



The Tunable W-Model Benchmark Problem

Thomas Weise & Zijun Wu · 汤卫思 & 吴自军

tweise@hfuu.edu.cn · <http://iao.hfuu.edu.cn>

Hefei University, South Campus 2
Faculty of Computer Science and Technology
Institute of Applied Optimization
230601 Shushan District, Hefei, Anhui, China
Econ. & Tech. Devel. Zone, Jinxiu Dadao 99

合肥学院 南艳湖校区/南2区
计算机科学与技术系
应用优化研究所
中国 安徽省 合肥市 蜀山区 230601
经济技术开发区 锦绣大道99号

July 24, 2018

- 1 Introduction
- 2 The *W-Model*
- 3 Experiment
- 4 Summary



website

These are the slides for paper ^[1] presented at the BB-DOB workshop at GECCO'2018.

1 Introduction

2 The *W-Model*

3 Experiment

4 Summary



website

What do we want from a Benchmark Problem?

What do we want from a Benchmark Problem?

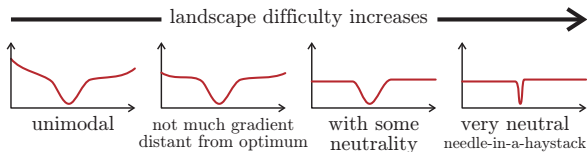
- The goal of benchmarking is to get a complete picture of the strengths and weaknesses of optimization methods.

What do we want from a Benchmark Problem?

- The goal of benchmarking is to get a complete picture of the strengths and weaknesses of optimization methods.
- Some frequently occurring problem characteristics cause difficulties to optimization algorithms ^[2, 3]

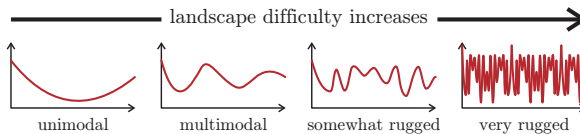
What do we want from a Benchmark Problem?

- The goal of benchmarking is to get a complete picture of the strengths and weaknesses of optimization methods.
- Some frequently occurring problem characteristics cause difficulties to optimization algorithms ^[2, 3]
 - neutrality



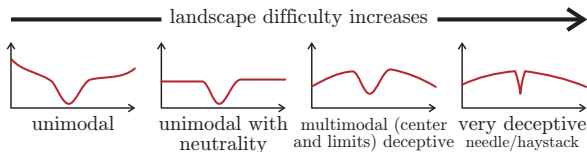
What do we want from a Benchmark Problem?

- The goal of benchmarking is to get a complete picture of the strengths and weaknesses of optimization methods.
- Some frequently occurring problem characteristics cause difficulties to optimization algorithms ^[2, 3]
 - neutrality
 - ruggedness



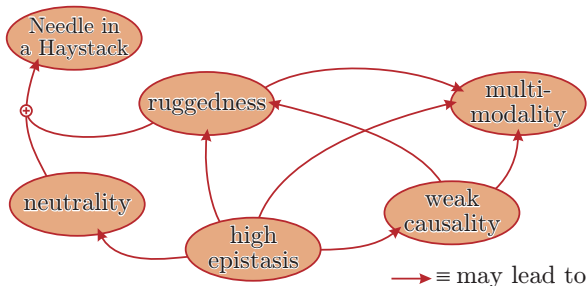
What do we want from a Benchmark Problem?

- The goal of benchmarking is to get a complete picture of the strengths and weaknesses of optimization methods.
- Some frequently occurring problem characteristics cause difficulties to optimization algorithms ^[2, 3]
 - neutrality
 - ruggedness
 - deceptiveness



What do we want from a Benchmark Problem?

- The goal of benchmarking is to get a complete picture of the strengths and weaknesses of optimization methods.
- Some frequently occurring problem characteristics cause difficulties to optimization algorithms ^[2, 3]
 - neutrality
 - ruggedness
 - deceptiveness
 - epistasis



A Benchmark Problem should...

A Benchmark Problem should. . .

- ① include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.

A Benchmark Problem should. . .

- ① include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- So are classical problems like the Traveling Salesman Problem (TSP) ^[4, 5] or the Maximum Satisfiability Problem (SAT) ^[6, 7] good candidates?

A Benchmark Problem should. . .

- ① include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- So are classical problems like the Traveling Salesman Problem (TSP) ^[4, 5] or the Maximum Satisfiability Problem (SAT) ^[6, 7] good candidates?
- Not really. It is not that easy to understand how hard, difficult, rugged, deceptive, or epistatic a TSP or SAT problem is. . .

A Benchmark Problem should. . .

- ① include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- ② exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.

A Benchmark Problem should. . .

- ① include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- ② exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.
- ③ have a problem hardness determined directly by tunable parameters.

A Benchmark Problem should. . .

- ① include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- ② exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.
- ③ have a problem hardness determined directly by tunable parameters.
- ④ have known optima and the range of the objective function should be known.

A Benchmark Problem should...

- ① include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- ② exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.
- ③ have a problem hardness determined directly by tunable parameters.
- ④ have known optima and the range of the objective function should be known.
- ⑤ be easy to understand and fast to compute.

A Benchmark Problem should...

- 1 include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- 2 exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.
- 3 have a problem hardness determined directly by tunable parameters.
- 4 have known optima and the range of the objective function should be known.
- 5 be easy to understand and fast to compute.
- 6 fit to a standard representation from discrete optimization, i.e., either bit strings or permutations (of fixed length).

A Benchmark Problem should...

- ➊ include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- ➋ exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.
- ➌ have a problem hardness determined directly by tunable parameters.
- ➍ have known optima and the range of the objective function should be known.
- ➎ be easy to understand and fast to compute.
- ➏ fit to a standard representation from discrete optimization, i.e., either bit strings or permutations (of fixed length).
- ➐ allow the creation of easy and hard, small and large instances.

A Benchmark Problem should...

- ① include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- ② exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.
- ③ have a problem hardness determined directly by tunable parameters.
- ④ have known optima and the range of the objective function should be known.
- ⑤ be easy to understand and fast to compute.
- ⑥ fit to a standard representation from discrete optimization, i.e., either bit strings or permutations (of fixed length).
- ⑦ allow the creation of easy and hard, small and large instances.
- ⑧ be replicable, i.e., allow to derive problem instances deterministically from very few parameters.

A Benchmark Problem should...

- ③ include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- ② exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.
- ③ have a problem hardness determined directly by tunable parameters.
- ④ have known optima and the range of the objective function should be known.
- ⑤ be easy to understand and fast to compute.
- ⑥ fit to a standard representation from discrete optimization, i.e., either bit strings or permutations (of fixed length).
- ⑦ allow the creation of easy and hard, small and large instances.
- ⑧ be replicable, i.e., allow to derive problem instances deterministically from very few parameters.
- ⑨ be theoretically tractable.

A Benchmark Problem should...

- ① include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- ② exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.
- ③ have a problem hardness determined directly by tunable parameters.
- ④ have known optima and the range of the objective function should be known.
- ⑤ be easy to understand and fast to compute.
- ⑥ fit to a standard representation from discrete optimization, i.e., either bit strings or permutations (of fixed length).
- ⑦ allow the creation of easy and hard, small and large instances.
- ⑧ be replicable, i.e., allow to derive problem instances deterministically from very few parameters.
- ⑨ be theoretically tractable.
- ⑩ have components which can be combined with other, existing problems.

A Benchmark Problem should...

- ① include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- ② exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.
- ③ have a problem hardness determined directly by tunable parameters.
- ④ have known optima and the range of the objective function should be known.
- ⑤ be easy to understand and fast to compute.
- ⑥ fit to a standard representation from discrete optimization, i.e., either bit strings or permutations (of fixed length).
- ⑦ allow the creation of easy and hard, small and large instances.
- ⑧ be replicable, i.e., allow to derive problem instances deterministically from very few parameters.
- ⑨ be theoretically tractable.
- ⑩ have components which can be combined with other, existing problems.
- ⑪ be extensible to other domains, e.g., variable-length representations, multi-objective domains, ...

A Benchmark Problem should...

- ❶ include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- ❷ exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.
- ❸ have a problem hardness determined directly by tunable parameters.
- ❹ have known optima and the range of the objective function should be known.
- ❺ be easy to understand and fast to compute.
- ❻ fit to a standard representation from discrete optimization, i.e., either bit strings or permutations (of fixed length).
- ❼ allow the creation of easy and hard, small and large instances.
- ❽ be replicable, i.e., allow to derive problem instances deterministically from very few parameters.
- ❾ be theoretically tractable.
- ❿ have components which can be combined with other, existing problems.
- ⓫ be extensible to other domains, e.g., variable-length representations, multi-objective domains, ...
- ⓬ have an available reference implementation with utilities and tests showing that/whether the implementation (or any implementation of the problem) is identical to the problem definition in a publication, maybe even an experiment execution environment.

A Benchmark Problem should...

- ① include problems which exhibit the difficult features ruggedness, epistasis, neutrality, and deceptiveness in different strengths and in different combinations.
- ② exhibit these difficult features in degrees which are obvious, easy to understand, and ideally tunable.
- ③ have a problem hardness determined directly by tunable parameters.
- ④ have known optima and the range of the objective function should be known.
- ⑤ be easy to understand and fast to compute.
- ⑥ fit to a standard representation from discrete optimization, i.e., either bit strings or permutations (of fixed length).
- ⑦ allow the creation of easy and hard, small and large instances.
- ⑧ be replicable, i.e., allow to derive problem instances deterministically from very few parameters.
- ⑨ be theoretically tractable.
- ⑩ have components which can be combined with other, existing problems.
- ⑪ be extensible to other domains, e.g., variable-length representations, multi-objective domains, ...
- ⑫ have an available reference implementation with utilities and tests showing that/whether the implementation (or any implementation of the problem) is identical to the problem definition in a publication, maybe even an experiment execution environment.
- ⑬ have available example data sets with results from example experiments.

- Benchmark Model ^[1, 8, 9] defined over $\{0, 1\}^n$.

1 original bit string (here: variable-length)
 x 0101 0110 0000 1110 1000 0

2 Introduction of Neutrality
 $\mu=2$ 01 01 01 10 00 00 11 10 10 00 0
 $\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$
 $u_2(g)$ 1 1 1 1 0 0 1 1 1 0 \times

3 Introduction of Epistasis
 $v=4$ 1111 0011 10 insufficient bits,
 $\downarrow \downarrow \downarrow$ at the end, use
 $e_1 \downarrow e_2 \downarrow e_3$ $\eta=2$ instead of
 1110 0110 11 $\eta=4$

4 Multi-Objectivity
 $m=2, n=6$ 1110011011 padding:
 (x_1, x_2) 110110 101010 $x^*[5]=0$
 x_1 x_2

5 Objective Values
 $n=6$ 110110 101010
 $\downarrow \downarrow$
 $f(x_1)=3$ $f(x_2)=6$

6 Introduction of Ruggedness
 $\gamma=12, n=6$ $f(x_1)=3$ $f(x_2)=6$
 $\gamma'=9$ $\downarrow \downarrow$
 $r_{12}[f(x_1)]=3$ $r_{12}[f(x_2)]=5$

- Benchmark Model [1, 8, 9] defined over $\{0, 1\}^n$.
- Fulfills all 13 requirements above.

1 original bit string (here: variable-length)
 x 0101 0110 0000 1110 1000 0

2 Introduction of Neutrality
 $\mu=2$ 01 01 01 10 00 00 11 10 10 00 0
 $\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$
 $u_2(g)$ 1 1 1 1 0 0 1 1 1 0 \times

3 Introduction of Epistasis
 $v=4$ 1111 0011 10 insufficient bits,
 $\downarrow \downarrow \downarrow$ at the end, use
 $e_1 \downarrow e_2 \downarrow e_3$ $\eta=2$ instead of
 1110 0110 11 $\eta=4$

4 Multi-Objectivity
 $m=2, n=6$ 1110011011 padding:
 (x_1, x_2) 110110 101010 $x^*[5]=0$
 x_1 x_2

5 Objective Values
 $n=6$ 110110 101010
 $\downarrow \downarrow$
 $f(x_1)=3$ $f(x_2)=6$

6 Introduction of Ruggedness
 $\gamma=12, n=6$
 $\gamma'=9$ $f(x_1)=3$ $f(x_2)=6$
 $\downarrow \downarrow$
 $r_{12}[f(x_1)]=3$ $r_{12}[f(x_2)]=5$

1 original bit string (here: variable-length)
 x 0101 0110 0000 1110 1000 0

2 Introduction of Neutrality
 $\mu=2$ 01 01 01 10 00 00 11 10 10 00 0
 $u_2(g)$ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
 1 1 1 1 0 0 1 1 1 0 ×

3 Introduction of Epistasis
 $v=4$ 1111 0011 10 insufficient bits,
 e_1 ↓ e_2 ↓ e_3 ↓ at the end, use
 1110 0110 11 $\eta=2$ instead of
 $\eta=4$

4 Multi-Objectivity
 $m=2, n=6$ 1110011011 padding:
 (x_1, x_2) 110110 101010 $x^*[5]=0$
 x_1 x_2

5 Objective Values
 $n=6$ 110110 101010
 $f(x_1)=3$ $f(x_2)=6$

6 Introduction of Ruggedness
 $\gamma=12, n=6$ $f(x_1)=3$ $f(x_2)=6$
 $\gamma'=9$ ↓ ↓
 $r_{12}[f(x_1)]=3$ $r_{12}[f(x_2)]=5$

- Benchmark Model [1, 8, 9] defined over $\{0, 1\}^n$.
- Fulfills all 13 requirements above.
- Neutrality, Epistasis, Ruggedness/Deceptiveness (and multi-objectivity) implemented as separate, parameterized layers which could also be plugged on top of other problems.

1 original bit string (here: variable-length)
 x 0101 0110 0000 1110 1000 0

2 Introduction of Neutrality
 $\mu=2$ 01 01 01 10 00 00 11 10 10 00 0
 $u_i(g)$ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
 1 1 1 1 0 0 1 1 1 0 ×

3 Introduction of Epistasis
 $v=4$ 1111 0011 10 insufficient bits,
 e_i ↓ ↓ ↓ at the end, use
 1110 0110 11 $\eta=2$ instead of
 $\eta=4$

4 Multi-Objectivity
 $m=2, n=6$ 1110011011 padding:
 (x_1, x_2) 110110 101010 $x^*[5]=0$
 x_1 x_2

5 Objective Values
 $n=6$ 110110 101010
 $f(x_1)=3$ $f(x_2)=6$

6 Introduction of Ruggedness
 $\gamma=12, n=6$ $f(x_1)=3$ $f(x_2)=6$
 $\gamma'=9$ ↓ ↓
 $r_{12}[f(x_1)]=3$ $r_{12}[f(x_2)]=5$

- Benchmark Model [1, 8, 9] defined over $\{0, 1\}^n$.
- Fulfills all 13 requirements above.
- Neutrality, Epistasis, Ruggedness/Deceptiveness (and multi-objectivity) implemented as separate, parameterized layers which could also be plugged on top of other problems.
- Problem instance completely defined by five parameters n, μ, ν, γ , and m

1 original bit string (here: variable-length)
 x 0101 0110 0000 1110 1000 0

2 Introduction of Neutrality
 $\mu=2$ 01 01 01 10 00 00 11 10 10 00 0
 $u_i(g)$ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
 1 1 1 1 0 0 1 1 1 0 ×

3 Introduction of Epistasis
 $v=4$ 1111 0011 10
 e_i ↓ ↓ ↓ insufficient bits,
 1110 0110 11 at the end, use
 $\eta=2$ instead of
 $\eta=4$

4 Multi-Objectivity
 $m=2, n=6$ 1110011011 padding:
 (x_1, x_2) 110110 101010 $x^*[5]=0$
 x_1 x_2

5 Objective Values
 $n=6$ 110110 101010
 $f(x_1)=3$ $f(x_2)=6$

6 Introduction of Ruggedness
 $\gamma=12, n=6$ $f(x_1)=3$ $f(x_2)=6$
 $\gamma'=9$ ↓ ↓
 $r_{12}[f(x_1)]=3$ $r_{12}[f(x_2)]=5$

- Benchmark Model [1, 8, 9] defined over $\{0, 1\}^n$.
- Fulfills all 13 requirements above.
- Neutrality, Epistasis, Ruggedness/Deceptiveness (and multi-objectivity) implemented as separate, parameterized layers which could also be plugged on top of other problems.
- Problem instance completely defined by five parameters n, μ, ν, γ , and m
- Known global optimum:
 $x^* = 0101010101010 \dots 01$ of length n with objective value $f(x^*) = 0$.

1 original bit string (here: variable-length)
 x 0101 0110 0000 1110 1000 0

2 Introduction of Neutrality
 $\mu=2$ 01 01 01 10 00 00 11 10 10 00 0
 $u_2(g)$ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
 1 1 1 1 0 0 1 1 1 0 ×

3 Introduction of Epistasis
 $v=4$ 1111 0011 10
 e_1 ↓ e_2 ↓ e_3 ↓ insufficient bits,
 1110 0110 11 at the end, use
 $\eta=2$ instead of
 $\eta=4$

4 Multi-Objectivity
 $m=2, n=6$ 1110011011 padding:
 (x_1, x_2) 110110 101010 $x^*[5]=0$
 x_1 x_2

5 Objective Values
 $n=6$ 110110 101010
 $f(x_1)=3$ $f(x_2)=6$

6 Introduction of Ruggedness
 $\gamma=12, n=6$ $f(x_1)=3$ $f(x_2)=6$
 $\gamma'=9$ $r_{12}[f(x_1)]=3$ $r_{12}[f(x_2)]=5$

- Benchmark Model [1, 8, 9] defined over $\{0, 1\}^n$.
- Fulfills all 13 requirements above.
- Neutrality, Epistasis, Ruggedness/Deceptiveness (and multi-objectivity) implemented as separate, parameterized layers which could also be plugged on top of other problems.
- Problem instance completely defined by five parameters n, μ, ν, γ , and m
- Known global optimum:
 $x^* = 0101010101010 \dots 01$ of length n with objective value $f(x^*) = 0$.
- Computing of objective function f is in $\mathcal{O}(m * n * \nu^2)$.

1 original bit string (here: variable-length)
 x 0101 0110 0000 1110 1000 0

2 Introduction of Neutrality
 $\mu=2$ 01 01 01 10 00 00 11 10 10 00 0
 $\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$
 $u_2(g)$ 1 1 1 1 0 0 1 1 1 0 \times

3 Introduction of Epistasis
 $\nu=4$ 1111 0011 10
 $\downarrow \downarrow \downarrow$ insufficient bits, at the end, use $\eta=2$ instead of $\eta=4$
 1110 0110 11

4 Multi-Objectivity
 $m=2, n=6$ 11100111
 (x_1, x_2) 110110 101010 padding: $x^*[5]=0$
 x_1 x_2

5 Objective Values
 $n=6$ 110110 101010
 $\downarrow \downarrow$
 $f(x_1)=3$ $f(x_2)=6$

6 Introduction of Ruggedness
 $\gamma=12, n=6$ $f(x_1)=3$ $f(x_2)=6$
 $\gamma'=9$ $\downarrow \downarrow$
 $r_{12}[f(x_1)]=3$ $r_{12}[f(x_2)]=5$

- Benchmark Model ^[1, 8, 9] defined over $\{0, 1\}^n$.
- Fulfills all 13 requirements above.
- ...
- Problem instance completely defined by five parameters n, μ, ν, γ , and m
- Known global optimum:
 $x^* = 0101010101010 \dots 01$ of length n with objective value $f(x^*) = 0$.
- Computing of objective function f is in $\mathcal{O}(m * n * \nu^2)$.
- Reference implementation ^[10] with many unit tests, parallel experiment execution environment, and example algorithms at http://github.com/thomasWeise/BBD0B_W_Model

1 original bit string (here: variable-length)
 x 0101 0110 0000 1110 1000 0

2 Introduction of Neutrality
 $\mu=2$ 01 01 01 10 00 00 11 10 10 00 0
 $\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$
 $u_2(g)$ 1 1 1 1 0 0 1 1 1 0 \times

3 Introduction of Epistasis
 $v=4$ 1111 0011 10
 $\downarrow \downarrow \downarrow$ insufficient bits, at the end, use $\eta=2$ instead of $\eta=4$
 1110 0110 11

4 Multi-Objectivity
 $m=2, n=6$ 1110011011 padding: $X^*[5]=0$
 (x_1, x_2) 110110 101010
 x_1 x_2

5 Objective Values
 $n=6$ 110110 101010
 $\downarrow \downarrow$
 $f(x_1)=3$ $f(x_2)=6$

6 Introduction of Ruggedness
 $\gamma=12, n=6$ $f(x_1)=3$ $f(x_2)=6$
 $\gamma'=9$ $\downarrow \downarrow$
 $r_{12}[f(x_1)]=3$ $r_{12}[f(x_2)]=5$

- Benchmark Model [1, 8, 9] defined over $\{0, 1\}^n$.
- Fulfills all 13 requirements above.
- ...
- Computing of objective function f is in $\mathcal{O}(m * n * v^2)$.
- Reference implementation [10] with many unit tests, parallel experiment execution environment, and example algorithms at http://github.com/thomasWeise/BBD0B_W_Model
- Huge example experiment [10] obtained from this implementation with 45 GB of algorithm traces at [doi:10.5281/zenodo.1256883](https://doi.org/10.5281/zenodo.1256883).

These are the slides for paper ^[1] presented at the BB-DOB workshop at GECCO'2018.

1 Introduction

2 The *W-Model*

3 Experiment

4 Summary



website

- Goal: minimize the Hamming distance $h(x, x^*)$ to $x^* = (0101\dots)$ of length n .

5

$n=6$

Objective Values

110110



$f(x_1)=3$

101010



$f(x_2)=6$

- Goal: minimize the Hamming distance $h(x, x^*)$ to $x^* = (0101\dots)$ of length n .
- Very similar to OneMax problem ^[11–14].

5

$n=6$

Objective Values

110110



$f(x_1)=3$

101010



$f(x_2)=6$

- Goal: minimize the Hamming distance $h(x, x^*)$ to $x^* = (0101\dots)$ of length n .
- Very similar to OneMax problem ^[11–14].
- Computing of objective function is in $\mathcal{O}(n)$ for bit strings with length $l(x) = n$.

5

$n=6$

Objective Values

110110



$f(x_1)=3$

101010



$f(x_2)=6$

- Goal: minimize the Hamming distance $h(x, x^*)$ to $x^* = (0101\dots)$ of length n .
- Very similar to OneMax problem ^[11–14].
- Computing of objective function is in $\mathcal{O}(n)$ for bit strings with length $l(x) = n$.
- **Fixed-Length Search Space:** bit strings of length n .

5

 $n=6$

Objective Values

110110 $f(x_1)=3$ 101010 $f(x_2)=6$

- Goal: minimize the Hamming distance $h(x, x^*)$ to $x^* = (0101\dots)$ of length n .
- Very similar to OneMax problem ^[11–14].
- Computing of objective function is in $\mathcal{O}(n)$ for bit strings with length $l(x) = n$.
- **Fixed-Length Search Space:** bit strings of length n .
- **Variable-Length Search Space:** overly long strings ($l(x) > n$) are cut after position n , strings that are too short are padded with $\overline{x^*}$.

5

Objective Values

$n=6$

110110



$f(x_1)=3$

101010



$f(x_2)=6$

- The application of a search operator is *neutral* if it yields no change in objective value ^[15, 16].

1	original bit string (here: variable-length)										
	x 0101 0110 0000 1110 1000 0										
2	Introduction of Neutrality										
$\mu=2$	01	01	01	10	00	00	11	10	10	00	0
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
$u_2(g)$	1	1	1	1	0	0	1	1	1	0	×

- The application of a search operator is *neutral* if it yields no change in objective value ^[15, 16].
- Usually negative impact on performance ^[17, 18].

1	original bit string (here: variable-length)										
	x	0	1	0	1	0	1	1	0	0	0
		0	1	0	1	1	0	0	0	0	0

2	Introduction of Neutrality										
$\mu=2$	0	1	0	1	0	1	0	0	0	0	0
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
$u_2(g)$	1	1	1	1	0	0	1	1	1	0	×

- The application of a search operator is *neutral* if it yields no change in objective value ^[15, 16].
- Usually negative impact on performance ^[17, 18].
- Transformation $u(x)$ shortens x by factor μ by computing majority value for blocks of length μ , set bit to 1 in draws for even μ (no average effect as $x^* = 0101\dots$).

1

original bit string (here: variable-length)

x

0101 0110 0000 1110 1000 0

2

Introduction of Neutrality

μ=2

01 01 01 10 00 00 11 10 10 00 0

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

↓

u₂(g)

1 1 1 1 0 0 1 1 1 0 ×

- The application of a search operator is *neutral* if it yields no change in objective value ^[15, 16].
- Usually negative impact on performance ^[17, 18].
- Transformation $u(x)$ shortens x by factor μ by computing majority value for blocks of length μ , set bit to 1 in draws for even μ (no average effect as $x^* = 0101\dots$).
- **Fixed-Length Search Space:** bit string length now $\mu * n$.

1	original bit string (here: variable-length)												
	x	0101 0110 0000 1110 1000 0											
2	Introduction of Neutrality												
	$\mu=2$	01	01	01	10	00	00	11	10	10	00	0	
		↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
	$u_2(g)$	1	1	1	1	0	0	1	1	1	0	×	

- The application of a search operator is *neutral* if it yields no change in objective value ^[15, 16].
- Usually negative impact on performance ^[17, 18].
- Transformation $u(x)$ shortens x by factor μ by computing majority value for blocks of length μ , set bit to 1 in draws for even μ (no average effect as $x^* = 0101\dots$).
- **Fixed-Length Search Space:** bit string length now $\mu * n$.
- **Variable-Length Search Space:** if $l(x)$ no multiple of μ , ignore last $l(x) \bmod \mu$ bits.

1	original bit string (here: variable-length)											
	x	0101 0110 0000 1110 1000 0										
2	Introduction of Neutrality											
	$\mu=2$	01	01	01	10	00	00	11	10	10	00	0
		↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
	$u_2(g)$	1	1	1	1	0	0	1	1	1	0	×

- The interaction between biological genes is epistatic if the effect on the fitness from altering one gene depends on the allelic state of other genes ^[19, 20].

- Two decision variables (here: bits) **interact epistatically**, if the contribution of one of these variables to the objective value depends on the value of the other variable ^[9, 20–22].

- Two decision variables (here: bits) **interact epistatically**, if the contribution of one of these variables to the objective value depends on the value of the other variable ^[9, 20–22].
- *W-Model*: Bijective function e_ν translates a bit string x of length ν to a bit string $e_\nu(x)$ of the same length in $\mathcal{O}(\nu^2)$ steps.

- Two decision variables (here: bits) **interact epistatically**, if the contribution of one of these variables to the objective value depends on the value of the other variable ^[9, 20–22].
- *W-Model*: Bijective function e_ν translates a bit string x of length ν to a bit string $e_\nu(x)$ of the same length in $\mathcal{O}(\nu^2)$ steps.

$$h(x_1, x_2) = 1 \Rightarrow h(e_\nu(x_1), e_\nu(x_2)) \geq \nu - 1 \quad \forall x_1, x_2 \in \{0, 1\}^\nu \quad (1)$$

- Two decision variables (here: bits) **interact epistatically**, if the contribution of one of these variables to the objective value depends on the value of the other variable ^[9, 20–22].
- *W-Model*: Bijective function e_ν translates a bit string x of length ν to a bit string $e_\nu(x)$ of the same length in $\mathcal{O}(\nu^2)$ steps.

$$h(x_1, x_2) = 1 \Rightarrow h(e_\nu(x_1), e_\nu(x_2)) \geq \nu - 1 \quad \forall x_1, x_2 \in \{0, 1\}^\nu \quad (1)$$

- A change of one bit in a bit string x leads to the change of at least $\nu - 1$ bits in the corresponding mapping $e_\nu(x)$.

- Two decision variables (here: bits) **interact epistatically**, if the contribution of one of these variables to the objective value depends on the value of the other variable ^[9, 20–22].
- W-Model*: Bijective function e_ν translates a bit string x of length ν to a bit string $e_\nu(x)$ of the same length in $\mathcal{O}(\nu^2)$ steps.

$$h(x_1, x_2) = 1 \Rightarrow h(e_\nu(x_1), e_\nu(x_2)) \geq \nu - 1 \quad \forall x_1, x_2 \in \{0, 1\}^\nu \quad (1)$$

- A change of one bit in a bit string x leads to the change of at least $\nu - 1$ bits in the corresponding mapping $e_\nu(x)$.

$$e_\nu(x) = \begin{cases} e_\nu(x)[i] = \bigotimes_{\substack{\forall j \in \mathbb{N}_0: 0 \leq j < \nu, \\ j \neq (i-1) \bmod \nu}} x[j] & \forall x : 0 \leq x < 2^{\nu-1} \\ \overline{e_\nu(x - 2^{\nu-1})} & \text{otherwise} \end{cases} \quad (2)$$

- Two decision variables (here: bits) **interact epistatically**, if the contribution of one of these variables to the objective value depends on the value of the other variable ^[9, 20–22].
- W-Model*: Bijective function e_ν translates a bit string x of length ν to a bit string $e_\nu(x)$ of the same length in $\mathcal{O}(\nu^2)$ steps.

$$h(x_1, x_2) = 1 \Rightarrow h(e_\nu(x_1), e_\nu(x_2)) \geq \nu - 1 \quad \forall x_1, x_2 \in \{0, 1\}^\nu \quad (1)$$

- A change of one bit in a bit string x leads to the change of at least $\nu - 1$ bits in the corresponding mapping $e_\nu(x)$.

$$e_\nu(x) = \begin{cases} e_\nu(x)[i] = \bigotimes_{\substack{\forall j \in \mathbb{N}_0: 0 \leq j < \nu, \\ j \neq (i-1) \bmod \nu}} x[j] & \forall x : 0 \leq x < 2^{\nu-1} \\ \overline{e_\nu(x - 2^{\nu-1})} & \text{otherwise} \end{cases} \quad (2)$$

- Each bit in x **influences** the value of $\nu - 1$ bits in $e_\nu(x)$.

$$e_\nu(x) = \begin{cases} e_\nu(x)[i] = \bigotimes_{\substack{\forall j \in \mathbb{N}_0: 0 \leq j < \nu, \\ j \neq (i-1) \bmod \nu}} x[j] & \forall x : 0 \leq x < 2^{\nu-1} \\ \overline{e_\nu(x - 2^{\nu-1})} & \text{otherwise} \end{cases} \quad (2)$$

- Each bit in x influences the value of $\nu - 1$ bits in $e_\nu(x)$.

z	$e_4(z)$
0000	0000
0001	1101
0010	1011
0100	0111
1000	1111

$h=1$ on the left, $h \geq 3$ on the right

z	$e_4(z)$
1111	1110
0111	0001
1011	1001
1101	0101
1110	0011

$h=1$ on the left, $h \geq 3$ on the right

z	$e_4(z)$
0011	0110
0101	1010
0110	1100
1001	0010
1010	0100
1100	1000

$$e_{\nu}(x) = \begin{cases} e_{\nu}(x)[i] = \bigotimes_{\substack{\forall j \in \mathbb{N}_0: 0 \leq j < \nu, \\ j \neq (i-1) \bmod \nu}} x[j] & \forall x : 0 \leq x < 2^{\nu-1} \\ \overline{e_{\nu}(x - 2^{\nu-1})} & \text{otherwise} \end{cases} \quad (2)$$

- Each bit in x influences the value of $\nu - 1$ bits in $e_{\nu}(x)$.
- Candidate solutions divided into blocks of length ν to be transformed separately, if block of length $l(x) \bmod \nu$ remains, it is transformed with $e_{l(x) \bmod \nu}$.

3

Introduction of Epistasis

$\nu=4$

1111 0011 10

$e_4 \downarrow$

$e_4 \downarrow$

$e_2 \downarrow$

1110 0110 11

insufficient bits,
at the end, use
 $\nu=2$ instead of
 $\nu=4$

- Many optimization problems are multi-objective, i.e., involve multiple, possible conflicting criteria ^[23–25]

5

$n=6$

Objective Values

110110



$f(x_1)=3$

101010



$f(x_2)=6$

- Many optimization problems are multi-objective, i.e., involve multiple, possible conflicting criteria ^[23–25]
- We simply interleave m instances of the *W-Model* to get an m -objective problem.

5

$n=6$

Objective Values

110110

101010



$f(x_1)=3$

$f(x_2)=6$

- Many optimization problems are multi-objective, i.e., involve multiple, possible conflicting criteria ^[23–25]
- We simply interleave m instances of the *W-Model* to get an m -objective problem.
- Disagreement in the orthogonal objective functions can be simulated via epistasis $\nu > 2$.

5

$n=6$

Objective Values

110110

101010



$f(x_1)=3$

$f(x_2)=6$

- Strong causality means that small changes in a candidate solution lead to small changes in the objective value ^[26, 27].

- Strong causality means that small changes in a candidate solution lead to small changes in the objective value ^[26, 27].
- Rugged fitness landscape: small changes in candidate solution \implies large changes in objective value.

- Strong causality means that small changes in a candidate solution lead to small changes in the objective value ^[26, 27].
- Rugged fitness landscape: small changes in candidate solution \implies large changes in objective value.
- Deceptive fitness landscape: following changes towards declining objective function leads away from optimum.

- Strong causality means that small changes in a candidate solution lead to small changes in the objective value [26, 27].
- Rugged fitness landscape: small changes in candidate solution \implies large changes in objective value.
- Deceptive fitness landscape: following changes towards declining objective function leads away from optimum.
- Epistasis is a source of ruggedness and deceptiveness.

- Strong causality means that small changes in a candidate solution lead to small changes in the objective value [26, 27].
- Rugged fitness landscape: small changes in candidate solution \implies large changes in objective value.
- Deceptive fitness landscape: following changes towards declining objective function leads away from optimum.
- Epistasis is a source of ruggedness and deceptiveness.
- For $\nu = 1$ in *W-Model*: Change one bit in x leads to change of 1 in objective value.

- Strong causality means that small changes in a candidate solution lead to small changes in the objective value ^[26, 27].
- Rugged fitness landscape: small changes in candidate solution \implies large changes in objective value.
- Deceptive fitness landscape: following changes towards declining objective function leads away from optimum.
- Epistasis is a source of ruggedness and deceptiveness.
- For $\nu = 1$ in *W-Model*: Change one bit in x leads to change of 1 in objective value.
- If we improve candidate solution from worst possible to best bit-by-bit, we traverse objective values $(n, n - 1, \dots, 2, 1, 0)$.

- Strong causality means that small changes in a candidate solution lead to small changes in the objective value ^[26, 27].
- Rugged fitness landscape: small changes in candidate solution \implies large changes in objective value.
- Deceptive fitness landscape: following changes towards declining objective function leads away from optimum.
- Epistasis is a source of ruggedness and deceptiveness.
- For $\nu = 1$ in *W-Model*: Change one bit in x leads to change of 1 in objective value.
- If we improve candidate solution from worst possible to best bit-by-bit, we traverse objective values $(n, n - 1, \dots, 2, 1, 0)$.
- If we can exchange the values in this sequence, this will increase the ruggedness or cause deceptiveness!

- Strong causality means that small changes in a candidate solution lead to small changes in the objective value [26, 27].
- Rugged fitness landscape: small changes in candidate solution \implies large changes in objective value.
- Deceptive fitness landscape: following changes towards declining objective function leads away from optimum.
- Epistasis is a source of ruggedness and deceptiveness.
- For $\nu = 1$ in *W-Model*: Change one bit in x leads to change of 1 in objective value.
- If we improve candidate solution from worst possible to best bit-by-bit, we traverse objective values $(n, n - 1, \dots, 2, 1, 0)$.
- If we can exchange the values in this sequence, this will increase the ruggedness or cause deceptiveness!
- The ruggedness Δ of a permutation r be $\Delta(r) = \sum_{i=0}^{n-1} |r_i - r_{i+1}|$.

- If we improve candidate solution from worst possible to best bit-by-bit, we traverse objective values $(n, n-1, \dots, 2, 1, 0)$.
- The ruggedness Δ of a permutation r be $\Delta(r) = \sum_{i=0}^{n-1} |r_i - r_{i+1}|$.

- If we improve candidate solution from worst possible to best bit-by-bit, we traverse objective values $(n, n-1, \dots, 2, 1, 0)$.
- The ruggedness Δ of a permutation r be $\Delta(r) = \sum_{i=0}^{n-1} |r_i - r_{i+1}|$.
- Original sequence above has $\Delta(n \dots 0) = n$ and maximum possible value is $\hat{\Delta} = \frac{n(n+1)}{2}$.

- If we improve candidate solution from worst possible to best bit-by-bit, we traverse objective values $(n, n-1, \dots, 2, 1, 0)$.
- The ruggedness Δ of a permutation r be $\Delta(r) = \sum_{i=0}^{n-1} |r_i - r_{i+1}|$.
- Original sequence above has $\Delta(n \dots 0) = n$ and maximum possible value is $\hat{\Delta} = \frac{n(n+1)}{2}$.
- We define mappings based on permutations $r_{\gamma'}$ with the following features

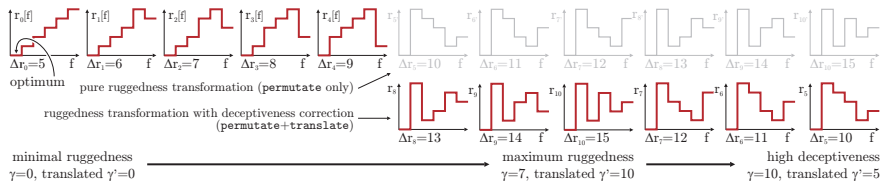
- If we improve candidate solution from worst possible to best bit-by-bit, we traverse objective values $(n, n-1, \dots, 2, 1, 0)$.
- The ruggedness Δ of a permutation r be $\Delta(r) = \sum_{i=0}^{n-1} |r_i - r_{i+1}|$.
- Original sequence above has $\Delta(n \dots 0) = n$ and maximum possible value is $\hat{\Delta} = \frac{n(n+1)}{2}$.
- We define mappings based on permutations $r_{\gamma'}$ with the following features:
 - ① They are bijective (since they are permutations).

- If we improve candidate solution from worst possible to best bit-by-bit, we traverse objective values $(n, n-1, \dots, 2, 1, 0)$.
- The ruggedness Δ of a permutation r be $\Delta(r) = \sum_{i=0}^{n-1} |r_i - r_{i+1}|$.
- Original sequence above has $\Delta(n \dots 0) = n$ and maximum possible value is $\hat{\Delta} = \frac{n(n+1)}{2}$.
- We define mappings based on permutations $r_{\gamma'}$ with the following features:
 - ① They are bijective (since they are permutations).
 - ② They must preserve the optimal value, i.e., $r_{\gamma'}[0] = 0$.

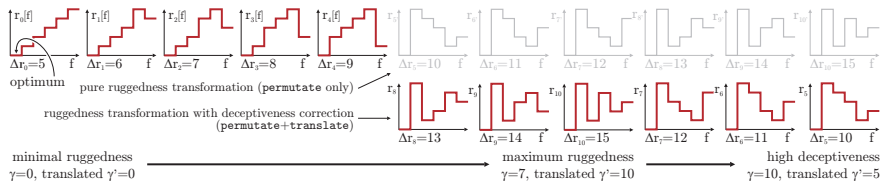
- If we improve candidate solution from worst possible to best bit-by-bit, we traverse objective values $(n, n-1, \dots, 2, 1, 0)$.
- The ruggedness Δ of a permutation r be $\Delta(r) = \sum_{i=0}^{n-1} |r_i - r_{i+1}|$.
- Original sequence above has $\Delta(n \dots 0) = n$ and maximum possible value is $\hat{\Delta} = \frac{n(n+1)}{2}$.
- We define mappings based on permutations $r_{\gamma'}$ with the following features:
 - ① They are bijective (since they are permutations).
 - ② They must preserve the optimal value, i.e., $r_{\gamma'}[0] = 0$.
 - ③ $\Delta(r_{\gamma'}) = n + \gamma'$.

- If we improve candidate solution from worst possible to best bit-by-bit, we traverse objective values $(n, n-1, \dots, 2, 1, 0)$.
- The ruggedness Δ of a permutation r be $\Delta(r) = \sum_{i=0}^{n-1} |r_i - r_{i+1}|$.
- Original sequence above has $\Delta(n \dots 0) = n$ and maximum possible value is $\hat{\Delta} = \frac{n(n+1)}{2}$.
- We define mappings based on permutations $r_{\gamma'}$ with the following features:
 - ① They are bijective (since they are permutations).
 - ② They must preserve the optimal value, i.e., $r_{\gamma'}[0] = 0$.
 - ③ $\Delta(r_{\gamma'}) = n + \gamma'$.
- With $\gamma' \in 0 \dots (\hat{\Delta} - n)$, we can fine-tune the ruggedness.

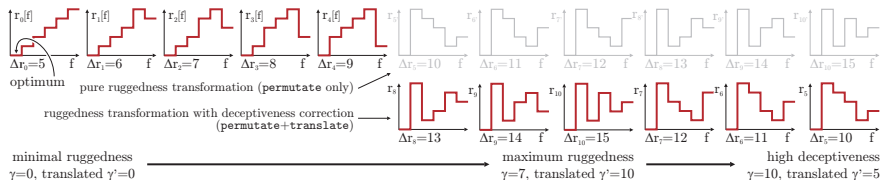
- Such permutations can be generated using the algorithm defined in ^[1]



- Such permutations can be generated using the algorithm defined in ^[1]
- Original algorithm produces alternating sequences of rugged and deceptive problems, the latter are much harder.



- Such permutations can be generated using the algorithm defined in [1]
- Original algorithm produces alternating sequences of rugged and deceptive problems, the latter are much harder.
- Re-arrangement into permutations $r_{\gamma'}$ which nicely blend from smooth to rugged to deceptive problems.



- Such permutations can be generated using the algorithm defined in ^[1]
- Original algorithm produces alternating sequences of rugged and deceptive problems, the latter are much harder.
- Re-arrangement into permutations $r_{\gamma'}$ which nicely blend from smooth to rugged to deceptive problems.
- Permutation serves as lookup-table to map objective values.

6

Introduction of Ruggedness

$$\begin{array}{ccc} \gamma=12, n=6 & & \\ \gamma'=9 & f(x_1)=3 & f(x_2)=6 \\ & \downarrow & \downarrow \\ & r_{12}[f(x_1)]=3 & r_{12}[f(x_2)]=5 \end{array}$$

1

original bit string (here: variable-length)

x 0101 0110 0000 1110 1000 0

2

Introduction of Neutrality

$\mu=2$ 01 01 01 10 00 00 11 10 10 00 0
 $\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$
 $u_2(g)$ 1 1 1 1 0 0 1 1 1 0 \times

3

Introduction of Epistasis

$v=4$ 1111 0011 10 insufficient bits,
 $\downarrow \downarrow \downarrow$ at the end, use
 $e_4 \downarrow e_4 \downarrow e_2 \leftarrow v=2$ instead of
 1110 0110 11 $v=4$

4

Multi-Objectivity

$m=2, n=6$ 1110011011 padding:
 (x_1, x_2) 110110 101010 $x^*[5]=0$
 x_1 x_2

5

Objective Values

$n=6$ 110110 101010
 $\downarrow \downarrow$
 $f(x_1)=3$ $f(x_2)=6$

6

Introduction of Ruggedness

$\gamma=12, n=6$
 $\gamma'=9$ $f(x_1)=3$ $f(x_2)=6$
 $\downarrow \downarrow$
 $r_{12}[f(x_1)]=3$ $r_{12}[f(x_2)]=5$

These are the slides for paper ^[1] presented at the BB-DOB workshop at GECCO'2018.

1 Introduction

2 The *W-Model*

3 Experiment

4 Summary



website

- Extensive experiments for the variable-length representation with multi-objectivity were performed in ^[28] and we use this data here.

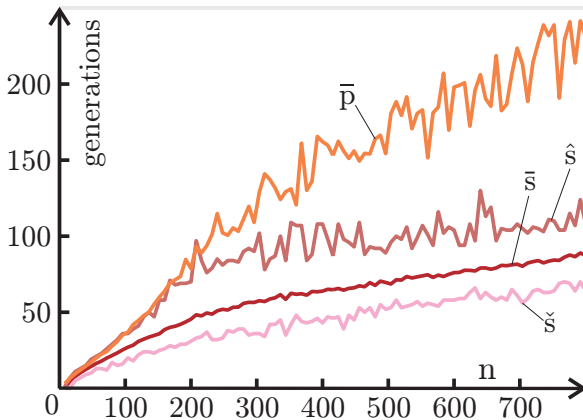
- Extensive experiments for the variable-length representation with multi-objectivity were performed in ^[28] and we use this data here.
- Now, a much bigger dataset for the fixed-length representation and a single-objective ($m = 1$) is available in ^[29].

- Extensive experiments for the variable-length representation with multi-objectivity were performed in ^[28] and we use this data here.
- Now, a much bigger dataset for the fixed-length representation and a single-objective ($m = 1$) is available in ^[29].
- In the experiment discussed here ^[28], we apply

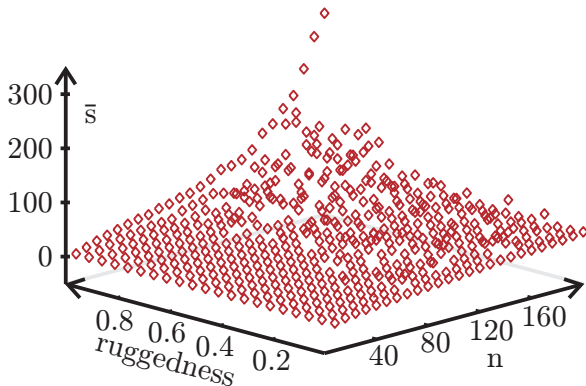
- Extensive experiments for the variable-length representation with multi-objectivity were performed in ^[28] and we use this data here.
- Now, a much bigger dataset for the fixed-length representation and a single-objective ($m = 1$) is available in ^[29].
- In the experiment discussed here ^[28], we apply
 - a standard multi-objective genetic algorithm with
 - population size 1000,
 - single-point crossover,
 - single-bit mutation,
 - tournament selection with tournament size 5,
 - Pareto ranking, and a
 - variable-length bit string genome with a maximum string length of 8000 bits.

- Extensive experiments for the variable-length representation with multi-objectivity were performed in ^[28] and we use this data here.
- Now, a much bigger dataset for the fixed-length representation and a single-objective ($m = 1$) is available in ^[29].
- In the experiment discussed here ^[28], we apply
 - a standard multi-objective genetic algorithm with
 - population size 1000,
 - single-point crossover,
 - single-bit mutation,
 - tournament selection with tournament size 5,
 - Pareto ranking, and a
 - variable-length bit string genome with a maximum string length of 8000 bits.
- We distinguish *success* (after s generations), i.e., finding a string x with $f(x) = 0$ (but which may be too long) and *perfection*, i.e., finding x^* (after $p \geq s$ generations).

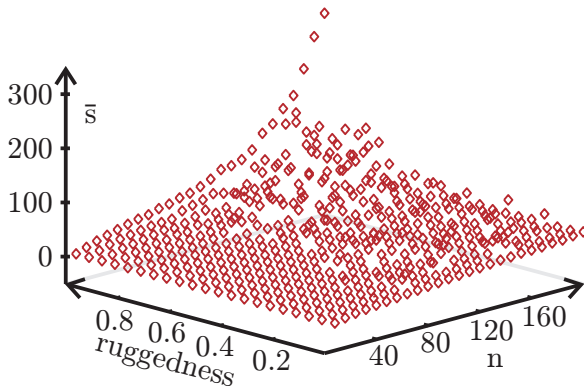
- The minimum, average, and maximum success generations \check{s} , \bar{s} , and \hat{s} measured rise almost linearly after the basic problem parameter n has exceeded 300 bits.



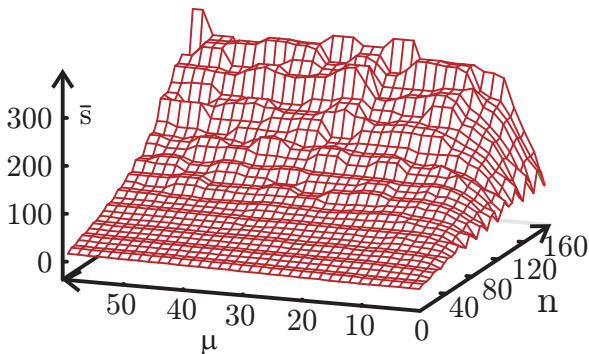
- Ruggedness parameter normalized into the range $[0, 1]$, because its range depends on n .



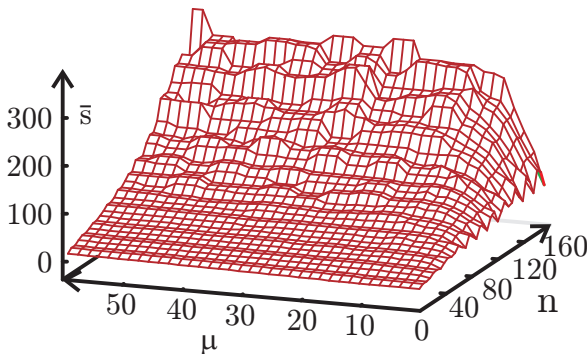
- Ruggedness parameter normalized into the range $[0, 1]$, because its range depends on n .
- Apart from a few peaks in the diagram occurring for $n > 70$, the problem hardness, as expected, increases very fast with the ruggedness.



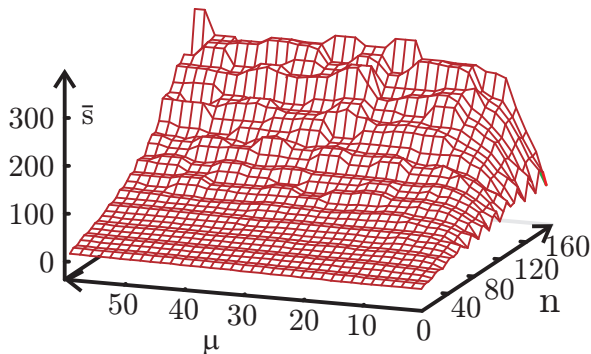
- Until a degree of $\mu \approx 10$, the problems rapidly gets harder with rising redundancy μ .



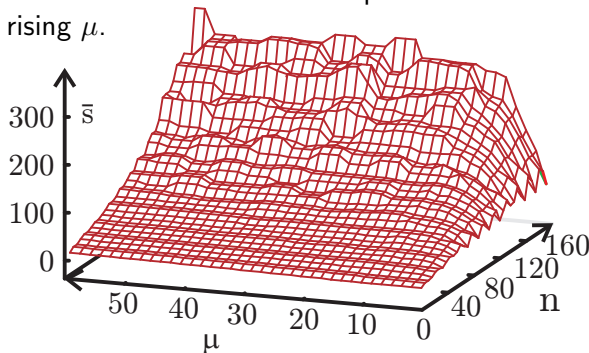
- Until a degree of $\mu \approx 10$, the problems rapidly gets harder with rising redundancy μ .
- From there on, a further increase of μ only leads to a very slow increase in hardness.



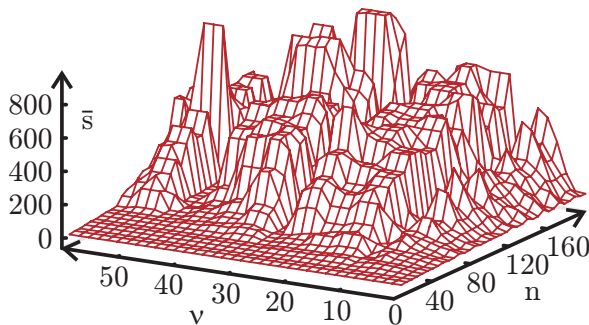
- Until a degree of $\mu \approx 10$, the problems rapidly gets harder with rising redundancy μ .
- From there on, a further increase of μ only leads to a very slow increase in hardness.
- Probably cause: for crossover, larger μ make no big difference.



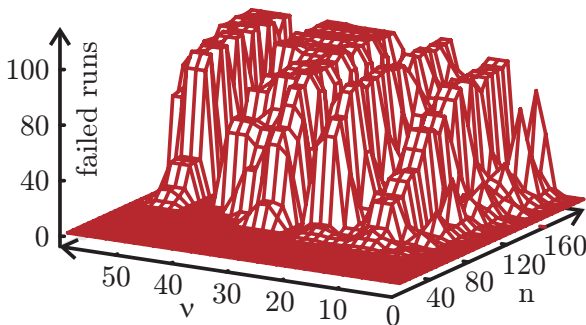
- Until a degree of $\mu \approx 10$, the problems rapidly gets harder with rising redundancy μ .
- From there on, a further increase of μ only leads to a very slow increase in hardness.
- Probably cause: for crossover, larger μ make no big difference.
- Experiments with lower crossover rate led to quick decrease of performance for rising μ .



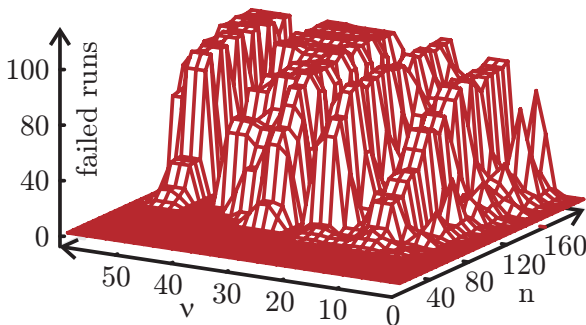
- Problem complexity steeply increases with rising epistasis (values of ν).



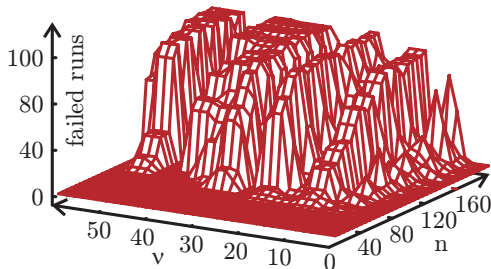
- Problem complexity steeply increases with rising epistasis (values of ν).
- Number of runs that cannot solve problem in 1000 generations quickly rises with ν .



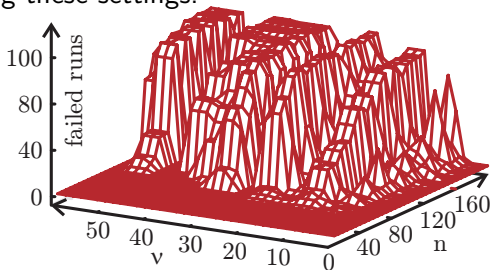
- Problem complexity steeply increases with rising epistasis (values of ν).
- Number of runs that cannot solve problem in 1000 generations quickly rises with ν .
- Problems for which $\nu = 2 + 4v : v \in \mathbb{N}$ are unexpectedly easy.



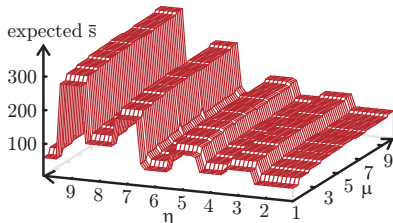
- Problem complexity steeply increases with rising epistasis (values of ν).
- Number of runs that cannot solve problem in 1000 generations quickly rises with ν .
- Problems for which $\nu = 2 + 4v : v \in \mathbb{N}$ are unexpectedly easy.
- The epistasis mapping e_ν decreases the Hamming distance for x_1, x_2 which have originally $h(x_1, x_2) = \nu/2$ in such cases ^[28].



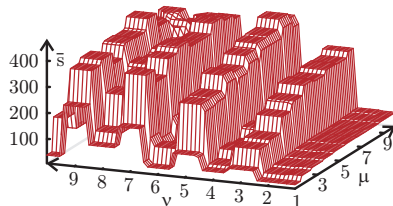
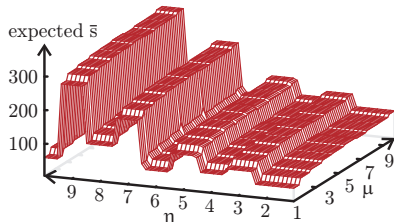
- Problem complexity steeply increases with rising epistasis (values of ν).
- Number of runs that cannot solve problem in 1000 generations quickly rises with ν .
- Problems for which $\nu = 2 + 4v : v \in \mathbb{N}$ are unexpectedly easy.
- The epistasis mapping e_ν decreases the Hamming distance for x_1, x_2 which have originally $h(x_1, x_2) = \nu/2$ in such cases ^[28].
- We suggest not using these settings.



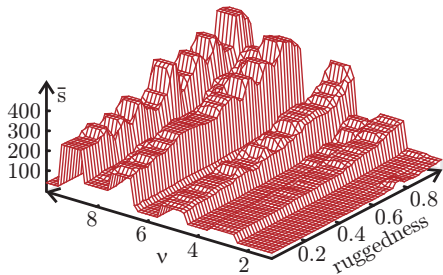
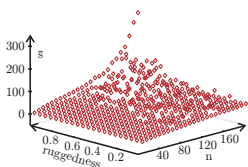
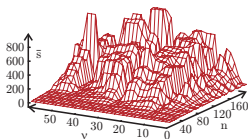
- Two separate experiments with either neutrality or epistasis added together: “expected \bar{s} ”



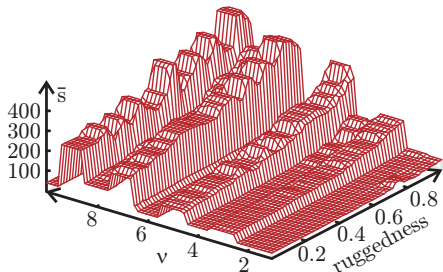
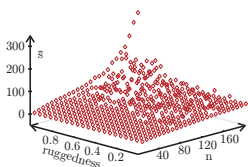
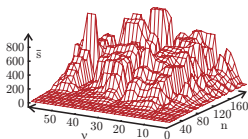
- Two separate experiments with either neutrality or epistasis added together: “expected \bar{s} ”
- Actual experiment with both neutrality and epistasis: shape similar to expectation, but 100 generations harder



- If both epistasis and ruggedness are used, results look similar to expectations of adding the results with only ruggedness to those with only epistasis.



- If both epistasis and ruggedness are used, results look similar to expectations of adding the results with only ruggedness to those with only epistasis.
- A rising ruggedness component leads, however, to over-proportional increases in \bar{s} .



These are the slides for paper ^[1] presented at the BB-DOB workshop at GECCO'2018.

- 1 Introduction
- 2 The *W-Model*
- 3 Experiment
- 4 Summary



website

- We have discussed several requirements for good discrete optimization benchmark problems.

- We have discussed several requirements for good discrete optimization benchmark problems.
- We have introduced the *W-Model* as an example problem that fulfills these requirements and suggest it for inclusion in the BB-DOB problem suite.

- We have discussed several requirements for good discrete optimization benchmark problems.
- We have introduced the *W-Model* as an example problem that fulfills these requirements and suggest it for inclusion in the BB-DOB problem suite.
- It can be applied with a fixed-length and a variable-length representation.

- We have discussed several requirements for good discrete optimization benchmark problems.
- We have introduced the *W-Model* as an example problem that fulfills these requirements and suggest it for inclusion in the BB-DOB problem suite.
- It can be applied with a fixed-length and a variable-length representation.
- Our old experiments with the variable-length representation show that it behaves well and as expected.

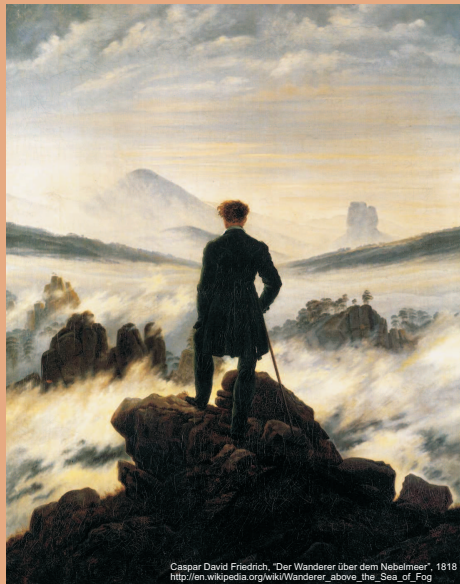
- We have discussed several requirements for good discrete optimization benchmark problems.
- We have introduced the *W-Model* as an example problem that fulfills these requirements and suggest it for inclusion in the BB-DOB problem suite.
- It can be applied with a fixed-length and a variable-length representation.
- Our old experiments with the variable-length representation show that it behaves well and as expected.
- A new implementation ^[10] is provided along with new results for the fixed-length representation in ^[29].

谢谢

Thank you

Thomas Weise & Zijun
Wu [汤卫思 & 吴自军]
tweise@hfu.edu.cn
<http://iao.hfu.edu.cn>

Hefei University, South Campus 2
Institute of Applied Optimization
Shushan District, Hefei, Anhui,
China





1. Thomas Weise and Zijun Wu. Difficult features of combinatorial optimization problems and the tunable *W-Model* benchmark problem for simulating them. In *Black Box Discrete Optimization Benchmarking (BB-DOB) Workshop in Companion Material Proceedings of the Genetic and Evolutionary Computation Conference (GECCO 2018), July 15–19, 2018, Kyoto, Japan*, pages 1769–1776. ACM. doi: 10.1145/3205651.3208240. ISBN: 978-1-4503-5764-7. Additional experimental results with the *W-Model*, which were not used in this paper, can be found in ^[29] at doi:10.5281/zenodo.1256883 based on implementation ^[10].
2. Thomas Weise, Raymond Chiong, and Ke Tang. Evolutionary optimization: Pitfalls and booby traps. *Journal of Computer Science and Technology (JCST)*, 27:907–936, 2012. doi: 10.1007/s11390-012-1274-4.
3. Thomas Weise, Michael Zapf, Raymond Chiong, and Antonio Jesús Nebro Urbaneja. Why is optimization difficult? In Raymond Chiong, editor, *Nature-Inspired Algorithms for Optimisation*, volume 193 of *Studies in Computational Intelligence*, chapter 1, pages 1–50. Springer-Verlag, Berlin/Heidelberg, 2009. ISBN 978-3-642-00266-3. doi: 10.1007/978-3-642-00267-0_1.
4. David Lee Applegate, Robert E. Bixby, Vašek Chvátal, and William John Cook. *The Traveling Salesman Problem: A Computational Study*. Princeton University Press, Princeton, NJ, USA, 2007.
5. Thomas Weise, Raymond Chiong, Ke Tang, Jörg Lässig, Shigeyoshi Tsutsui, Wenxiang Chen, Zbigniew Michalewicz, and Xin Yao. Benchmarking optimization algorithms: An open source framework for the traveling salesman problem. *IEEE Computational Intelligence Magazine (CIM)*, 9(3):40–52, August 2014. doi: 10.1109/MCI.2014.2326101.
6. *Proc. of the 18th Intl. Conf. on Theory and Applications of Satisfiability Testing (SAT 2015)*, volume 9340 of *Lecture Notes in Computer Science book series (LNCS)*, Cham, September 24–27, 2015. Springer. ISBN 978-3-319-24317-7. doi: 10.1007/978-3-319-24318-4.
7. Holger H. Hoos and Thomas Stützle. SATLIB: An online resource for research on SAT. In *SAT2000 – Highlights of Satisfiability Research in the Year 2000*, volume 63 of *Frontiers in Artificial Intelligence and Applications*, pages 283–292, Amsterdam, The Netherlands, 2000. IOS Press. ISBN 978-1-58603-061-2. URL <http://www.cs.ubc.ca/~hoos/Publ/sat2000-satlib.pdf>.
8. Thomas Weise, Stefan Niemczyk, Hendrik Skubch, Roland Reichle, and Kurt Geihs. A tunable model for multi-objective, epistatic, rugged, and neutral fitness landscapes. In *Genetic and Evolutionary Computation Conference*, pages 795–802, Atlanta, GA, USA, 2008. ACM Press.
9. Thomas Weise. *Global Optimization Algorithms – Theory and Application*. it-weise.de (self-published), Germany, 2009. URL <http://www.it-weise.de/projects/book.pdf>.

10. Thomas Weise. The *W-Model*, a tunable black-box discrete optimization benchmarking (bb-dob) problem, implemented for the bb-dob@gecco workshop, June 2018. URL http://github.com/thomasWeise/BBD0B_W_Model. Open source implementation *W-Model*^[1] provided in a GitHub repository, used in^[29].
11. David H. Ackley. *A Connectionist Machine for Genetic Hillclimbing*. PhD thesis, Carnegie Mellon University (CMU), Pittsburgh, PA, USA, 1987.
12. Dirk Thierens and David Edward Goldberg. Convergence models of genetic algorithm selection schemes. In *Proc. of the Third Conf. on Parallel Problem Solving from Nature (PPSN III)*, volume 866/1994 of *Lecture Notes in Computer Science (LNCS)*, pages 119–129, Berlin, Germany, October 9–14, 1994. Springer-Verlag GmbH. ISBN 0387584846. doi: 10.1007/3-540-58484-6_256.
13. Brad L. Miller and David Edward Goldberg. Genetic algorithms, selection schemes, and the varying effects of noise. *Evolutionary Computation*, 4(2):113–131, 1996. doi: 10.1162/evco.1996.4.2.113.
14. Tobias Blickle and Lothar Thiele. A comparison of selection schemes used in genetic algorithms. TIK-Report 11, ETH Zürich, Department of Electrical Engineering, Computer Engineering and Networks Laboratory (TIK), Zürich, Switzerland, December 1995. URL <ftp://ftp.tik.ee.ethz.ch/pub/publications/TIK-Report11.ps>.
15. Christian M. Reidys and Peter F. Stadler. Neutrality in fitness landscapes. *Journal of Applied Mathematics and Computation*, 117(2–3):321–350, 2001. doi: 10.1016/S0096-3003(99)00166-6.
16. Lionel Barnett. Ruggedness and neutrality – the nkp family of fitness landscapes. In *Proc. of the Sixth Intl. Conf. on Artificial Life (Artificial Life VI)*, volume 6 of *Complex Adaptive Systems*, pages 18–27, Cambridge, MA, USA, June 27–29, 1998. MIT Press. ISBN 0-262-51099-5.
17. Joshua D. Knowles and Richard A. Watson. On the utility of redundant encodings in mutation-based evolutionary search. In *Proc. of the 7th Intl. Conf. on Parallel Problem Solving from Nature (PPSN VII)*, volume 2439 of *Lecture Notes in Computer Science (LNCS)*, pages 88–98, Berlin, Germany, September 7–11, 2002. Springer-Verlag GmbH. ISBN 3-540-44139-5. doi: 10.1007/3-540-45712-7_9.
18. Franz Rothlauf. *Representations for Genetic and Evolutionary Algorithms*, volume 104 of *Studies in Fuzziness and Soft Computing*. Springer-Verlag, Berlin/Heidelberg, second edition, 2006. doi: 10.1007/3-540-32444-5.
19. Jay L. Lush. Progeny test and individual performance as indicators of an animal's breeding value. *Journal of Dairy Science (JDS)*, 18(1):1–19, 1935.
20. Lee Altenberg. Nk fitness landscapes. In Thomas Bäck, David B. Fogel, and Zbigniew Michalewicz, editors, *Handbook of Evolutionary Computation*, chapter B2.7.2. Oxford University Press, Oxford, England, UK, 1997.

21. Yuval Davidor. Epistasis variance: A viewpoint on GA-hardness. In *Proc. of the First Workshop on Foundations of Genetic Algorithms (FOGA'90)*, pages 23–35, San Francisco, CA, USA, July 15–18, 1990. Morgan Kaufmann Publishers Inc. ISBN 1-55860-170-8.
22. Bart Naudts and Alain Verschoren. Epistasis on finite and infinite spaces. In *Proc. of the 8th Intl. Conf. on Systems Research, Informatics and Cybernetics (InterSymp'96)*, pages 19–23, Tecumseh, ON, Canada, August 14–18, 1996. Intl. Institute for Advanced Studies in Systems Research and Cybernetic (IIAS).
23. Kalyanmoy Deb. *Multi-Objective Optimization Using Evolutionary Algorithms*. Wiley Interscience Series in Systems and Optimization. John Wiley & Sons Ltd., New York, NY, USA, 2001. ISBN 047187339X.
24. Carlos Artemio Coello Coello. An updated survey of evolutionary multiobjective optimization techniques: State of the art and future trends. In *Congress on Evolutionary Computation (CEC)*, pages 3–13, Piscataway, NJ, USA, July 6–9, 1999. IEEE Press. doi: 10.1109/CEC.1999.781901.
25. Carlos M. Fonseca and Peter J. Fleming. Multiobjective optimization and multiple constraint handling with evolutionary algorithms – part i: A unified formulation. *IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans*, 28(1):26–37, 1998.
26. Charles Campbell Palmer and Aaron Kershenbaum. Representing trees in genetic algorithms. In *Proc. of the 1st IEEE Conf. on Evolutionary Comput. (CEC'94)*, volume 1, pages 379–384, Piscataway, NJ, USA, June 27–29, 1994. IEEE Comp. Soc. ISBN 0-7803-1899-4.
27. Charles Campbell Palmer. *An approach to a problem in network design using genetic algorithms*. PhD thesis, Polytechnic University, New York, NY, 1994.
28. Stefan Niemczyk. Ein benchmark problem für globale optimierungsverfahren. Bachelor's thesis, Distributed Systems Group, University of Kassel, May 2008. Supervisor: Thomas Weise.
29. Thomas Weise. Results for several simple algorithms on the *W-Model* for black-box discrete optimization benchmarking, June 2018. URL <https://doi.org/10.5281/zenodo.1256883>. Based on the implementation of *W-Model*^[1] given in^[10].