Combining Two Local Searches with Crossover: An Efficient Hybrid Algorithm for the Traveling Salesman Problem

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Outline

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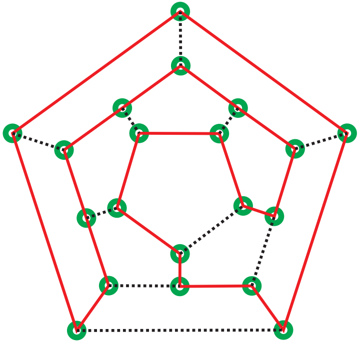
Traveling Salesman Problem

- Traveling Salesman Problem (TSP): Given a collection of n cities and the travel distance between them, solving a TSP means to find the shortest round-trip tour through all cities and back to the starting point.



Traveling Salesman Problem

- Given: A cost matrix $D = (D_{i,j})$, where $D_{i,j}$ is the cost of traveling from city i to j.
  Target: Find a permutation t of the integers from 1 to *n* minimizing the sum $D_{t[1],t[2]} + D_{t[2],t[3]} + \cdots + D_{t[n],t[1]}$.
- In this paper, we focus on symmetric TSPs, where $D_{i,j} = D_{j,i}$ holds.
- Prominent $\mathcal{NP}$-hard problem in Combinatorial Optimization.

Experimentation Environment

- *TSP Suite*: A holistic benchmark environment for algorithms solving the TSP written in Java. It offers integrated support for implementing, testing, benchmarking and comparing algorithms.
- Benchmark: *TSPLIB* contains 110 symmetric TSP instances whose city scale is range from 14 to 85900.

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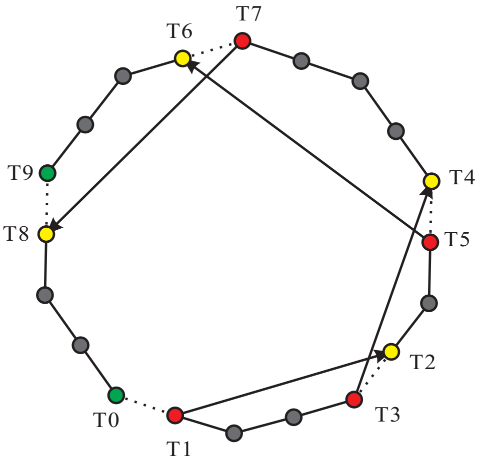
Lin-Kernighan Algorithm

The LK10 is an improved LK heuristic algorithm introduced in [5].



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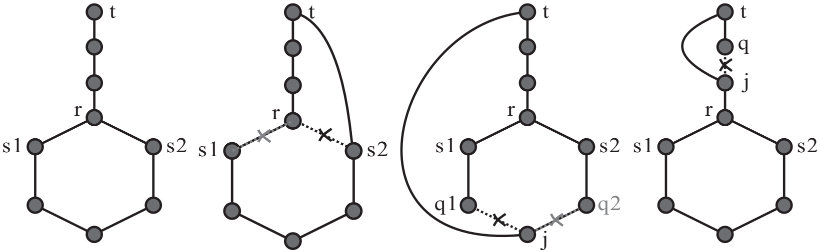
Ejection Chain Method

FSM** is an improved Ejection Chain Method [1, 3].

- It iteratively improves a stem-and-cycle reference structure ($S\&C$) by applying two rules.



(a) $S\&C$ (b) Trial (c) Rule 1 (d) Rule 2

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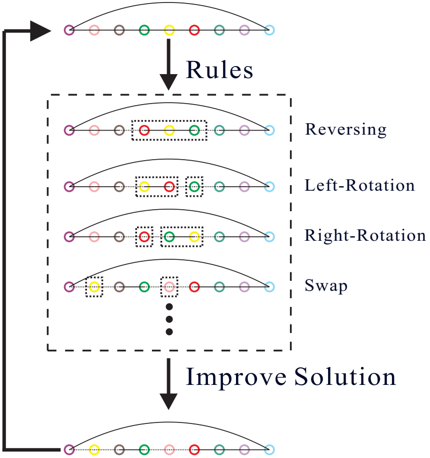
Multi-Neighborhood Search

Multi-Neighborhood Search (MNS) is an efficient local search algorithm introduced in [4].



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Heuristic Crossover Operator

1. It first selects a random city as the current (starting) city of the offspring tour.
2. Second, it considers the four (directed) edges incident to the current city. Over these edges, a probability distribution is defined based on their cost. The probability associated with an edge incident to a previously visited city is equal to zero.
3. An edge is selected based on this distribution. If none of the parental edges leads to an unvisited city, a random edge is selected.
4. The step 2 and 3 are repeated until a complete tour has been constructed.

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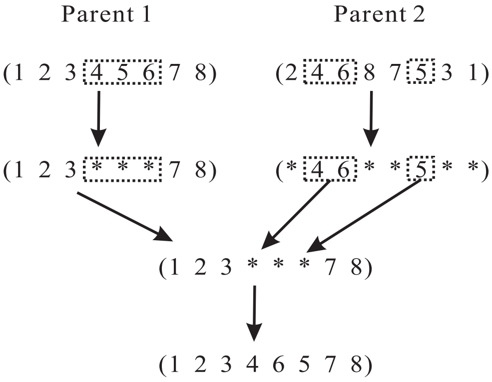
Order Based Crossover Operator

The Order Based Crossover Operator (OX2) selects (at random) several positions in a parent tour and the order of the cities in the selected positions of this parent is imposed on the other parent.



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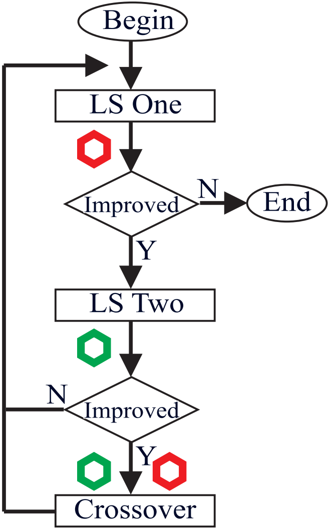
Motivation

- Different local search algorithms have different features, use different data structures, and different search moves neighborhoods. Combining two different local searches means combining their different strengths.
- Crossover helps the search to escape from local optima, while retaining good building blocks.

LS-LS-X Hybrids

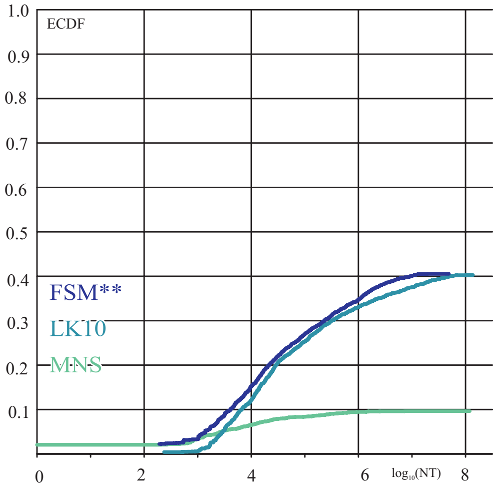


Performance Measure and Time

- Normalized Runtime (*NT*): measured runtime normalized with machine- and problem-depending performance factor.
- Empirical cumulative distribution function (ECDF) returns the fraction of runs that have reached a given goal error F_t (normally, $F_t = 0$) for a time measure such as *NT* or *FE*. It is plotted over the runtime. The earlier and the higher the ECDF rises, the better is the algorithm.

LS algorithms Performance

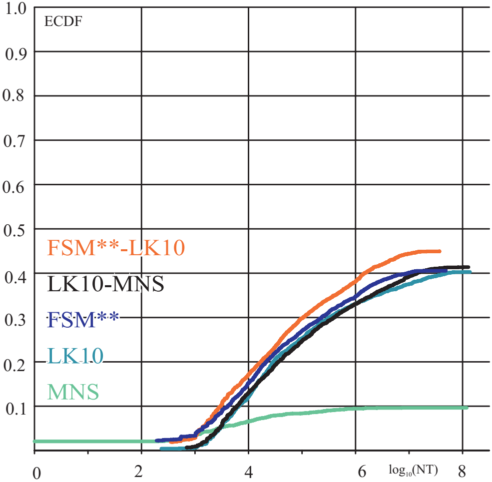
FSM**, LK10 and MNS algorithms



(e) ECDF for NT and F_t=0

LS-LS Hybrids Performance

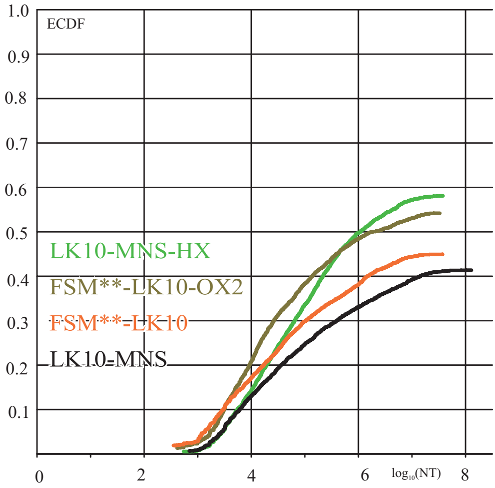
LS-LS hybrids：FSM**-LK10，LK10-MNS



(f) ECDF for NT and F_t=0

LS-LS-X Hybrids Performance

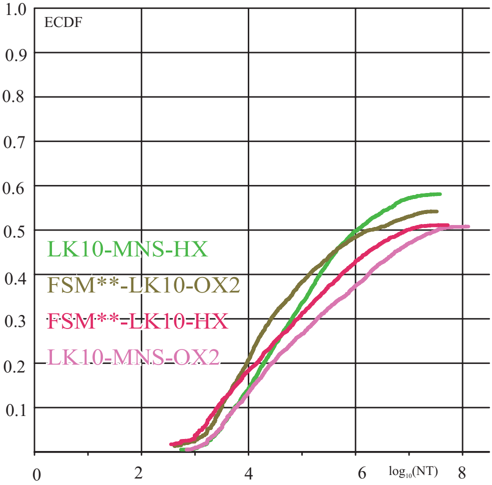
LS-LS-X hybrids：LK10-MNS-HX，FSM**-LK10-OX2



(g) ECDF for NT and F_t=0

LS-LS-X Hybrids Performance

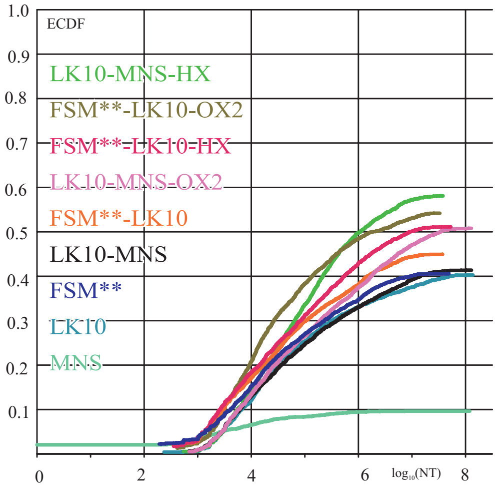
Different LS-LS has different suitable Crossover Operator



(h) ECDF for NT and F_t=0

LS-LS-X Hybrids

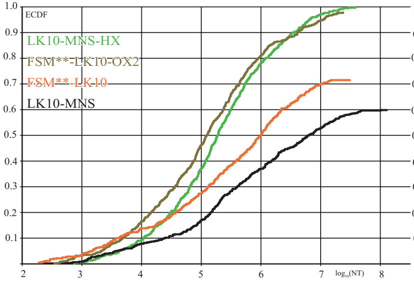
All tested LS-LS-X hybrids



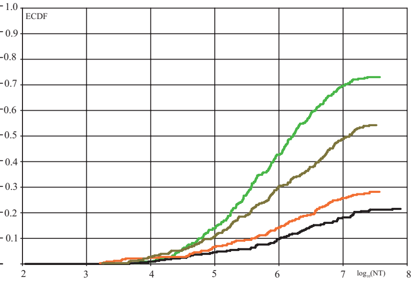
(i) ECDF for NT and F_t=0

LS-LS-X Hybrids

City Scale from 128 to 255 and 256 to 511:



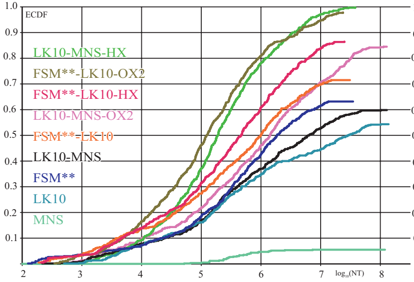
(j) F_t=0 for $128 \leq n \leq 255$



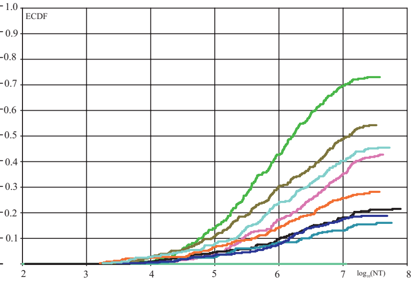
(k) F_t=0 for $256 \leq n \leq 511$

LS-LS-X Hybrids

City Scale from 128 to 255 and 256 to 511:



(l) F_t=0 for $128 \leq n \leq 255$



(m) F_t=0 for $256 \leq n \leq 511$

LS-LS-X Hybrids

LS-LS-X Experiment Result：

LK10-MNS-HX *(rank 1)*, LK10-MNS-MPX *(2)*, FSM**-LK10-OX2 *(3)*, FSM**-LK10-CX *(4.5)*, LK10-MNS-OX2 *(4.5)*, FSM**-LK10-HX *(6)*, FSM**-LK10-MPX *(7)*, FSM**-LK10 *(8.5)*, LK10-MNS *(8.5)*, LK10-MNS-CX *(10)*.

图: LS-LS-X hybrid algorithms ranking from best to worst. The different algorithm types LS-LS hybrid and LS-LS-X hybrid are highlighted.

Outline

1 Research Content

2 Local Search Algorithm

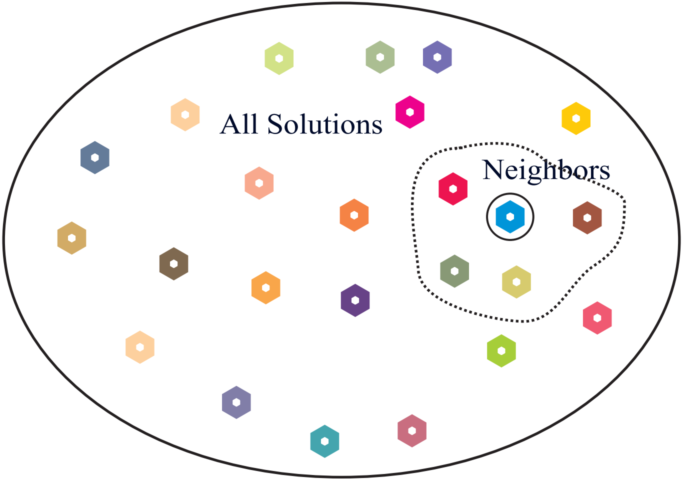
3 Crossover Operator

4 Hybrid Algorithms for the TSP
- LS-LS-X Hybrids
- Hybrid Global Search with Local Search

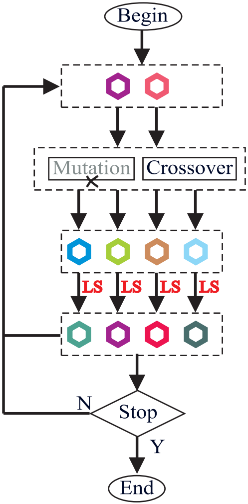
5 Conclusion

Local Search (LS)

- Feature： Fast convergence, but may get trapped by local optima

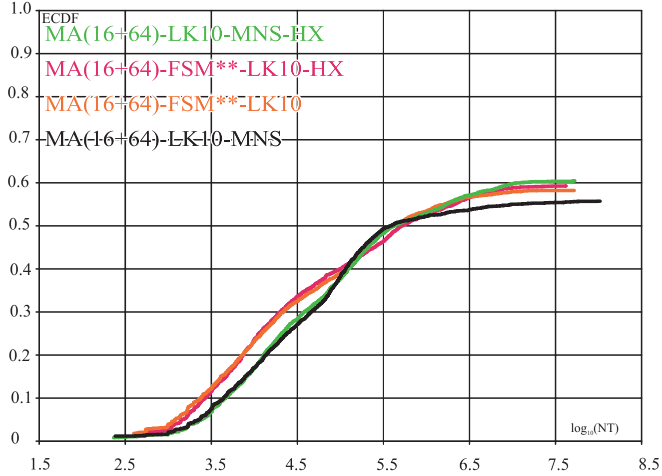


Hybrid Evolutionary Algorithm with LS



Hybrid Evolutionary Algorithm with LS

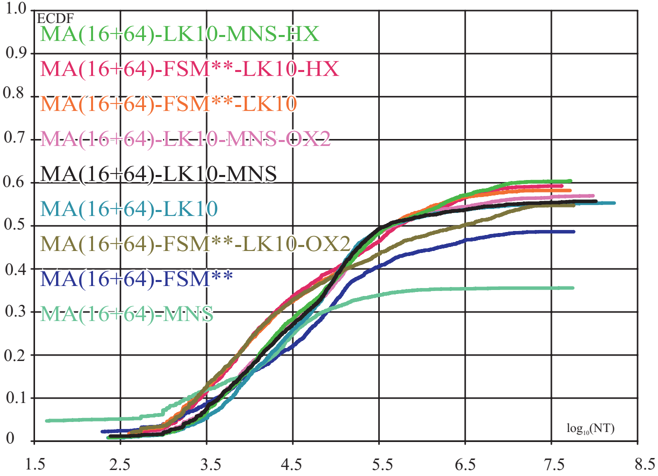
EA-LS-LS-X experiment result:



(a) ECDF for NT and F_t=0

Hybrid Evolutionary Algorithm with LS

EA-LS-LS-X experiment result:



(b) ECDF for NT and F_t=0

Hybrid Evolutionary Algorithm with LS

EA-LS-LS-X experiment result:

MA(2+8)-LK10-MNS-HX *(rank 1)*, MA(2+4)-LK10-MNS-HX *(2)*,
MA(16+64)-LK10-MNS-HX *(3)*, MA(16+64)-LK10-MNS *(4)*,
MA(16+64)-LK10-MNS-OX2 *(5)*, MA(16+64)-FSM**-LK10-HX *(6)*,
MA(16+64)-FSM**-LK10 *(7)*, MA(16+64)-LK10 *(8)*,
MA(2+8)-FSM**-LK10-OX2 *(9)*, MA(2+4)-FSM**-LK10-OX2 *(10)*,
MA(2+8)-FSM**-LK10-HX *(11)*, MA(2+4)-LK10-MNS-OX2 *(12)*,
MA(2+8)-LK10-MNS-OX2 *(13)*, MA(16+64)-FSM**-LK10-OX2 *(14)*,
MA(2+8)-LK10-MNS *(15)*, MA(2+4)-FSM**-LK10-HX *(16)*,
MA(2+4)-LK10-MNS *(17)*, MA(2+8)-FSM**-LK10 *(18)*,
MA(16+64)-FSM** *(19)*, MA(2+4)-LK10 *(20)*, MA(2+8)-LK10 *(21)*,
MA(2+4)-FSM**-LK10 *(22)*, MA(16+64)-MNS *(23)*, MA(2+8)-FSM** *(24)*,
MA(2+4)-FSM** *(25)*, MA(2+8)-MNS *(26)*, MA(2+4)-MNS *(27)*.

图: EA hybrid algorithms ranking from best to worst.

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Conclusion

- The new LS-LS-X hybrids are better than their pure LS algorithm and LS-LS hybrid components.
- The new EC-LS-LS-X hybrids outperform the LS-LS-X algorithms as well as EC-LS and EC-LS-LS hybrids. MA(2+4)-LK10-MNS-HX becomes the new most powerful hybrid EA algorithm in the huge collection of algorithms and experimental results of the popular *TSP Suite*.
- Different LS-LS hybrids have different suitable crossover operators.

References

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Thanks!

Appendix

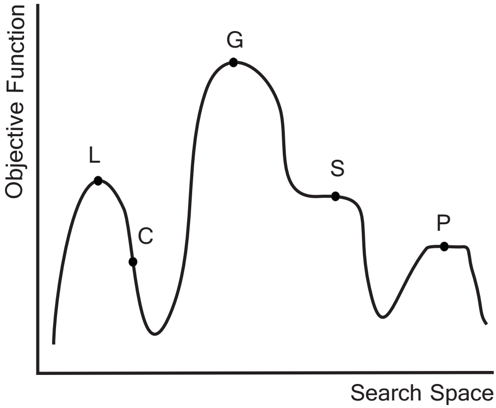
TSPLIB contains 110 symmetric TSP instances whose city scale is range from 14 to 85900.

Instance Scale	Number of instances
$0 \sim 2^4 - 1$	1
$2^4 \sim 2^5 - 1$	8
$2^5 \sim 2^6 - 1$	8
$2^6 \sim 2^7 - 1$	17
$2^7 \sim 2^8 - 1$	20
$2^8 \sim 2^9 - 1$	11
$2^9 \sim 2^{10} - 1$	13
$2^{10} \sim 2^{11} - 1$	15
$2^{11} \sim 2^{12} - 1$	6
$2^{12} \sim 2^{13} - 1$	4
$2^{13} \sim 2^{14} - 1$	4
$2^{14} \sim 2^{15} - 1$	1
$2^{15} \sim 2^{16} - 1$	1
$2^{16} \sim 2^{17} - 1$	1

表: Distribution of Symmetric Instances in *TSPLIB*

Global Search VS Local Search

Global Search VS Local Search:



(a) Global Search VS Local Search