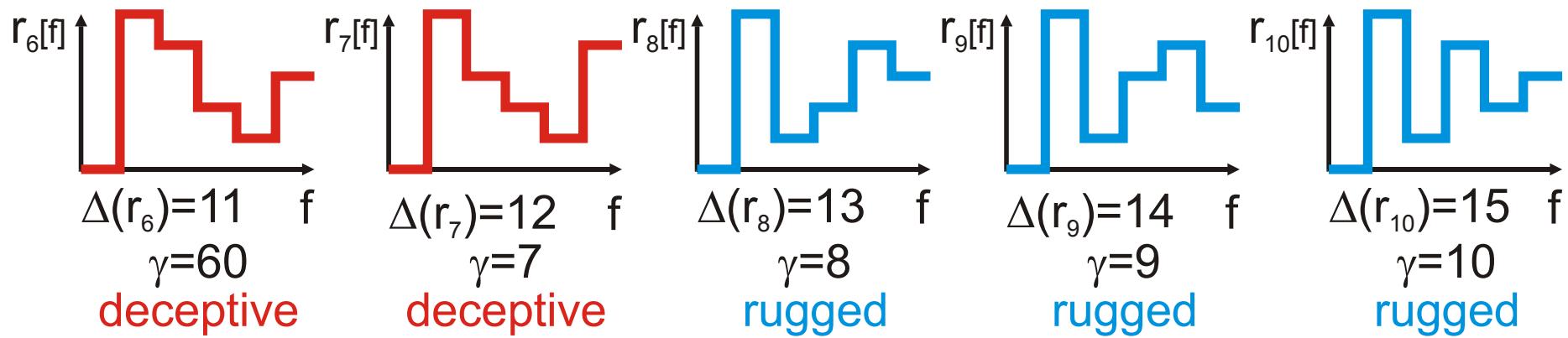
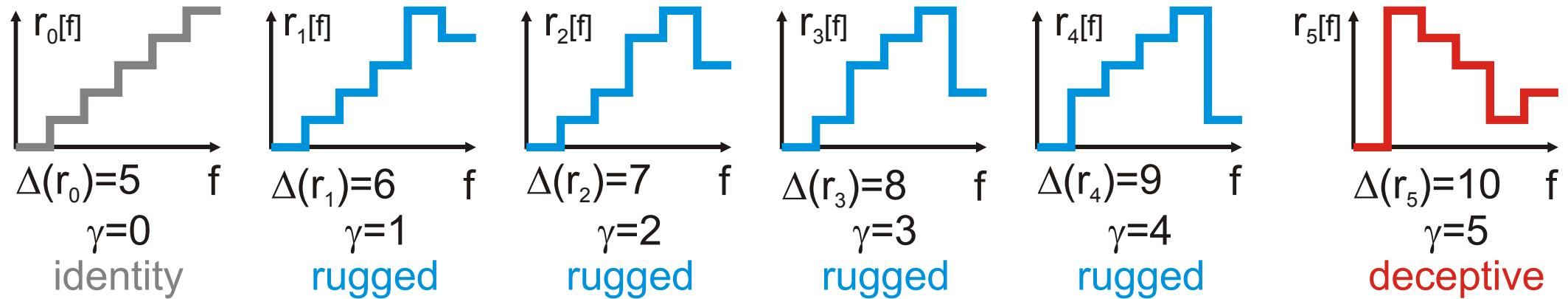
# Model Problem – Ruggedness

- introduce ruggedness via weak causality
- objective values: 0 is best,  $q \approx (n - o)t$  is worst

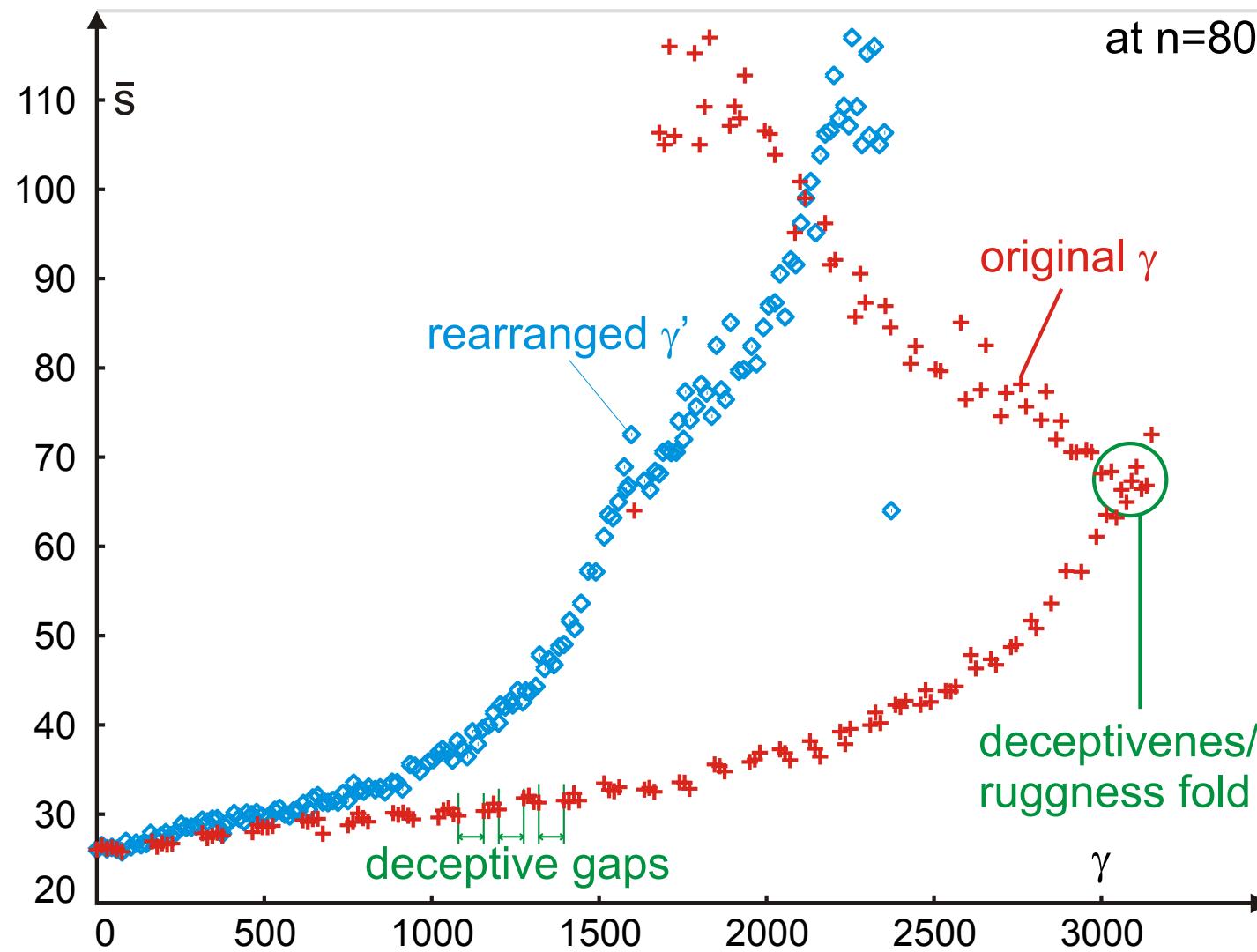


$$\gamma = \frac{q(q-1)}{2} \Rightarrow \Delta(r_\gamma) = q + \frac{q(q-1)}{2} = \frac{q(q+1)}{2}$$

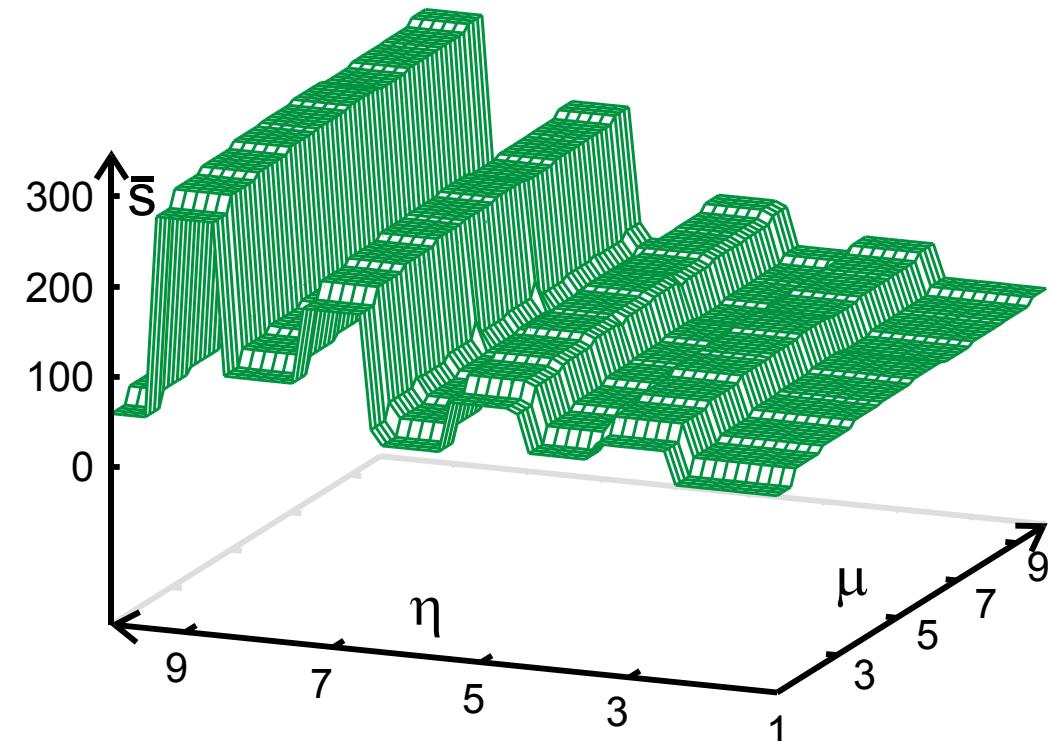
# Model Problem – Ruggedness



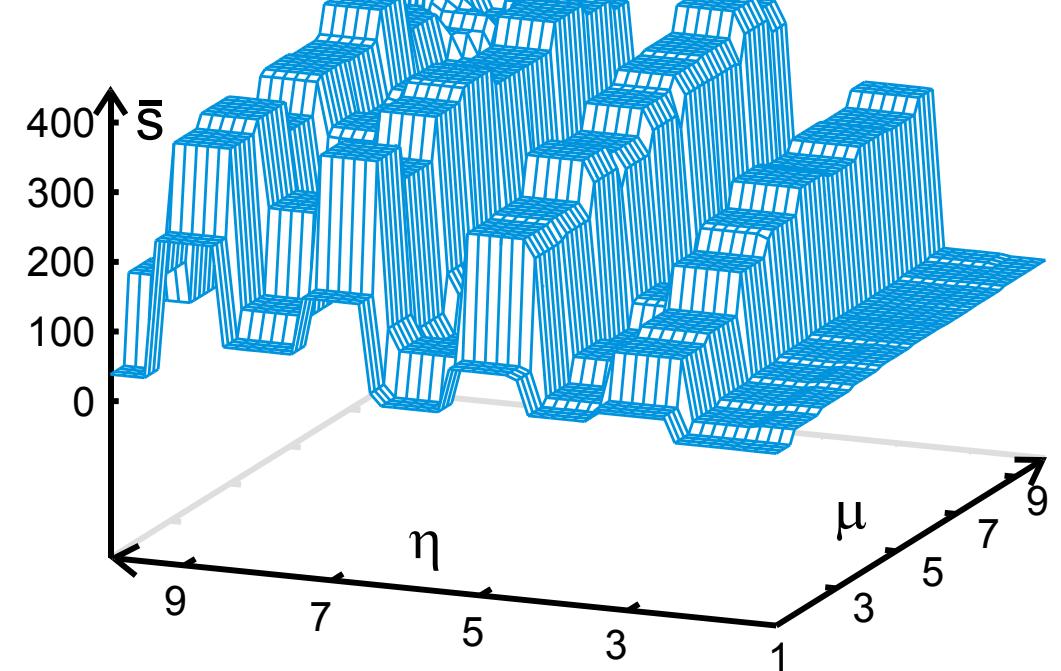
# Model Problem – Ruggedness



# Combinations – Epistasis/Neutrality

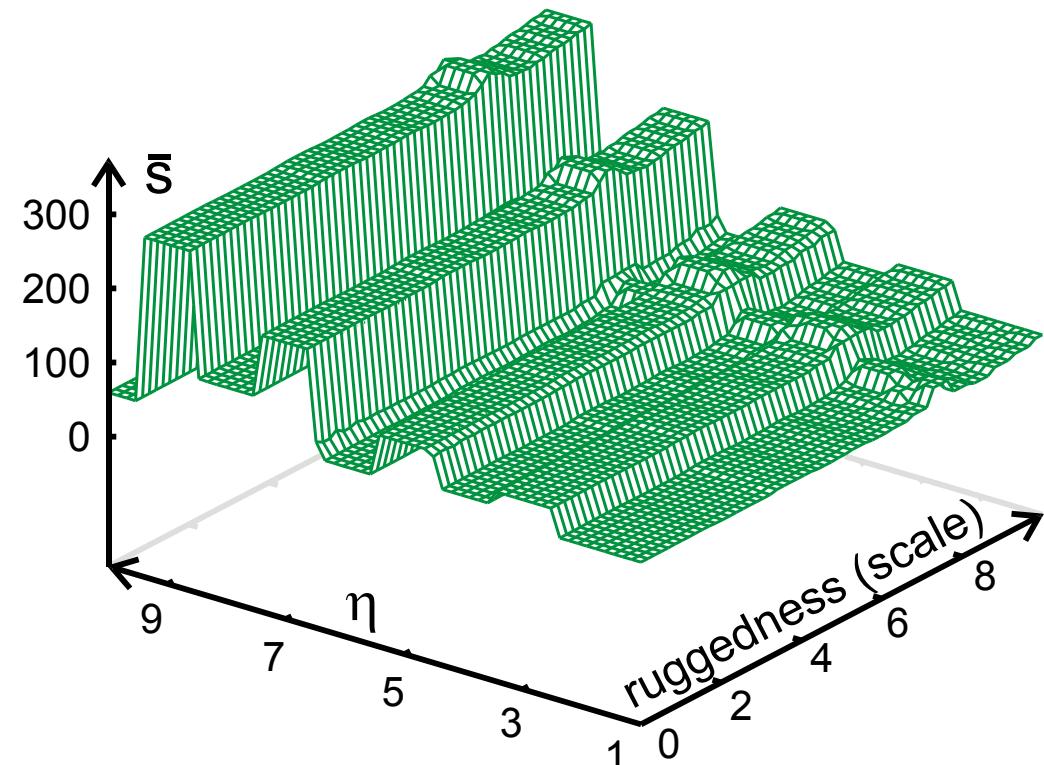


expected outcome

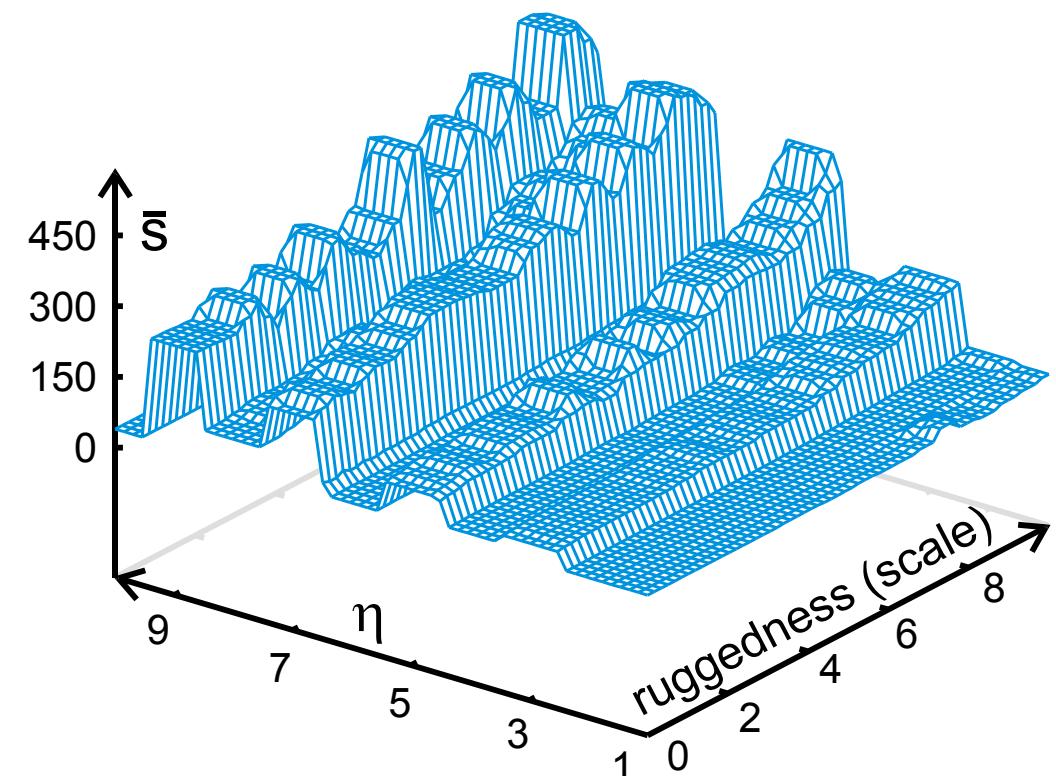


experimental outcome

# Combinations – Epistasis/Ruggedness



expected outcome



experimental outcome

# Conclusions

- model problem with tunable fitness landscape features
- additionally: scale from 0 to 10 for all features
- shown that the presented approaches are viable
- extremely fast evaluation ( $ps=1000$ ,  $0.1s < t/gen < 0.3s$ ,)
- improvements required, e.g. for epistasis/ruggedness investigation on EA parameter influence on search
- Online available:  
<http://iao.hfuu.edu.cn/research/publications>

# Thanks for your kind attention!

## You may ask questions now.

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[tweise@gmx.de](mailto:tweise@gmx.de)

# Model Problem

## Genotype

 $g \in \mathbb{G}$ 

010001100000111010000

## Introduction of Neutrality

 $\mu=2$ 
 $u_2(g) \in \mathbb{G}$ 

01 00 01 10 00 00 11 10 10 00 0  
 ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓  
 1 0 1 1 0 0 1 1 1 0 ✗

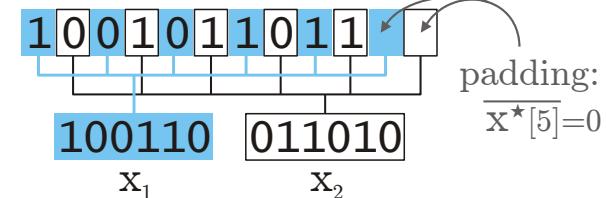
## Introduction of Epistasis

 $\eta=4$ 

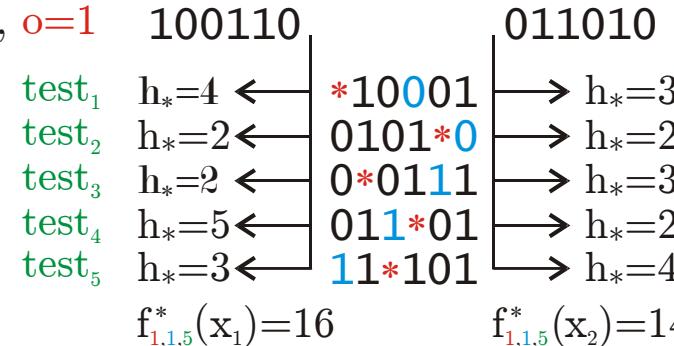
1011 0011 10  
 $e_4 \downarrow$   $e_4 \downarrow$   $e_2 \downarrow$   
 1001 0110 11

insufficient bits,  
 at the end, use  
 $\eta=2$  instead of  
 $\eta=4$

## Multi-Objectivity and Phenotype

 $m=2, n=6$ 
 $(x_1, x_2) \in \mathbb{X}$ 


## Introduction of Overfitting

 $t=5, \epsilon=1, o=1$ 


## Introduction of Ruggedness

 $\gamma'=34$ 
 $\gamma=57, q=25$   
 $(\Delta(r_{\gamma=57})=82)$ 

$$f_{1,1,5}(x_1)=16 \quad f_{1,1,5}(x_2)=14$$

$$r_{57}[f_{1,1,5}(x_1)]=17 \quad r_{57}[f_{1,1,5}(x_2)]=15$$

# Related Work I

- Kauffmann's NK fitness landscapes (Rugged/Epistatic)
- $N$  bits, each contributes to total fitness (sum of contribs)
- each contribution influenced by  $K$  neighboring genes
- S.A. Kauffman. *Adaptation on rugged fitness landscapes*. Lectures in the Sciences of Complexity: 1988 Compl. Sys. Summer School, ISBN 978-0201510157, 0201510154.
- S.A. Kauffman and S. Asher. *Towards a general theory of adaptive walks on rugged landscapes*. Journal of Theoretical Biology, 128(1):11–45, 1987
- S.A. Kauffman and E.D. Weinberger. *The NK model of rugged fitness landscapes and its application to maturation of the immune response*. Journal of Theoretical Biology, 141:211–245, 1989. [http://dx.doi.org/10.1016/S0022-5193\(89\)80019-0](http://dx.doi.org/10.1016/S0022-5193(89)80019-0)
- L. Altenberg. NK Fitness Landscapes. In Handbook of Evolutionary Computation, 1997. ISBN 0750303921. <http://citeseer.ist.psu.edu/704814.html> and <http://dynamics.org/Altenberg/FILES/LeeNKFL.pdf>

# Related Work II

- NKp landscape: add neutrality to NK landscape by setting a fraction  $p$  of contribution to zero
- L. Barnett. *Ruggedness and neutrality – the NKp family of fitness landscapes*. Artificial Life VI: Proceedings of the sixth international conference on Artificial life, 1998, ISBN 0-262-51099-5. See <http://citeseer.ist.psu.edu/barnett98ruggedness.html> and [ftp://ftp.informatics.sussex.ac.uk/pub/users/lionelb/publications/alife6\\_paper.pdf](ftp://ftp.informatics.sussex.ac.uk/pub/users/lionelb/publications/alife6_paper.pdf)
- NKq landscape: add neutrality to NK landscape by discretizing contributions into interval  $[0, q) : q \geq 1$
- M. E. J. Newman and R. Engelhardt. *Effect of neutral selection on the evolution of molecular species*. Proceedings of the Royal Society of London B, 256(1403):1333–1338, 1998. Online available at <http://citeseer.ist.psu.edu/202669.html> and <http://journals.royalsociety.org/content/5ttwagyubu88bang/fulltext.pdf>

# Related Work III

- Technological landscape: add neutrality to NK landscape by discretizing total fitness into  $M$  bins
- J. Lobo, J.H. Miller, and W. Fontana. *Neutrality in technological landscapes*. Santa Fe working paper, 2004. Online available at <http://citeseer.ist.psu.edu/lobo04neutrality.html> and <http://fontana.med.harvard.edu/www/Documents/WF/Papers/tech.neut.pdf>
- p-Spin model: complete evolutionary model for epistasis and ruggedness with selection/mutation,  $N$  bits, ***all*** sets of  $K$  genes contribute to total fitness
- C. Amitrano, L. Peliti, and M. Saber. Population dynamics in a spin-glass model of chemical evolution. *Journal of Molecular Evolution*, 29(6):513–525, 1989. Online available at <http://www.springerlink.com/content/n172u715884w0632/fulltext.pdf>

# Related Work IV

- ND fitness landscape:
  - neutrality:  $N$  bits,  $D$  is distribution of number of neutral neighbors of genotypes
  - deceptiveness: fitness of neutral networks set according to Trap Function
- W. Beaudoin, S. Verel, P. Collard, and C. Escazut. *Deceptiveness and neutrality the ND family of fitness landscapes*. In GECCO '06: Proceedings of the 8th Annual Conference on Genetic and Evolutionary Computation, 2006, ACM Press, ISBN 1-59593-186-4. Online available at <http://doi.acm.org/10.1145/1143997.1144091>
- K. Deb and D.E. Goldberg. *Analyzing deception in trap functions*. In Proceedings of the Second Workshop on Foundations of Genetic Algorithms, 1992, Morgan Kaufmann, ISBN 1-55860-263-1

# Related Work V

- Royal Road: search for schemas of doubling order
- neutrality via don't cares
- M. Mitchell, S. Forrest, and J.H. Holland. The Royal Road for Genetic Algorithms: Fitness Landscapes and GA Performance. In Toward a Practice of Autonomous Systems: Proceedings of the First European Conference on Artificial Life, 1991, ISBN 0-262-72019-1. See <http://citeseer.ist.psu.edu/mitchell91royal.html> and <http://web.cecs.pdx.edu/~mm/ecal92.pdf>

# Ruggedness-generating Algorithms

---

**Algorithm 19.4:**  $r_\gamma \leftarrow \text{buildRPermutation}(\gamma, \hat{f})$

---

**Input:**  $\gamma$ : the  $\gamma$  value  
**Input:**  $\hat{f}$ : the maximum objective value  
**Data:**  $i, j, d, \text{tmp}$ : temporary variables  
**Data:**  $k, \text{start}, r$ : parameters of the subalgorithm  
**Output:**  $r_\gamma$ : the permutation  $r_\gamma$

```

1 begin
2   Subalgorithm  $r \leftarrow \text{permutate}(k, r, \text{start})$ 
3   begin
4     if  $k > 0$  then
5       if  $k \leq (\hat{f} - 1)$  then
6          $r \leftarrow \text{permutate}(k - 1, r, \text{start})$ 
7          $\text{tmp} \leftarrow r[\hat{f}]$ 
8          $r[\hat{f}] \leftarrow r[\hat{f}-k]$ 
9          $r[\hat{f}-k] \leftarrow \text{tmp}$ 
10      else
11         $i \leftarrow \lfloor \frac{\text{start}+1}{2} \rfloor$ 
12        if  $(\text{start} \bmod 2) = 0$  then
13           $i \leftarrow \hat{f} + 1 - i$ 
14           $d \leftarrow -1$ 
15        else
16           $d \leftarrow 1$ 
17        for  $j \leftarrow \text{start}$  up to  $\hat{f}$  do
18           $r[j] \leftarrow i$ 
19           $i \leftarrow i + d$ 
20         $r \leftarrow \text{permutate}(k - \hat{f} + \text{start}, r, \text{start} + 1)$ 
21      end
22     $r \leftarrow (0, 1, 2, \dots, \hat{f} - 1, \hat{f})$ 
23    return  $\text{permutate}(\gamma, r, 1)$ 
24 end

```

---



---

$\gamma \leftarrow \text{translate}(\gamma', \hat{f})$

---

**Input:**  $\gamma'$ : the raw  $\gamma$  value  
**Input:**  $\hat{f}$ : the maximum value of  $f_{\varepsilon, o, t}$   
**Data:**  $i, j, k, l$ : some temporary variables  
**Output:**  $\gamma$ : the translated  $\gamma$  value

```

1 begin
2    $l \leftarrow \frac{\hat{f}(\hat{f}-1)}{2}$ 
3    $i \leftarrow \left\lfloor \frac{\hat{f}}{2} \right\rfloor * \left\lfloor \frac{\hat{f}+1}{2} \right\rfloor$ 
4   if  $\gamma \leq \hat{f}^i$  then
5      $j \leftarrow \left\lfloor \frac{\hat{f}+2}{2} - \sqrt{\frac{\hat{f}^2}{4} + 1 - \gamma} \right\rfloor$ 
6      $k \leftarrow \gamma - j(\hat{f} + 2) + j^2 + \hat{f}$ 
7     return  $k + 2(j(\hat{f} + 2) - j^2 - \hat{f}) - j$ 
8   else
9      $j \leftarrow \left\lfloor \frac{(\hat{f} \bmod 2)+1}{2} + \sqrt{\frac{1-(\hat{f} \bmod 2)}{4} + \gamma - 1 - i} \right\rfloor$ 
10     $k \leftarrow \gamma - (j - (\hat{f} \bmod 2))(j - 1) - 1 - i$ 
11    return  $l - k - 2j^2 + j - (\hat{f} \bmod 2)(-2j + 1)$ 
12 end

```

---

# Neutrality

- Neutrality can have positive and negative effects
- Negative: - uniform redundancy
  - neutral networks with very high proportion of neutral search operation outcomes
- Beaudoin et al. *Deceptiveness and neutrality the ND family of fitness landscapes.*
- Positive: - neutral networks with moderate proportion of neutral search operations
- M. Shackleton, R. Shipman, and M. Ebner. An investigation of redundant genotype-phenotype mappings and their role in evolutionary search. Congress on Evolutionary Computation, pages 493–500, 2000. <http://citeseer.ist.psu.edu/409243.html>
- M. Toussaint and C. Igel. Neutrality: A Necessity for Self-Adaptation. Congress on Evolutionary Computation, pages 1354–1359, 2002.

# Strange Epistasis Behavior

- $e_\eta(z)_{[k]} = \begin{cases} \text{xor}_{\substack{\forall i: 0 \leq i < \eta, \\ i \neq (\textcolor{red}{k-1}) \% \eta}} z_{[i]} & \forall z: 0 \leq z < 2^{\eta-1} \\ \overline{e_\eta(z - 2^{\eta-1})_{[k]}} & \text{otherwise} \end{cases}$
- $e_\eta$  consists of two parts (second = negated first one)
- otherwise not bijective for even  $\eta$
- mappings with  $\eta = 2k + 2, k \in \mathbb{N}$  are much simpler
- then, there exist many pairs with

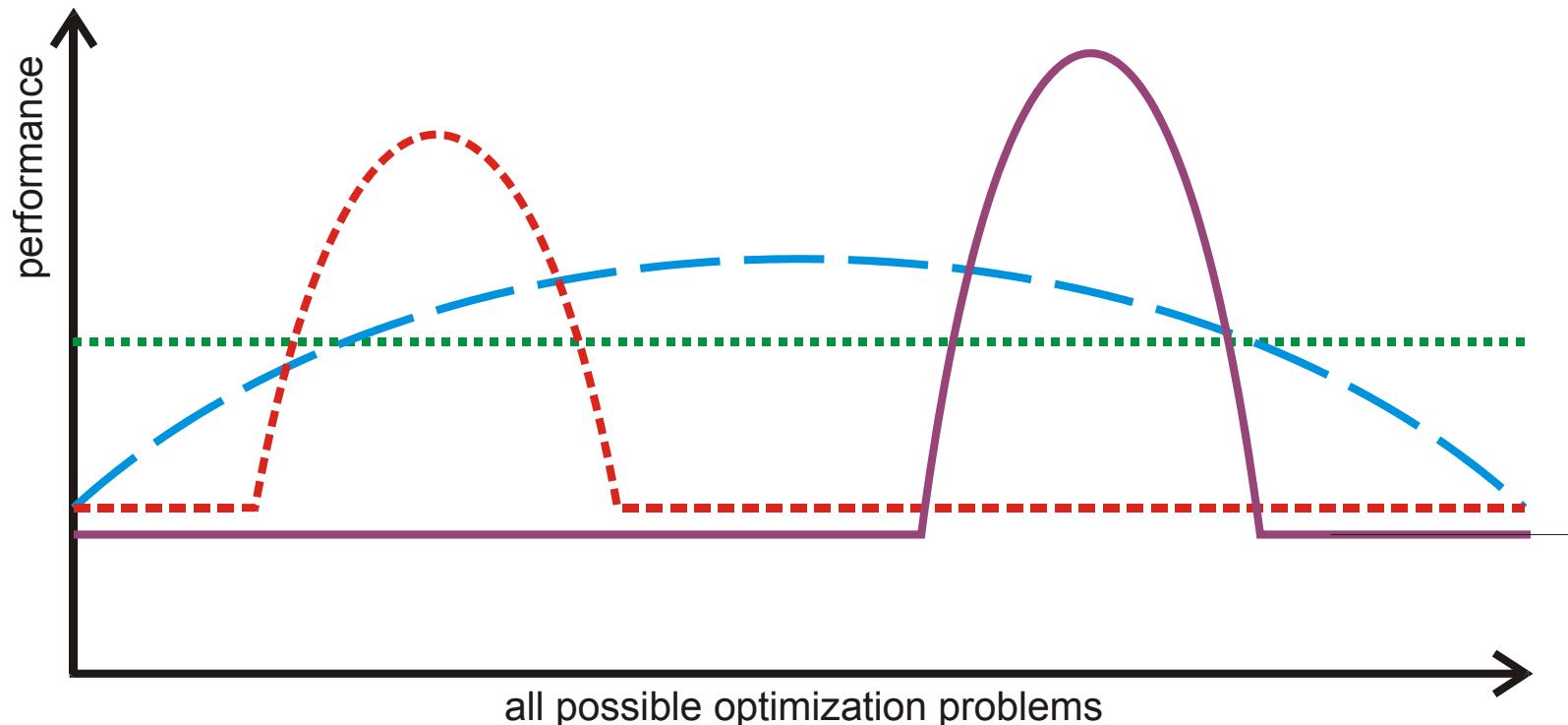
$$\left[ h(z_1, z_2) = \frac{1}{2}\eta \right] \wedge \left[ h(e_\eta(z_1), e_\eta(z_2)) \neq \frac{1}{2}\eta \right]$$

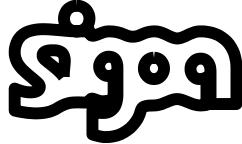
# Epistasis vs. Ruggedness

- Epistasis
  - explicit dependency between *groups of genes*
  - one configuration of the group  
⇒ one distinct fitness contribution (tendency)
- Ruggedness permutations
  - influence independent from gene groups ( $< n$ ), assume
    - + basic problem ( $n = 10$ ) + ruggedness
    - + individual with: 6 bits wrong, 4 bits correct
    - + toggling *any* correct bit ⇒ the same result  $x$
    - + toggling *any* wrong bit ⇒ same result  $y$
- Either no epistasis or weak epistasis involving all  $n$  genes...

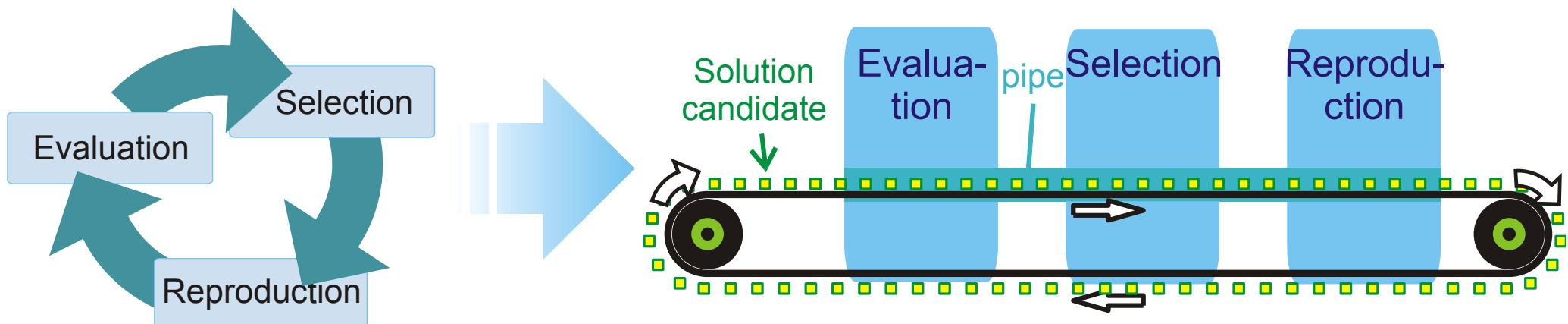
# No Free Lunch (NFL)

- All optimization algorithms have the same performance averaged over all possible optimization problems.
- D.H. Wolpert and W.G. Macready. No Free Lunch Theorems for Optimization. IEEE Trans. on Ev. Comp., 1(1):67–82, 1997. <http://citeseer.ist.psu.edu/wolpert96no.html>





- Simple Interface for Global Optimization
- versatile Java framework for global optimization
- especially suitable for problems involving complex simulations
- pipes and filters principle
- open source, <http://www.sigoa.org/>



Thomas Weise, Stefan Niemczyk, Hendrik Skubch,  
Roland Reichle, and Kurt Geihs

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