合肥学院
HEFEI UNIVERSITY

Solving Job Shop Scheduling Problems Without Using a Bias for Good Solutions

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Hefei, Anhui, China	中国安徽省合肥市

Outline

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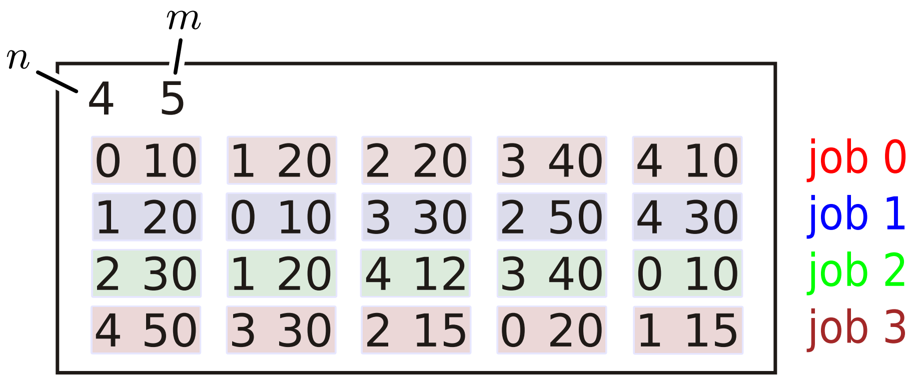

```
4   5
0 10   1 20   2 20   3 40   4 10
1 20   0 10   3 30   2 50   4 30
2 30   1 20   4 12   3 40   0 10
4 50   3 30   2 15   0 20   1 15
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- A JSSP instance defined by n jobs with operations that need to be processed by each of the m machines in a job-specific order.
- Each such operation of each job needs a specific time.
- The goal is to find the assignment of jobs to machines with the smallest possible makespan, i.e., schedule that finishes fastest.

Encoding and Solutions

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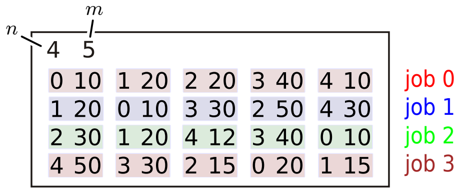

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0 10  1 20  2 20  3 40  4 10
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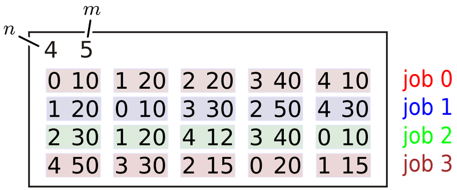

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example point in the search space:
permutation with repetitions:

x=(0, 2, 1, 0, 3, 1, 0, 1, 2, 3,
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Encoding and Solutions

n m

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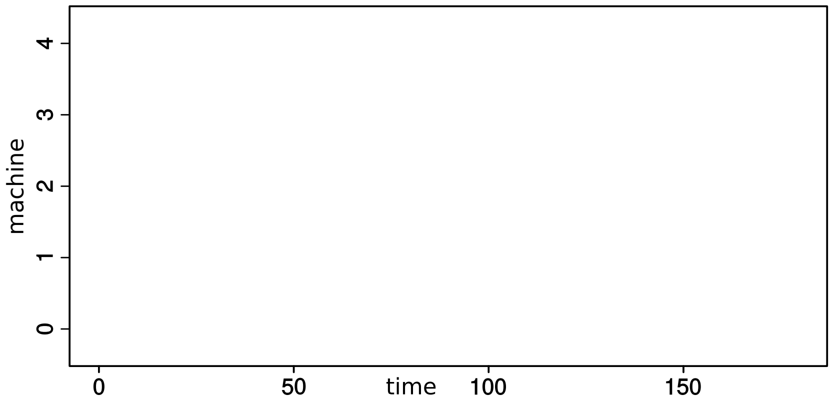

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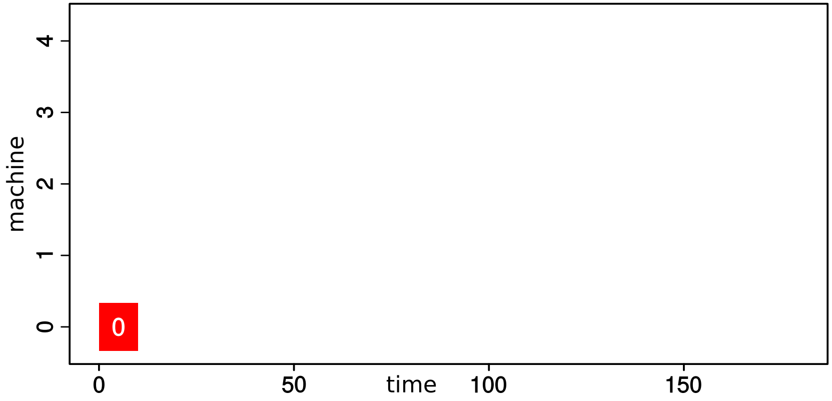

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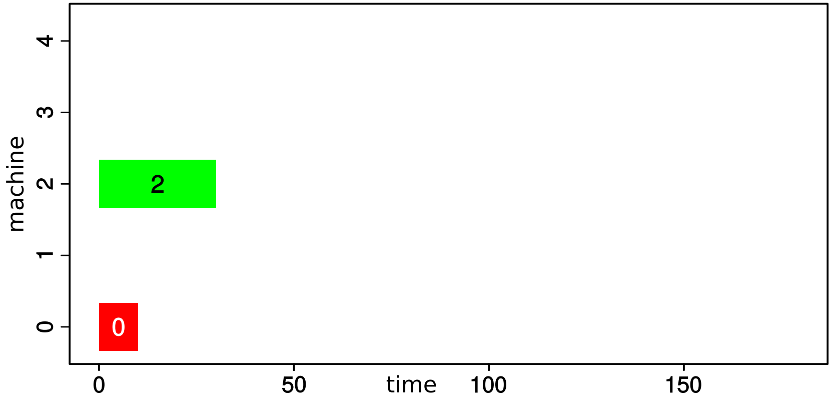

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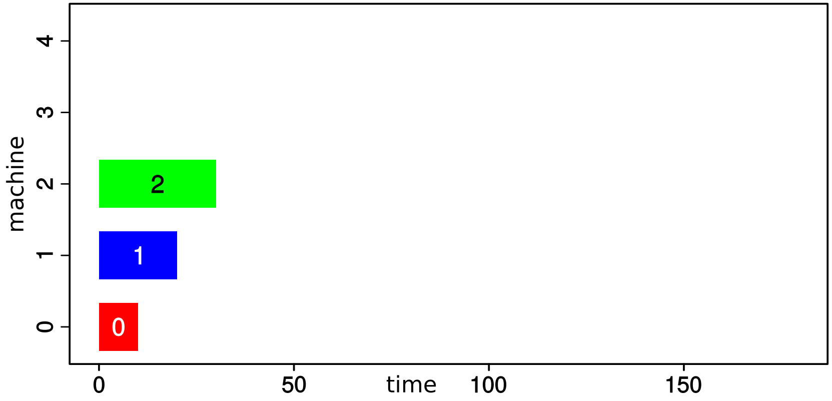

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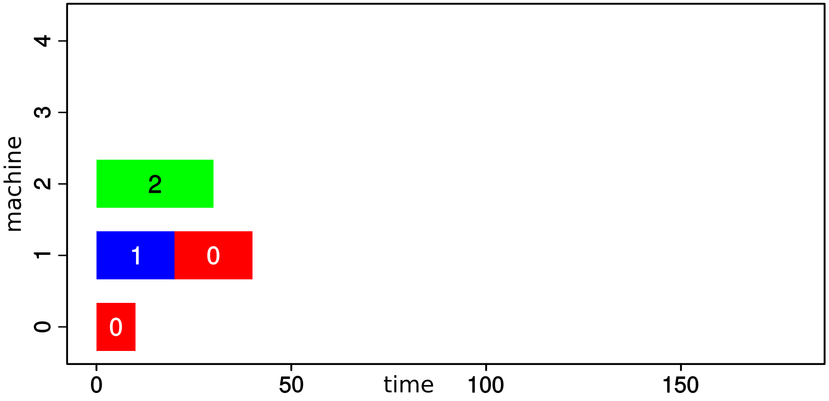

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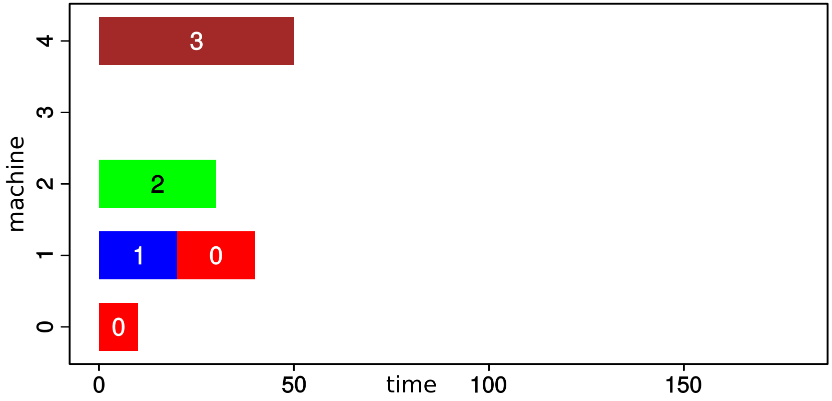

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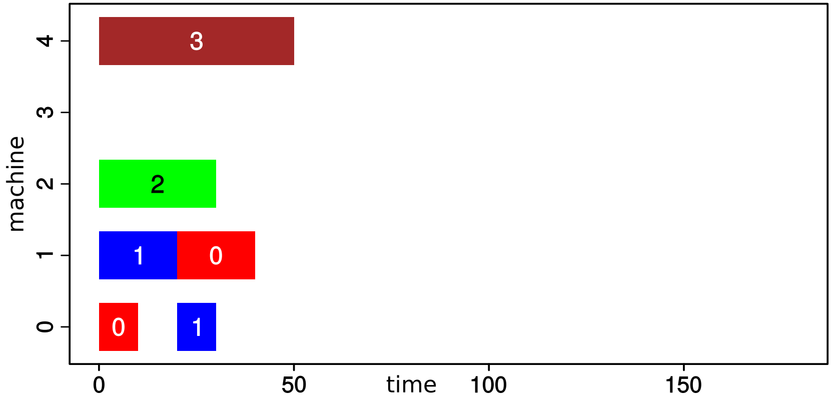

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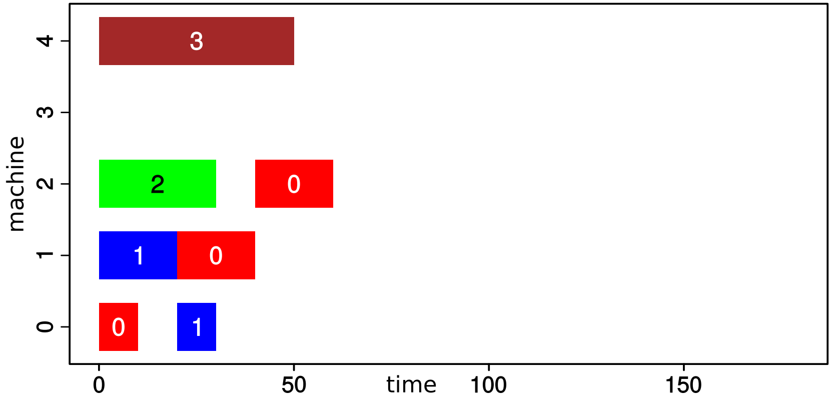

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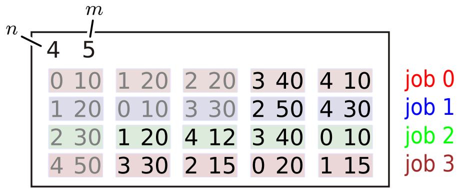

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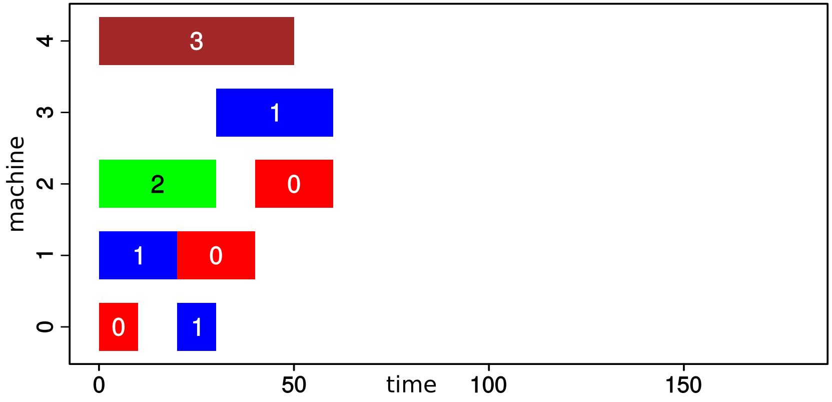

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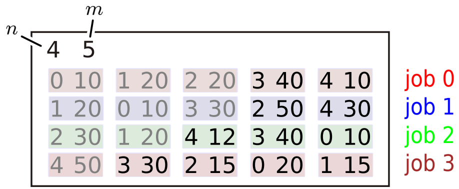

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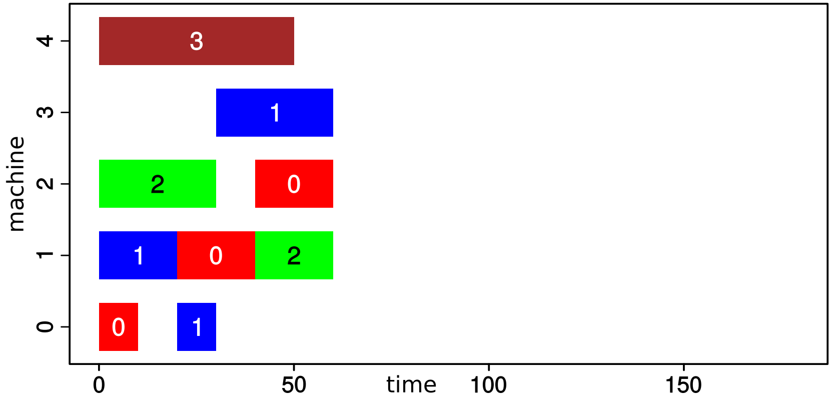

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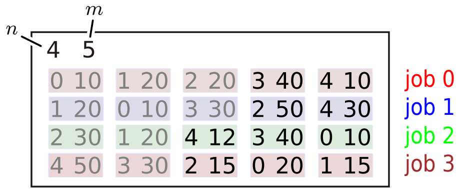

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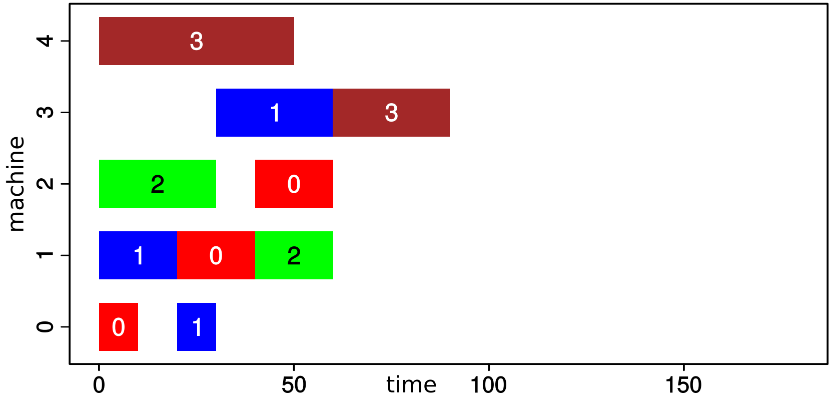

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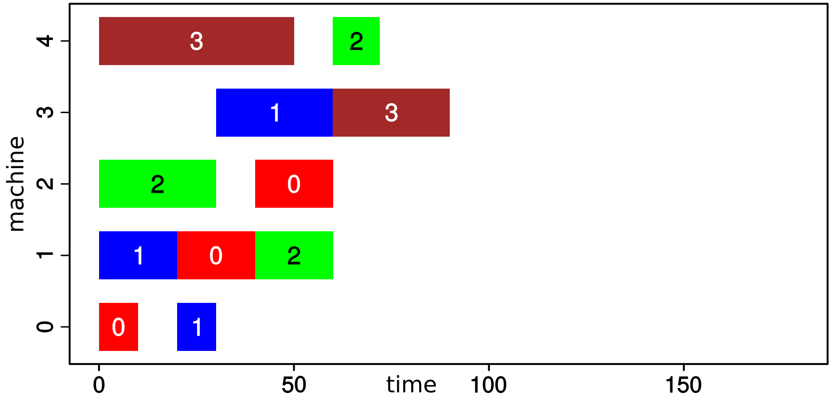

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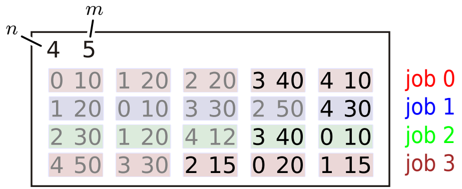

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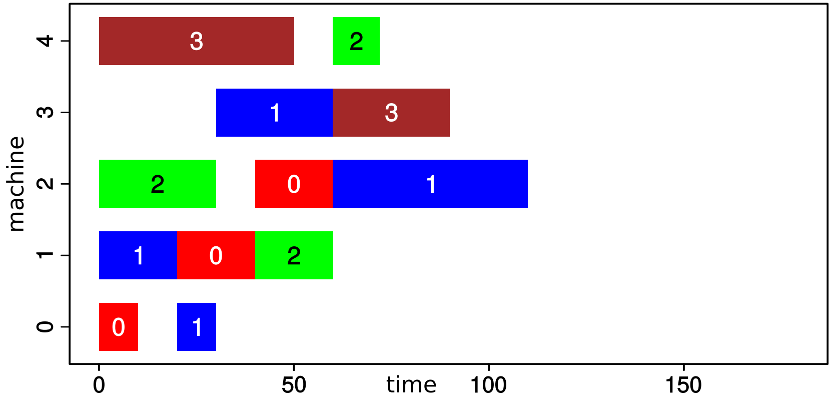

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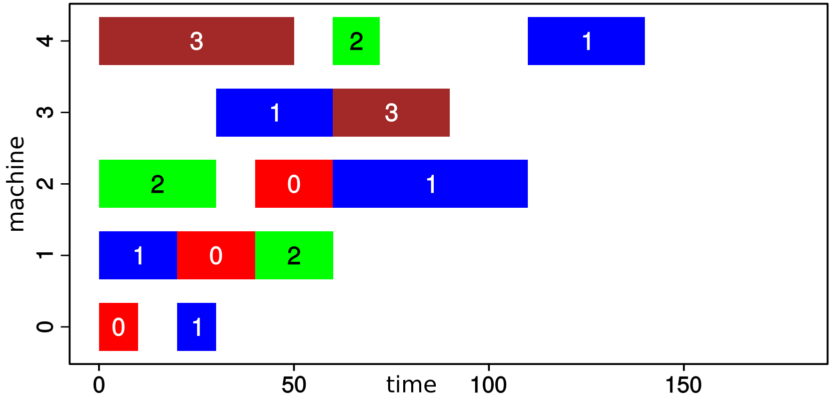

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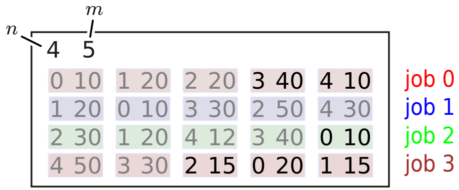

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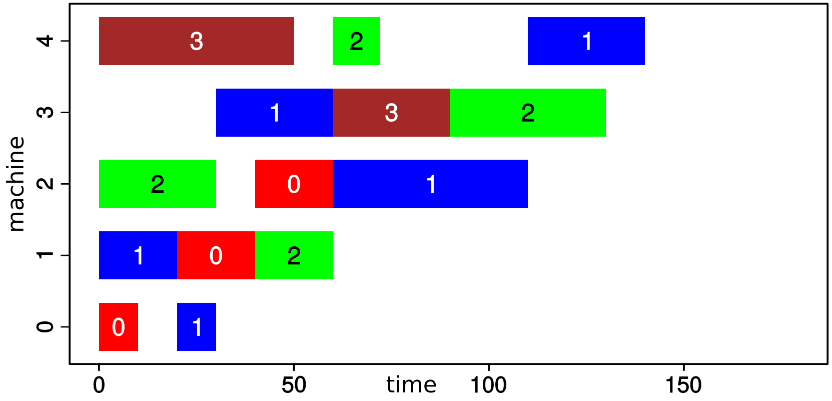

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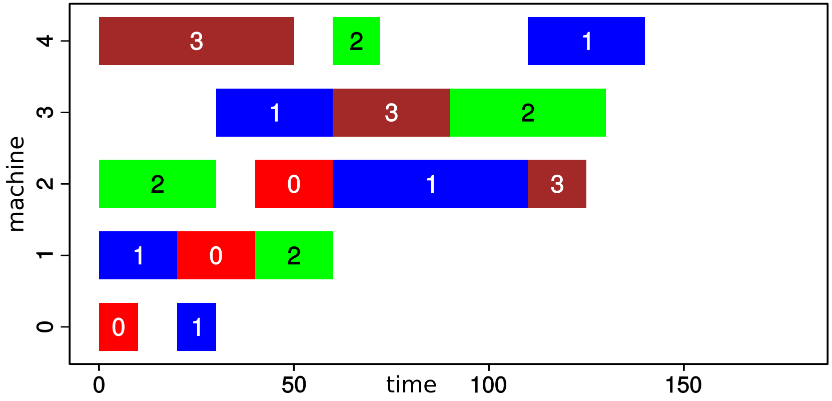

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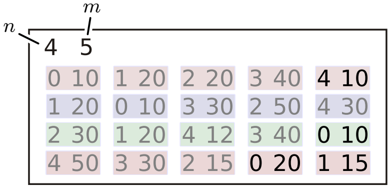

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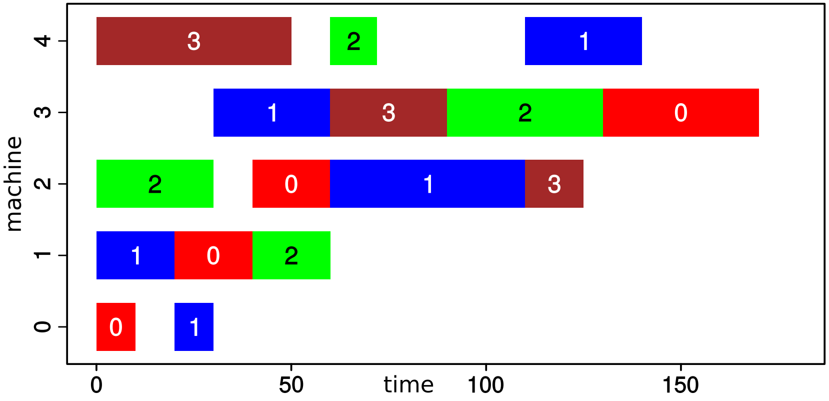

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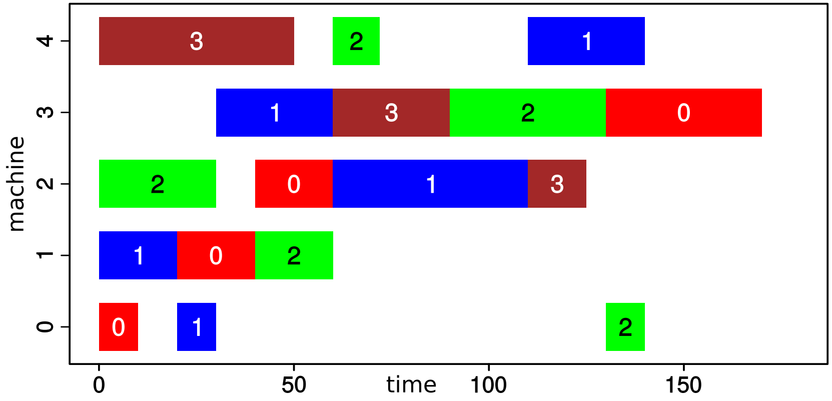

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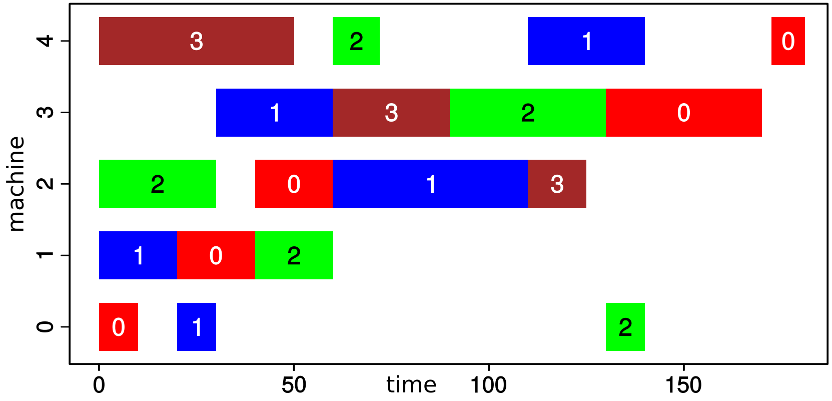

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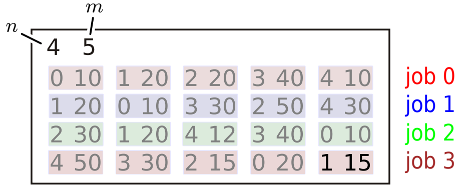

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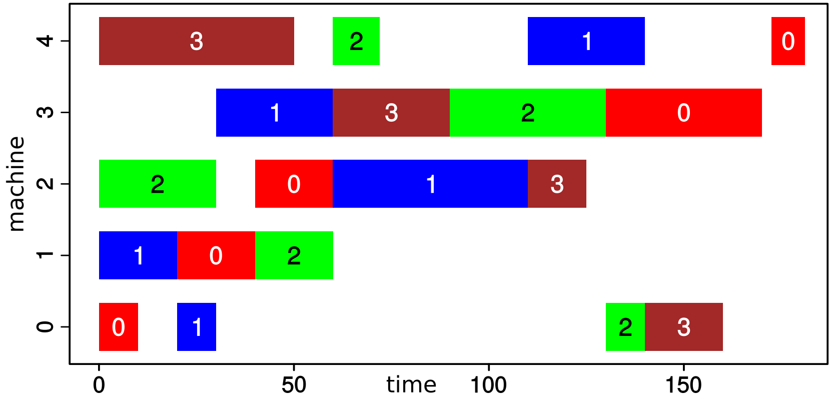

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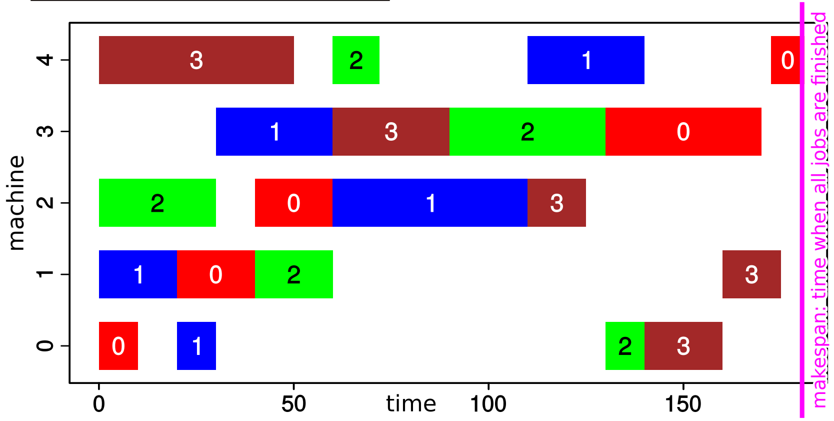

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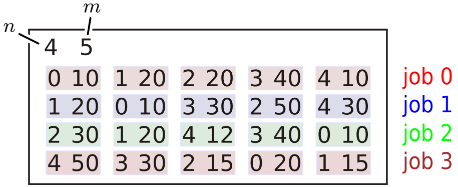

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example point in the search space: permutation with repetitions:

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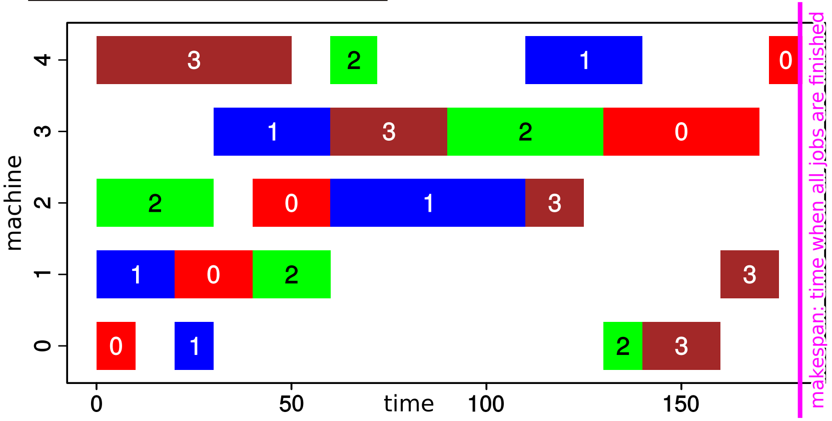

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- Unary search operator $move$ picks two random indices in the string with different job IDs and swaps these IDs.
- All of this is fairly standard ($\geq$ 27 year old stuff).

Frequency Fitness Assignment (FFA)

Solving Optimization Problems

- Input: objective function $f : \mathcal{X} \mapsto \mathcal{Y}$ that maps the elements of a solution space $\mathcal{X}$ to a subset $\mathcal{Y} \subseteq \mathcal{R}$ of the real numbers $\mathcal{R}$.

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- How do we find good solution?

Solving Optimization Problems

- Input: objective function $f : \mathcal{X} \mapsto \mathcal{Y}$ that maps the elements of a solution space $\mathcal{X}$ to a subset $\mathcal{Y} \subseteq \mathcal{R}$ of the real numbers $\mathcal{R}$.
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- We are looking for solutions with harder-to-find objective values.
- $\Longrightarrow$ Algorithms basing all decisions on FFA are not biased towards better solutions.
- Can optimization without bias for better solutions even work?

(1+1)-EA **and** (1+1)-FEA

- We plug FFA into the simplest possible evolutionary algorithm, the (1+1)-EA and we get the (1+1)-FEA.

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```
proc (1+1)-FEA($f : \mathcal{X} \mapsto 0..UB$)
    $H[0..UB] \leftarrow (0, 0, \cdots, 0)$;
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    randomly sample $x_c$ from $\mathcal{X}$; $y_c \leftarrow f(x_c)$;

    while $\neg$ terminate do
        $x_n \leftarrow move(x_c)$; $y_n \leftarrow f(x_n)$;

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    **return** $(x_b, y_b)$;
```

(b) (1+1)-FEA

(1+1)-EA and (1+1)-FEA

- We plug FFA into the simplest possible evolutionary algorithm, the (1+1)-EA and we get the (1+1)-FEA.

```
proc (1+1)-EA($f : \mathcal{X} \mapsto 0..UB$)

    randomly sample $x_c$ from $\mathcal{X}$; $y_c \leftarrow f(x_c)$;

    while ¬ terminate do
        $x_n \leftarrow move(x_c)$; $y_n \leftarrow f(x_n)$;


        if $y_n \leq y_c$ then $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;
    return ($x_c, y_c$);
```

(a) (1+1)-EA

```
proc (1+1)-FEA($f : \mathcal{X} \mapsto 0..UB$)
    $H[0..UB] \leftarrow (0, 0, \cdots, 0)$;
    randomly sample $x_c$ from $\mathcal{X}$; $y_c \leftarrow f(x_c)$;
    $x_b \leftarrow x_c$;
    while ¬ terminate do
        $x_n \leftarrow move(x_c)$; $y_n \leftarrow f(x_n)$;

        $H[y_c] \leftarrow H[y_c] + 1$; $H[y_n] \leftarrow H[y_n] + 1$;
        if $H[y_n] \leq H[y_c]$ then $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;
    return ($x_b, y_b$);
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        if $y_n \leq y_c$ then $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;
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    return $(x_b, y_b)$;
```

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(1+1)-EA and (1+1)-FEA

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proc (1+1)-EA($f : \mathcal{X} \mapsto 0..UB$)

    randomly sample $x_c$ from $\mathcal{X}$; $y_c \leftarrow f(x_c)$;

    while $\neg$ terminate do
        $x_n \leftarrow move(x_c)$; $y_n \leftarrow f(x_n)$;


        if $y_n \leq y_c$ then $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;
    return ($x_c, y_c$);
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```
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    randomly sample $x_c$ from $\mathcal{X}$; $y_c \leftarrow f(x_c)$;
    $x_b \leftarrow x_c$; $y_b \leftarrow y_c$;
    while $\neg$ terminate do
        $x_n \leftarrow move(x_c)$; $y_n \leftarrow f(x_n)$;
        if $y_n < y_b$ then
        $H[y_c] \leftarrow H[y_c] + 1$; $H[y_n] \leftarrow H[y_n] + 1$;
        if $H[y_n] \leq H[y_c]$ then $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;
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(b) (1+1)-FEA

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        if $y_n < y_b$ then $x_b \leftarrow x_n$; $y_b \leftarrow y_n$;
        $H[y_c] \leftarrow H[y_c] + 1$; $H[y_n] \leftarrow H[y_n] + 1$;
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```
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    $x_b \leftarrow x_c$; $y_b \leftarrow y_c$;
    while $\neg$ terminate do
        $x_n \leftarrow move(x_c)$; $y_n \leftarrow f(x_n)$;
        if $y_n < y_b$ then $x_b \leftarrow x_n$; $y_b \leftarrow y_n$;
        $H[y_c] \leftarrow H[y_c] + 1$; $H[y_n] \leftarrow H[y_n] + 1$;
        if $H[y_n] \leq H[y_c]$ then $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;
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```

(b) (1+1)-FEA

- Search is driven entirely by frequency H.

(1+1)-EA and (1+1)-FEA

- We plug FFA into the simplest possible evolutionary algorithm, the (1+1)-EA and we get the (1+1)-FEA.

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    return $(x_b, y_b)$;
```

(b) (1+1)-FEA

- Search is driven entirely by frequency H.
- Whether a solution is better or worse plays no role in the algorithm's decisions.

(1+1)-EA and (1+1)-FEA

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```
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    return $(x_c, y_c)$;
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```
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    return $(x_b, y_b)$;
```

(b) (1+1)-FEA

- Search is driven entirely by frequency H.
- Whether a solution is better or worse plays no role in the algorithm's decisions.
- Can (1+1)-FEA even work?

(1+1)-EA and (1+1)-FEA

- We plug FFA into the simplest possible evolutionary algorithm, the (1+1)-EA and we get the (1+1)-FEA.

```
 1: proc (1+1)-EA($f : \mathcal{X} \mapsto 0..UB$)
 2:
 3:     randomly sample $x_c$ from $\mathcal{X}$; $y_c \leftarrow f(x_c)$;
 4:
 5:     while ¬ terminate do
 6:         $x_n \leftarrow move(x_c)$; $y_n \leftarrow f(x_n)$;
 7:
 8:
 9:         if $y_n \leq y_c$ then $x_c \leftarrow x_n$; $y_c \leftarrow y_n$;
10:     return $(x_c, y_c)$;
```

(a) (1+1)-EA

```
 1: proc (1+1)-FEA($f : \mathcal{X} \mapsto 0..UB$)
 2:     $H[0..UB] \leftarrow (0, 0, \cdots, 0)$;
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```

(b) (1+1)-FEA

- Search is driven entirely by frequency H.
- Whether a solution is better or worse plays no role in the algorithm's decisions.
- Can (1+1)-FEA even work on a hard problem like the JSSP?

Experiment and Results

Experiment

- We use the 8 most common sets: `abz`[8], `dmu`[9], `ft`[10], `la`[11], `orb`[12], `swv`[13], `ta`[14], and `yn`[15].

Experiment

- We use the 8 most common sets: `abz`[8], `dmu`[9], `ft`[10], `la`[11], `orb`[12], `swv`[13], `ta`[14], and `yn`[15].
- We do 5 runs for each algorithm on each of these 242 instances.

Experiment

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- We do 5 runs for each algorithm on each of these 242 instances.
- Runtime limit: 2^{30} FEs ($\approx 10^9$ FEs)

Results

inst	EA			FEA			FEA vs. EA		
	best	*mean*	*conv*	*best*	*mean*	*conv*	*best*	*mean*	*conv*
`abz`	1	0	5	5	5	0	-0.3%	-1.2%	22.0
`dmu`	65	60	79	20	23	1	3.8%	3.4%	4.0
`ft`	2	1	2	3	3	1	-0.2%	-1.3%	5.0
`la`	32	23	38	35	37	2	-0.1%	-0.5%	32.0
`orb`	2	0	8	10	10	2	-1.5%	-3.6%	13.0
`swv`	12	10	19	13	15	1	0.1%	-0.6%	8.6
`ta`	59	52	80	29	33	0	1.1%	1.0%	20.0
`yn`	1	0	4	3	4	0	-0.5%	-0.6%	4.0

best (*mean*): number of instances algorithm reached the best (best average) solution

conv: number of instances algorithm reached end solution (stopped improving) fastest

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- (1+1)-FEA reaches best mean results in 6/8 benchmark sets and best results in 5/8.

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- (1+1)-FEA reaches best mean results in 6/8 benchmark sets and best results in 5/8.
- (1+1)-FEA always converge slower.

Results

inst	EA			FEA			FEA vs. EA		
	best	*mean*	*conv*	*best*	*mean*	*conv*	*best*	*mean*	*conv*
abz	1	0	5	5	5	0	-0.3%	-1.2%	22.0
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best (*mean*): number of instances algorithm reached the best (best average) solution

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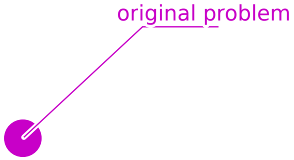
- (1+1)-FEA is at least as good and sometimes better than *all* mean and best results in [16] (2010), aLSGA [17] (2015), the EAS in [18] (2013), all GAs in [19] (2014), the GWO in [20] (2018), SAFA [21] (2018), HBFO[22] (2012), and all algorithms in[23] (2018).

Invariances

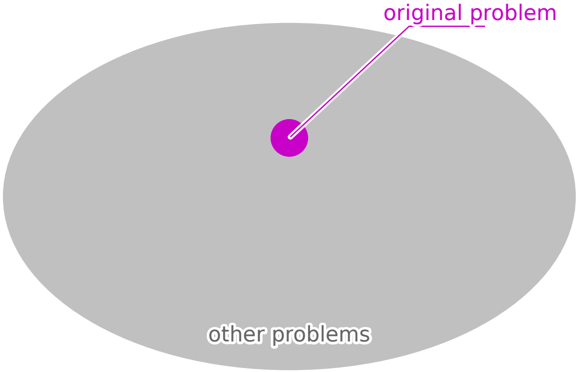
How should a good optimization algorithm behave?

- It should perform well.



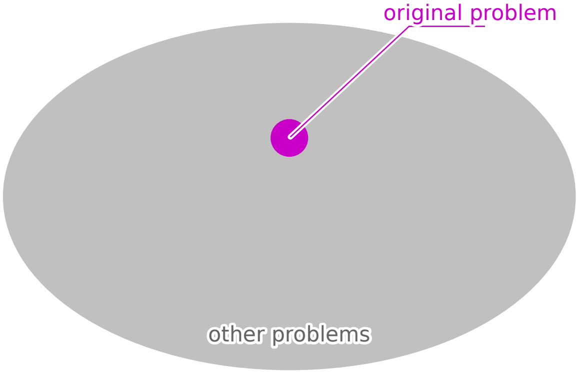
How should a good optimization algorithm behave?

- It should perform well.
- The performance observed on some problems should carry over to other problems.



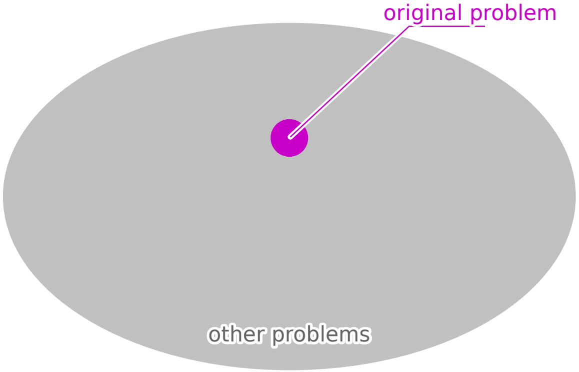
How should a good optimization algorithm behave?

- It should perform well on benchmarks.
- The performance observed on some (benchmark) problems should carry over to other (real-world) problems.



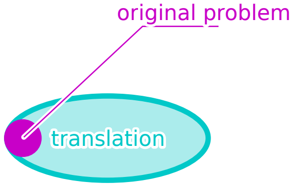
How should a good optimization algorithm behave?

- It should perform well on benchmarks.
- The performance observed on some (benchmark) problems should carry over to other (real-world) problems.
- The algorithm should be invariant under transformations of the original problem.



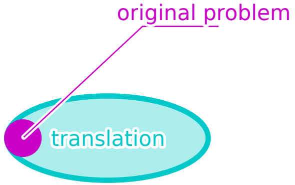
How should a good optimization algorithm behave?

- The algorithm should behave the same if we add an offset to the objective values.



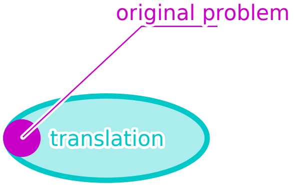
How should a good optimization algorithm behave?

- The algorithm should behave the same if we add an offset to the objective values.
- Then, we cannot use proportions of objective values in the decisions.



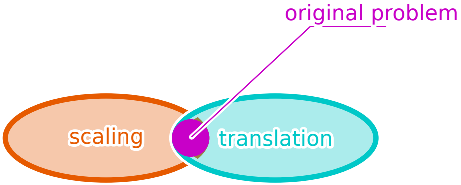
How should a good optimization algorithm behave?

- The algorithm should behave the same if we add an offset to the objective values.
- Then, we cannot use proportions of objective values in the decisions.
- The Genetic Algorithm with Roulette Wheel Selection is not translation invariant.



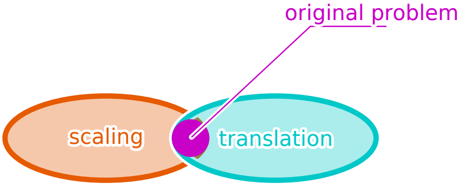
How should a good optimization algorithm behave?

- The algorithm should behave the same if we multiply the objective values by a positive number.



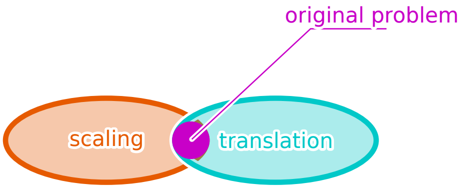
How should a good optimization algorithm behave?

- The algorithm should behave the same if we multiply the objective values by a positive number.
- Then, we cannot use absolute differences of objective values in the decisions.



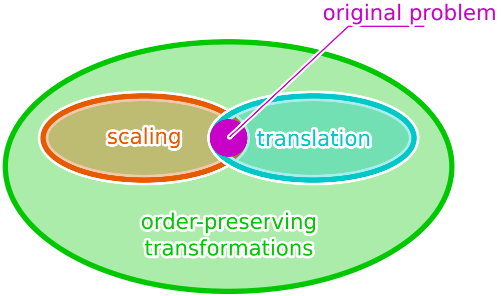
How should a good optimization algorithm behave?

- The algorithm should behave the same if we multiply the objective values by a positive number.
- Then, we cannot use absolute differences of objective values in the decisions.
- Simulated Annealing is not scale invariant.



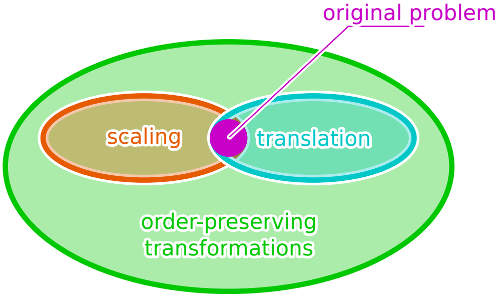
How should a good optimization algorithm behave?

- The algorithm should behave the same on problems where the order of the objective values is the same.



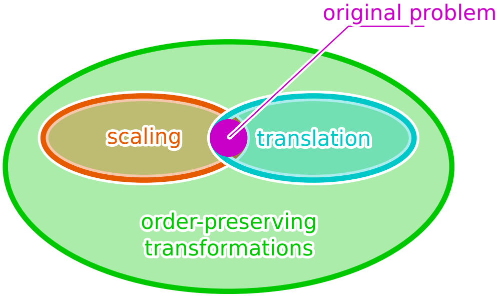
How should a good optimization algorithm behave?

- The algorithm should behave the same on problems where the order of the objective values is the same.
- Then, we can only compare whether solutions are better, worse, or as same as good.



How should a good optimization algorithm behave?

- The algorithm should behave the same on problems where the order of the objective values is the same.
- Then, we can only compare whether solutions are better, worse, or as same as good.
- The (1+1)-EA is invariant under order-preserving transformations.



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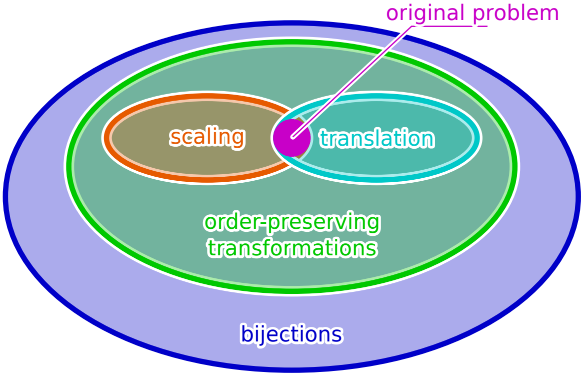

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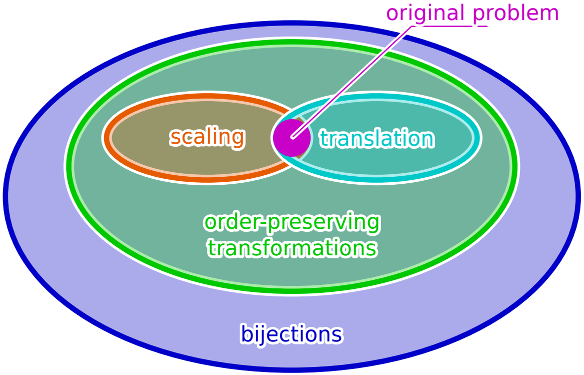

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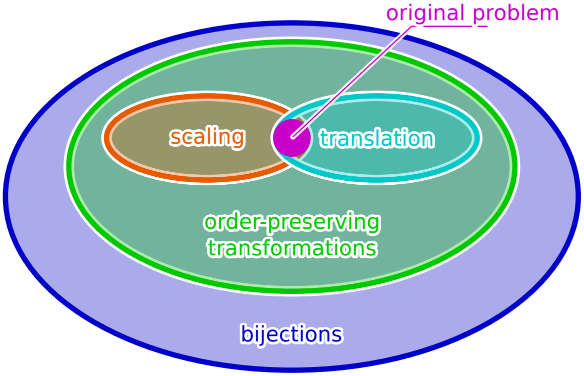

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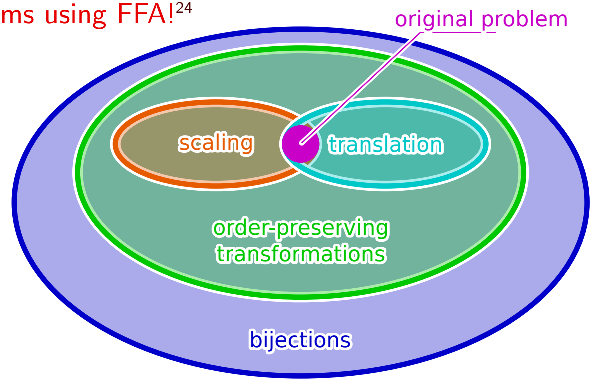

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- To give you a taste: encryption is a bijection, too. . .

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- Interesting: All algorithms using FFA are invariant under all bijections of the objective function value.

谢谢

Thank you

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