

# Hybrid PACO & Pheromone Initialization for VRPTWs

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  - vehicle capacity and time windows must not be violated



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  - $f_1$  often considered as more important, since using more vehicles costs more than driving a bit longer



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  - if vehicle capacity is exhausted or no other customer can be visited in time-window restriction, vehicle returns to  $c_0$
  - next vehicle is used, until all customers are satisfied



Related Work



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  - we hybridize the algorithm to further improve the result quality



#### • Solomon benchmark set [21]: 25-, 50-, and 100-customer instance sets

## **Existing ACO Methods**



- Solomon benchmark set <sup>[21]</sup>: 25-, 50-, and 100-customer instance sets
- 20 independent runs per instance, 300'000 FEs per run



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	In	Instances with 25 customers						Instances with 50 customers						ance	es w	ith 1	00 c	ustomers	Total						
			$f_1$		$f_2$				$f_1$				$f_2$	$f_1$					$f_2$	$f_1$			$f_2$		
Algorithm 1 vs. 2		-	+	0	-	+	0	-	+	0	-	$^+$	0	-	+	0	1	$^+$	0	-	+	0	-	+	0
ACS	IACO	0	40	16	0	52	4	0	40	16	1	51	4	0	44	12	0	54	2	0	124	44	1	157	10
ACS	MMAS	0	49	7	1	51	4	0	41	15	9	36	11	2	36	18	35	7	14	2	126	40	45	94	29
ACS	PACO-ABS	0	12	44	9	31	16	0	35	21	5	45	6	0	54	2	0	51	5	0	101	67	14	127	27
ACS	PACO-EBS	0	47	9	0	54	2	0	48	8	0	54	2	0	55	1	0	52	4	0	150	18	0	160	8
ACS	PACO-QBS	0	45	11	0	55	1	0	49	7	0	55	1	0	56	0	0	56	0	0	150	18	0	166	2
IACO	MMAS	0	17	39	13	28	15	9	7	40	38	7	11	35	1	20	56	0	0	44	25	99	107	35	26
IACO	PACO-ABS	0	12	44	9	31	16	0	35	21	5	45	6	0	54	2	0	51	5	0	101	67	14	127	27
IACO	PACO-EBS	0	9	47	9	28	19	0	31	25	4	43	9	0	42	14	2	39	15	0	82	86	15	110	43
IACO	PACO-QBS	0	13	43	8	34	14	0	37	19	4	49	3	0	54	2	0	52	4	0	104	64	12	135	21
MMAS	PACO-ABS	5	1	50	15	12	29	0	31	25	1	49	6	0	54	2	0	56	0	5	86	77	16	117	35
MMAS	PACO-EBS	3	1	52	19	12	25	0	29	27	0	49	7	0	53	3	0	56	0	3	89	82	19	117	32
MMAS	PACO-QBS	3	1	52	16	15	25	0	33	23	0	52	4	0	55	1	0	56	0	3	83	76	16	123	29
PACO-ABS	PACO-EBS	0	0	56	5	1	50	3	0	53	19	2	35	28	0	28	48	0	8	31	0	137	72	3	93
PACOABS	PACO-QBS	0	0	56	0	3	53	0	1	55	0	13	43	0	1	55	1	28	27	0	2	166	1	44	123
PACO-EBS	PACO-QBS	1	0	55	0	18	38	0	4	52	0	34	22	0	39	17	0	55	1	1	43	124	0	107	61

Mann-Whitey U test ( $\alpha = 0.02$ ) comparison results for ACO algorithms (– is better, + is worse).



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	In	Instances with 25 customers						Instances with 50 customers						Instances with 100 customers							Total					
			$f_1$		$f_2$			$f_1$					$f_2$	$f_1$					$f_2$	$f_1$			$f_2$			
Algorithm 1 vs. 2		-	+	0	-	+	0	-	+	0	-	+	0	-	+	0	-	+	0	-	+	0	-	+	0	
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#### ACS performs worst



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		Lo	cton		with	25 4	ustomers	Lo	cton		with	F0 /	ustomers	Inc			+h 1	00 6	ustomers	r –		T	otal		
			SLall	ces i	WILII	25 0	uscomers		SLan	Les V	with	30 0	Lustomers	IIIS	anc	es w	1111 1	00 0	uscomers			П	JLai		
			$f_1$				$f_2$		$f_1$				$f_2$		$f_1$				$f_2$		$f_1$			$f_2$	
Algorith	n 1 vs. 2	-	$^+$	0	-	+	0	-	$^+$	0	-	$^+$	0	-	+	0	-	$^+$	0	-	+	0	-	+	0
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PACO-EBS	PACO-QBS	1	0	55	0	18	38	0	4	52	0	34	22	0	39	17	0	55	1	1	43	124	0	107	61
				1		0 0	2								<hr/>				/ .						`

Mann-Whitey U test ( $\alpha = 0.02$ ) comparison results for ACO algorithms (– is better, + is worse).

#### ACS performs worst

• PACO with QBS rule performs best



- Solomon benchmark set <sup>[21]</sup>: 25-, 50-, and 100-customer instance sets
- 20 independent runs per instance, 300'000 FEs per run

		In	stan	ces v	vith	25 c	ustomers	In	stan	ces v	with	50 c	ustomers	Inst	ance	es w	ith 1	00 c	ustomers			Т	otal		
			$f_1$				fa		$f_1$				fa		$f_1$				fa		$f_1$			$f_2$	
Algorithr	n 1 vs. 2	-	+	0	-	+	0	-	+	0	-	+	0	-	+	0	-	+	0	-	+	0	-	+	0
ACS	IACO	0	40	16	0	52	4	0	40	16	1	51	4	0	44	12	0	54	2	0	124	44	1	157	10
ACS	MMAS	0	49	7	1	51	4	0	41	15	9	36	11	2	36	18	35	7	14	2	126	40	45	94	29
ACS	PACO-ABS	0	12	44	9	31	16	0	35	21	5	45	6	0	54	2	0	51	5	0	101	67	14	127	27
ACS	PACO-EBS	0	47	9	0	54	2	0	48	8	0	54	2	0	55	1	0	52	4	0	150	18	0	160	8
ACS	PACO-QBS	0	45	11	0	55	1	0	49	7	0	55	1	0	56	0	0	56	0	0	150	18	0	166	2
IACO	MMAS	0	17	39	13	28	15	9	7	40	38	7	11	35	1	20	56	0	0	44	25	99	107	35	26
IACO	PACO-ABS	0	12	44	9	31	16	0	35	21	5	45	6	0	54	2	0	51	5	0	101	67	14	127	27
IACO	PACO-EBS	0	9	47	9	28	19	0	31	25	4	43	9	0	42	14	2	39	15	0	82	86	15	110	43
IACO	PACO-QBS	0	13	43	8	34	14	0	37	19	4	49	3	0	54	2	0	52	4	0	104	64	12	135	21
MMAS	PACO-ABS	5	1	50	15	12	29	0	31	25	1	49	6	0	54	2	0	56	0	5	86	77	16	117	35
MMAS	PACO-EBS	3	1	52	19	12	25	0	29	27	0	49	7	0	53	3	0	56	0	3	89	82	19	117	32
MMAS	PACO-QBS		1	52	16	15	25	0	33	23	0	52	4	0	55	1	0	56	0	3	83	76	16	123	29
PACO-ABS	PACO-EBS		0	56	5	1	50	3	0	53	19	2	35	28	0	28	48	0	8	31	0	137	72	3	93
PACOABS	PACO-QBS		0	56	0	3	53	0	1	55	0	13	43	0	1	55	1	28	27	0	2	166	1	44	123
	PACO-QBS	1	0	55	0	18	38	0	4	52	0	34	22	0	39	17	0	55	1	1	43	124	0	107	61

Mann-Whitey U test ( $\alpha = 0.02$ ) comparison results for ACO algorithms (– is better, + is worse).

- ACS performs worst
- PACO with QBS rule performs best  $\Rightarrow$  use from now on

Hybrid PACO with Enhanced Pheromone Initialization for Solving the VRPTW, 2015-12-10, CIPLS Shi et al.



• Utilize static information from problem instance to initialize pheromones for PACO-QBS



- Utilize static information from problem instance to initialize pheromones for PACO-QBS
- Model service begin time  $b_i$  as random variable PD



- Utilize static information from problem instance to initialize pheromones for PACO-QBS
- Model service begin time  $b_i$  as random variable PD (for PD, we test normal, uniform, and power distribution PDFs)



- Utilize static information from problem instance to initialize pheromones for PACO-QBS
- Model service begin time  $b_i$  as random variable PD
- Define VE as a function which is larger if  $c_i$  and  $c_j$  are close and if  $c_j$  would be serviced at the end of its time window if visited directly after  $c_i$



- Utilize static information from problem instance to initialize pheromones for PACO-QBS
- Model service begin time  $b_i$  as random variable PD
- Define VE as a function which is larger if  $c_i$  and  $c_j$  are close

• Set 
$$\tau_{ij}^0 \approx \max\left\{\frac{1}{n}, \int_{e_i}^{l_i} PD(x) * VE(i, j, x)dx\right\}$$



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• Experiments with PACO-QBS and the three different probability distribution models show...



- Utilize static information from problem instance to initialize pheromones for PACO-QBS
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		In	star	ices	wit	h 25	customers	In	star	nces	wit	h 50	customers	In	stan	ces \	with	n 100	) customers			То	otal		
			$f_1$				$f_2$		$f_1$				$f_2$		$f_1$				$f_2$		$f_1$			$f_2$	
Algorith	m 1 vs. 2	-	+	0	-	+	0	1	$^+$	0	-	+	0	-	+	0	1	+	0	-	+	0	-	+	0
Nolni	Normal	0	2	54	4	26	26	4	7	45	6	32	18	8	14	34	4	32	20	12	23	133	14	90	64
Nolni	Power	0	3	53	1	33	22	2	7	47	3	35	18	5	13	38	4	34	18	7	23	138	8	102	58
Nolni	Uniform	0	2	54	1	39	16	1	7	48	4	40	12	3	15	38	2	42	12	4	24	140	7	121	40
Normal	Power	0	0	56	1	13	42	0	2	54	1	8	47	0	1	55	1	6	49	0	3	165	3	27	138
Normal	Uniform	0	0	56	0	19	37	0	3	53	1	13	42	0	3	53	0	15	41	0	6	162	1	47	120
Power	Uniform	0	0	56	4	0	52	0	0	56	3	1	52	0	0	56	9	0	47	0	0	168	16	1	151



- Utilize static information from problem instance to initialize pheromones for PACO-QBS
- Model service begin time  $b_i$  as random variable PD
- Define VE as a function which is larger if  $c_i$  and  $c_j$  are close

• Set 
$$\tau_{ij}^0 \approx \max\left\{\frac{1}{n}, \int_{e_i}^{l_i} PD(x) * VE(i, j, x)dx\right\}$$

 Experiments with PACO-QBS and the three different probability distribution models show that phromone-initialized PACO performs significantly better

		In	star	nces	wit	h 25	customers	In	star	nces	wit	h 50	customers	In	stan	ces ۱	vitł	n 100	) customers			To	otal		
			$f_1$				$f_2$		$f_1$				$f_2$		$f_1$				$f_2$		$f_1$			$f_2$	
Algorith	m 1 vs. 2			0	-	+	0	-	+	0	-	+	0	1	+	0	1	+	0	-	+	0	-	+	0
Nolni	Normal	0	2	54	4	26	26	4	7	45	6	32	18	8	14	34	4	32	20	12	23	133	14	90	64
Nolni	Power	0	3	53	1	33	22	2	7	47	3	35	18	5	13	38	4	34	18	7	23	138	8	102	58
Nolni	Uniform	0	2	54	1	39	16	1	7	48	4	40	12	3	15	38	2	42	12	4	24	140	7	121	40
Normal	Power	0	0	56	1	13	42	0	2	54	1	8	47	0	1	55	1	6	49	0	3	165	3	27	138
Normal	Uniform	0	0	56	0	19	37	0	3	53	1	13	42	0	3	53	0	15	41	0	6	162	1	47	120
Power	Uniform	0	0	56	4	0	52	0	0	56	3	1	52	0	0	56	9	0	47	0	0	168	16	1	151



- Utilize static information from problem instance to initialize pheromones for PACO-QBS
- Model service begin time  $b_i$  as random variable PD
- Define VE as a function which is larger if  $c_i$  and  $c_j$  are close

• Set 
$$\tau_{ij}^0 \approx \max\left\{\frac{1}{n}, \int_{e_i}^{l_i} PD(x) * VE(i, j, x)dx\right\}$$

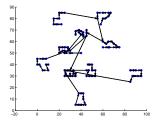
• Experiments with PACO-QBS and the three different probability distribution models show that phromone-initialized PACO performs significantly better and power distributed *b* performs best

		In	star	ices	wit	h 25	customers	In	star	nces	wit	h 50	customers	In	stan	ces v	vitł	n 100	) customers			To	otal		
			$f_1$				$f_2$		$f_1$				$f_2$		$f_1$				$f_2$		$f_1$			$f_2$	
Algorith	m 1 vs. 2	-	$^+$	0	-	+	0	1	$^+$	0	-	+	0	-	+	0	-	+	0	-	+	0	-	+	0
Nolni	Normal	0	2	54	4	26	26	4	7	45	6	32	18	8	14	34	4	32	20	12	23	133	14	90	64
Nolni	Power	0	3	53	1	33	22	2	7	47	3	35	18	5	13	38	4	34	18	7	23	138	8	102	58
Nolni	Uniform	0	2	54	1	39	16	1	7	48	4	40	12	3	15	38	2	42	12	4	24	140	7	121	40
Normal	Power	0	0	56	1	13	42	0	2	54	1	8	47	0	1	55	1	6	49	0	3	165	3	27	138
Normal	Uniform	0	0	56	0	19	37	0	3	53	1	13	42	0	3	53	0	15	41	0	6	162	1	47	120
Power	Uniform	0	0	56	4	0	52	0	0	56	3	1	52	0	0	56	9	0	47	0	0	168	16	1	151



 Ideally, initialization should assign pheromones such that the edges with the strongest pheromones form larger tour components

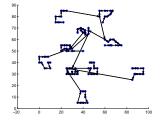
- Ideally, initialization should assign pheromones such that the edges with the strongest pheromones form larger tour components
- This works especially for instances where customers are clustered



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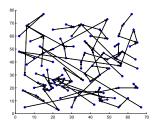
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- Ideally, initialization should assign pheromones such that the edges with the strongest pheromones form larger tour components
- This works especially for instances where customers are clustered, but not if customers and time windows are completely random

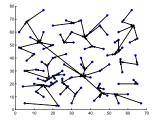


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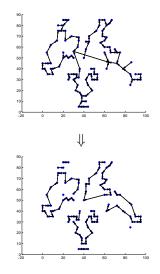


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- Method 1: Change VE to put more pheromones on shorter edges



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- Method 2: Keep initialized pheromone only on one edge per node; two choices maximum or difference selection (see paper)

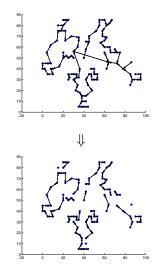


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#### maximum selection

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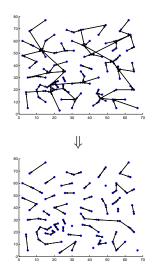


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difference selection

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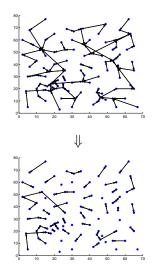


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#### maximum selection

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difference selection

## Hybridize with Local Search

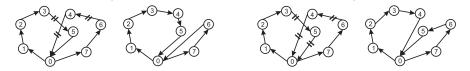


• Homberger and Gehring <sup>[12]</sup> proposed a hybrid metaheuristic that randomly selects one neighborhood from  $\{N_{1-insert}, N_{1-exchange}, N_{2-opt}\}$  to refine solutions with local search

## Hybridize with Local Search

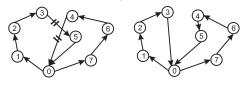


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 $N_{1-insert}$ 

 $N_{1-exchange}$ 



 $N_{2-opt}$ 

## Hybridize with Local Search



- Homberger and Gehring <sup>[12]</sup> proposed a hybrid metaheuristic that randomly selects one neighborhood from  $\{N_{1-insert}, N_{1-exchange}, N_{2-opt}\}$  to refine solutions with local search
- We adopt this mechanism into PI-PACO.



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- We adopt this mechanism into PI-PACO.
- Hybrid PI-PACO with difference selection achieves better results than hybrid PACO without pheromone initialization

Туре	Goal	Chen and	Sodsoon and	hybrid PACO	hybrid PI-PACO maximum	hybrid PI-PACO difference
		Ting <sup>[14]</sup>	Changyom <sup>[22]</sup>		selection	selection
R1	$f_1$	12.83	13.83	12.83	12.92	12.75
	$f_2$	1203.56	1259.19	1204.06	1205.11	1203.67
C1	$f_1$	10	10	10	10	10
	$f_2$	828.76	838.12	828.61	828.60	828.55
RC1	$f_1$	12.50	12.63	12.75	12.63	12.38
	$f_2$	1363.84	1436.58	1381.42	1380.78	1380.54
R2	$f_1$	3.09	3.82	3.45	3.64	3.54
	$f_2$	932.23	980.98	1005.35	995.03	1006.38
C2	$f_1$	3	3	3	3	3
	$f_2$	589.86	591.13	590.71	589.93	589.86
RC2	$f_1$	3.75	4.5	4.13	4.38	4.13
	$f_2$	1079.81	1141.63	1113.59	1156.20	1109.8

Hybrid PACO with Enhanced Pheromone Initialization for Solving the VRPTW, 2015-12-10, CIPLS Shi et al.



- Homberger and Gehring <sup>[12]</sup> proposed a hybrid metaheuristic
- We adopt this mechanism into PI-PACO.
- Hybrid PI-PACO with difference selection achieves better results than hybrid PACO without pheromone initialization
- It outperforms the hybrid algorithm by Chen and Ting <sup>[14]</sup> on problem type C1 and achieves similar results on problem type C2

Туре	Goal	Chen and	Sodsoon and	hybrid PACO	hybrid PI-PACO maximum	hybrid PI-PACO difference
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- Homberger and Gehring <sup>[12]</sup> proposed a hybrid metaheuristic
- We adopt this mechanism into PI-PACO.
- Hybrid PI-PACO with difference selection achieves better results than hybrid PACO without pheromone initialization
- It is similar to the MMAS-VRPTW <sup>[22]</sup> but outperforms it on all except R2 instances in terms of the distance

Type	Goal	Chen and	Sodsoon and	hybrid PACO	hybrid PI-PACO maximum	hybrid PI-PACO difference
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## PACO best ACO for VRPTW





- PACO best ACO for VRPTW
- Pheromone matrix initialization makes it better



- PACO best ACO for VRPTW
- Pheromone matrix initialization makes it better
- Hybridization + pheromone matrix initialization is best



- PACO best ACO for VRPTW
- Pheromone matrix initialization makes it better
- Hybridization + pheromone matrix initialization is best
- Concept should be tested in offer domains, such as quadratic assignment poblems



# 谢谢!

## Thank you.

Wei Shi^1, Thomas Weise<sup>1</sup>, Raymond Chiong<sup>2</sup>, and Bülent  $\mbox{\it Catay}^3$ 

<sup>1</sup> University of Science and Technology of China,

<sup>2</sup> The University of Newcastle, Australia

<sup>3</sup> Sabanci University, Turkey







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