

Modeling Optimization Algorithm Runtime Behavior and its Applications

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GECCO JULY 2017

Introduction & Approach

Data:

- ~ sequence of tuples (t_i, q_i) , where t_i relating an elapsed amount of time and to the quality q_i of the best solution discovered with t_i

$$\text{Models Fitting Quality } \Phi: \Phi(M) = 1/n_s (\sum_{i=1:n_s} (M(t_i) - q_i)^2 / q_i)$$

Curve Fitting Method:

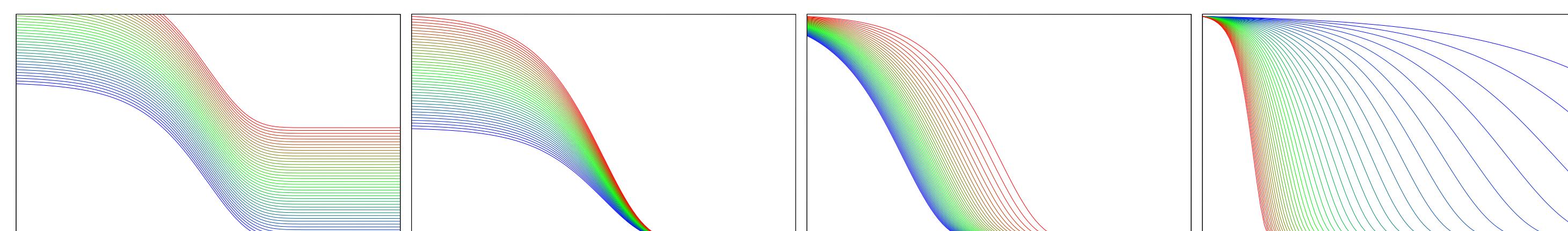
- ~ Levenberg-Marquardt algorithm
- ~ intelligent initialization strategy
- ~ multiple restarts
- ~ models:

Name	Shortcut	Formula
Decay Model	DCM	$A + B * \exp(C * t^D)$
Logistic Model	LGM	$A + B / (1 + \exp(C * \ln(t) + D))$
Exp-Linear Model	ELM	$A + B * \exp(C * \ln(t+D))$
Gompertz Model	GPM	$A + B * \exp(C * \exp(D * t))$

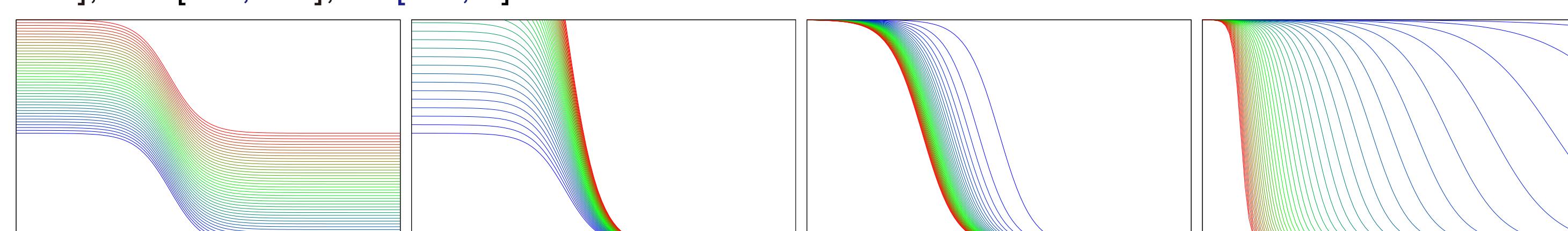
ANNs:

- ~ represent a much wider range of behavior
- ~ have significantly more parameters
- ~ the semantics of these parameters are too complex to be manually interpreted
- ~ might violate the constraint that any reasonable optimization process will never forget the best solution encountered
- ~ in our experiment we use feed-forward ANNs with a single hidden layer.
- ~ implemented in R

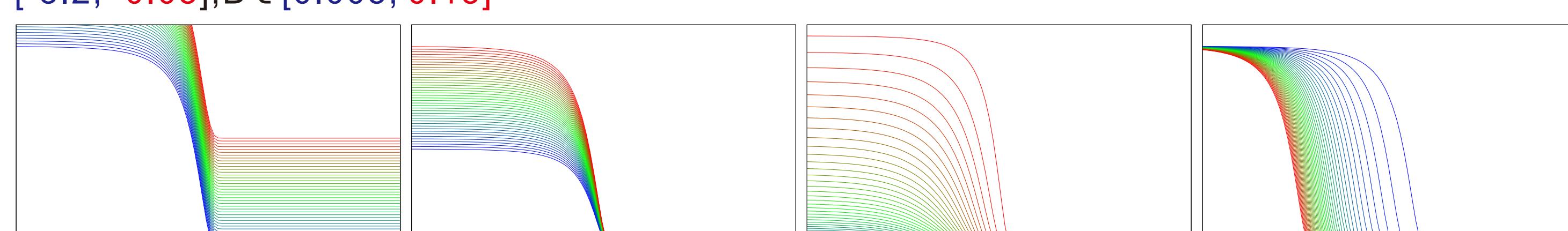
a. Generalized exponential decay (DCMP) $y = M(t) = A + B * \exp(C * t^D)$ for $A \in [0, 50]$, $B \in [50, 100]$, $C \in [-0.1, -0.01]$, $D \in [0.3, 3]$



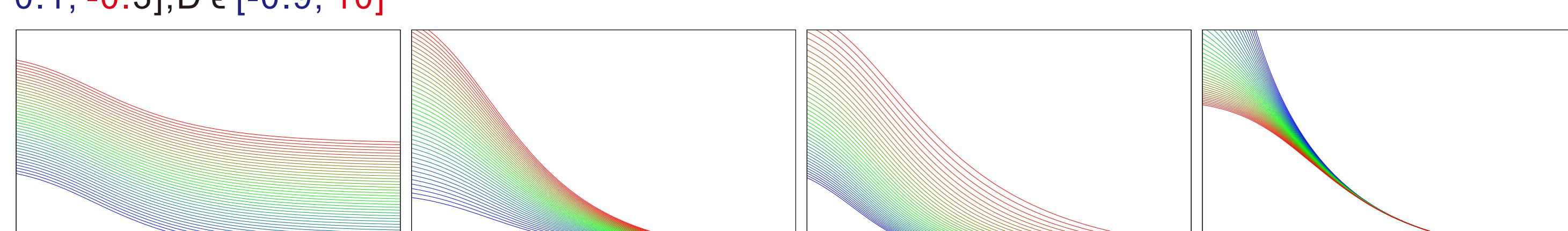
b. Generalized logistic model (LGMP) $y = M(t) = A + B / (1 + \exp(C * \ln(t) + D))$ for $A \in [0, 50]$, $B \in [50, 200]$, $C \in [0.5, 0.8]$, $D \in [0.3, 3]$



c. Gompertz function (GPMP) $y = M(t) = A + B * \exp(C * \exp(D * t))$ for $A \in [0, 50]$, $B \in [50, 100]$, $C \in [-3.2, -0.05]$, $D \in [0.005, 0.15]$



d. Exp-linear model (ELMP) $y = M(t) = A + B * \exp(C * \ln(t+D))$ for $A \in [0, 50]$, $B \in [50, 200]$, $C \in [-0.1, -0.3]$, $D \in [-0.9, 10]$



Case Study: MAX-SAT

Problem Introduction & Experiment Setup:

- ~ six setups with hill climber algorithms: 3 search operator (1, 2, m-bit flip) and whether or not restarts are applied
- ~ 20 independent runs for each algorithm-instance combination
- ~ Maximum Satisfiability Problem (MAX-SAT) with n variables as case study

Model Parameters vs. Instance Features:

- ~ analysis relationship between model parameters and algorithm configuration
- ~ the smaller parameter A the better asymptotic results
- ~ parameter B has an upward trend for all algorithms with a rising number of variables/clauses of the MAX-SAT instances
- ~ parameter C and D both have approximately a negative linear association with the instance scale

Algorithm and Instance Classification:

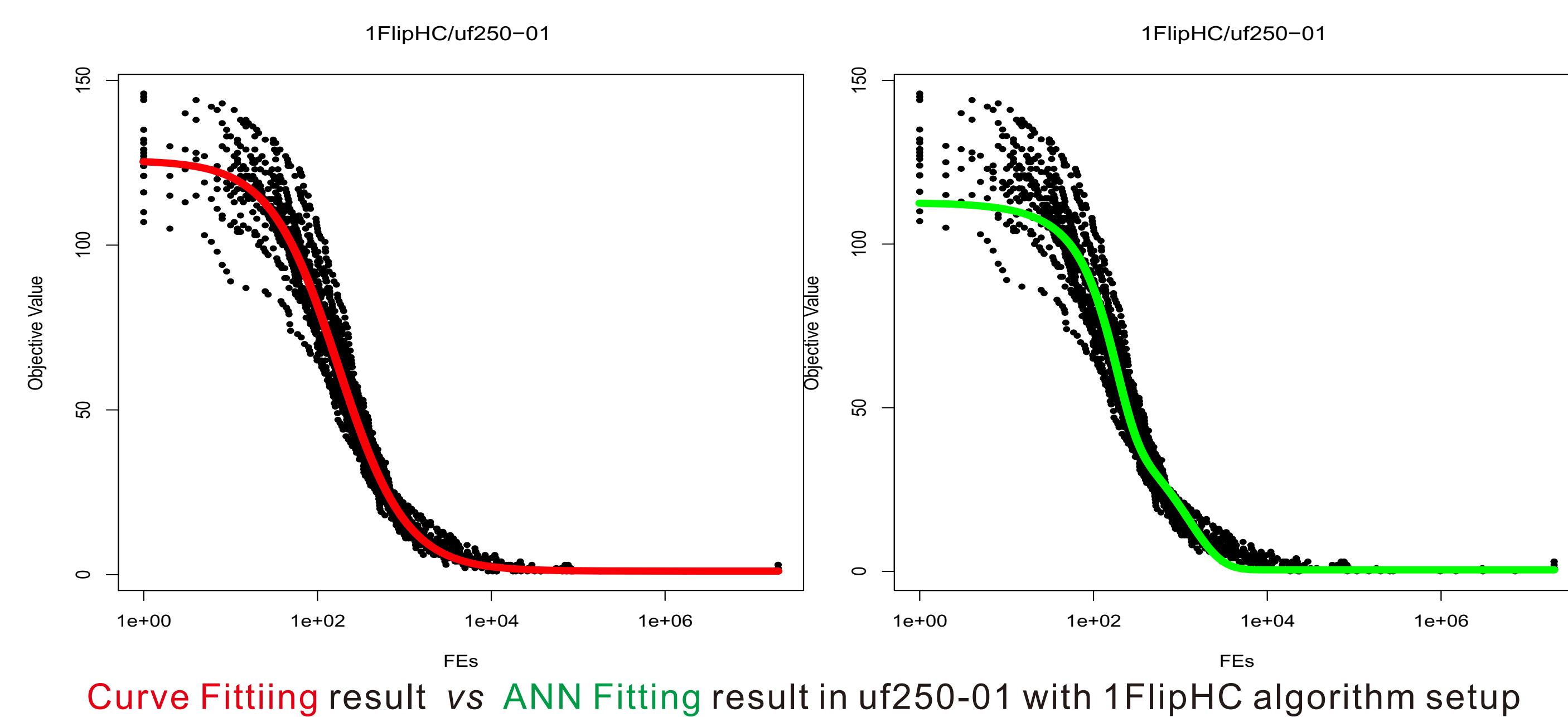
- ~ samples feature: A, B, C, D and Φ
- ~ sample class: 6 in total, each algorithm setup as one class.
- ~ classifier: ANN with backpropagation, Support Vector Machine (SVM) with linear kernel, Gradient Boosting Decision Tree (GBDT)

Model Parameter Prediction:

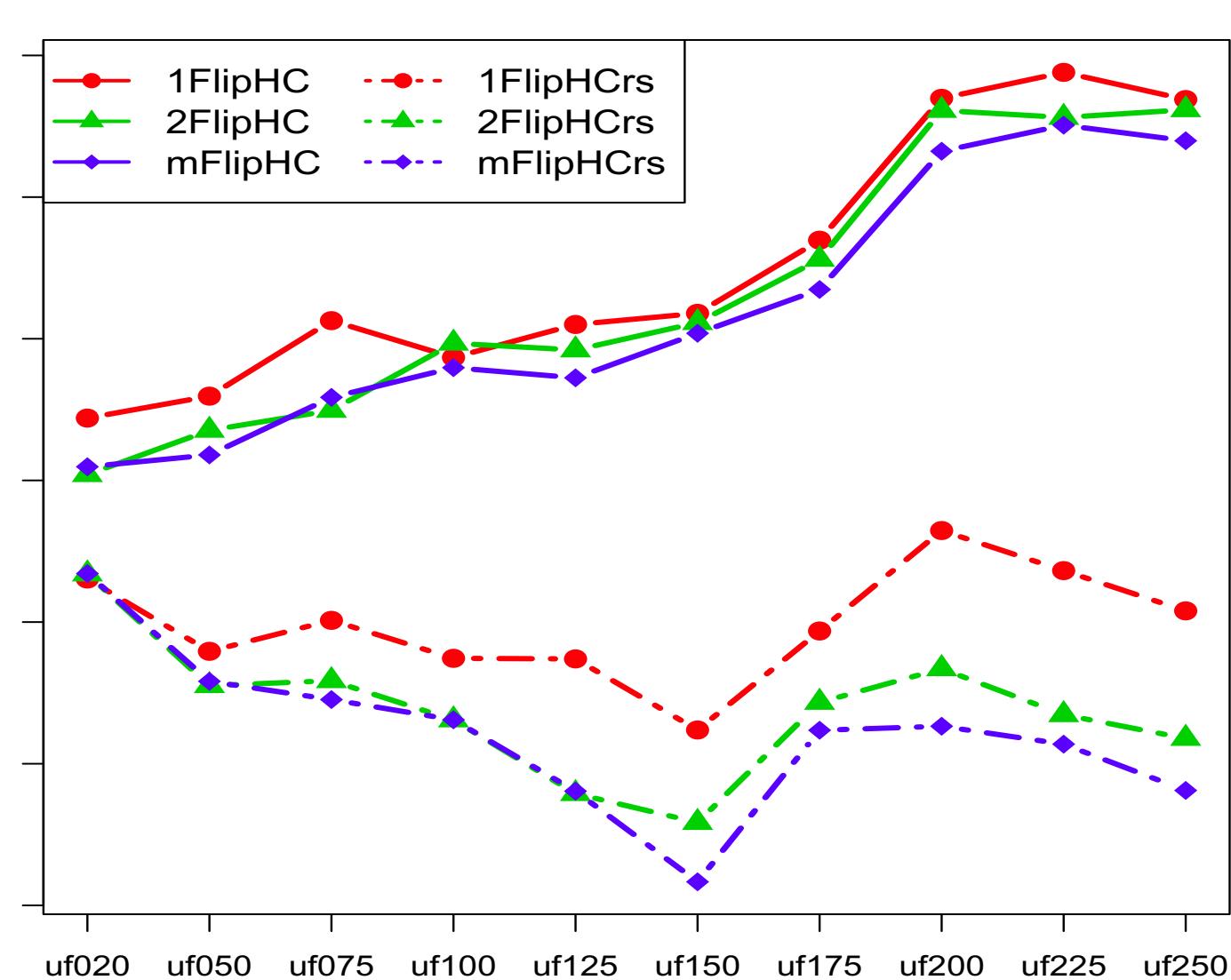
- ~ the model parameters are clearly related to the instance features.
- ~ ANN directly predicts model parameters based on instance features.
- ~ first omit two instance scale parameters results and use ANN predict it and than compared them with the omitted true parameters by calculating the Φ
- ~ ANN with linear activation function and grid search for weight decay
- ~ Root Mean Square Error (RMSE) for choosing optimal model

Predictor of Future Progress:

- ~ representing algorithm behavior as function can compute, for any point in time, which solution quality the algorithm likely has obtained
- ~ used for prediction future progress of algorithms in the running optimization process.
- ~ two methods, ANN and Curve Fitting
- ~ $(\text{train}_t, \text{test}_t) = (50, 100)$ stands for predicting the complete algorithm behavior during the first 50 in the future based on the data points collected



Curve Fitting result vs ANN Fitting result in uf250-01 with 1FlipHC algorithm setup



Trends for the mean values of the four parameters of the fitted LGMP models over the number n of variables of the problem instances for the six algorithms.



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