Metaheuristic Optimization

19. Estimation of Distribution Algorithms

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Outline

1. Introduction
2. Univariate EDAs
3. Multivariate EDAs
4. Tree-based EDA
5. Summary

website

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- Estimation of Distribution Algorithms (EDAs) are similar, but also different:
  - EAs try to evolve a solution
  - EDAs build a model of what a perfect solution should look like

Idea

- Information in a population can be represented by statistical model

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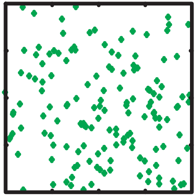

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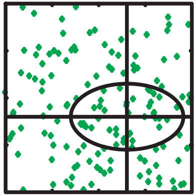

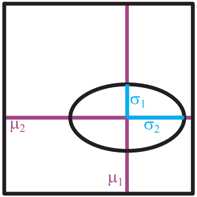

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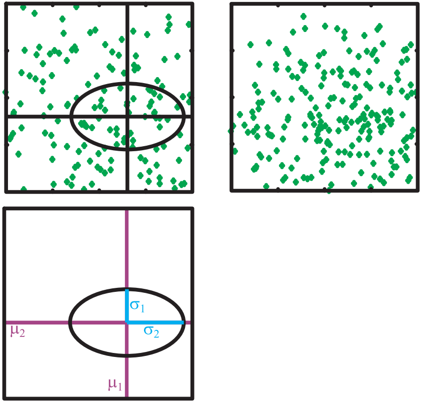

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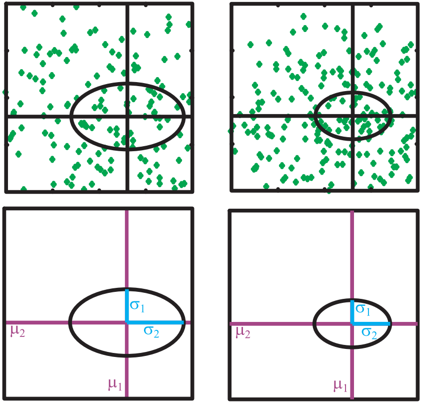

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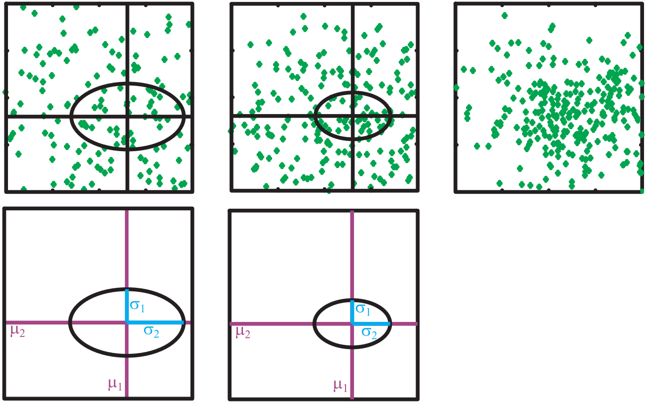

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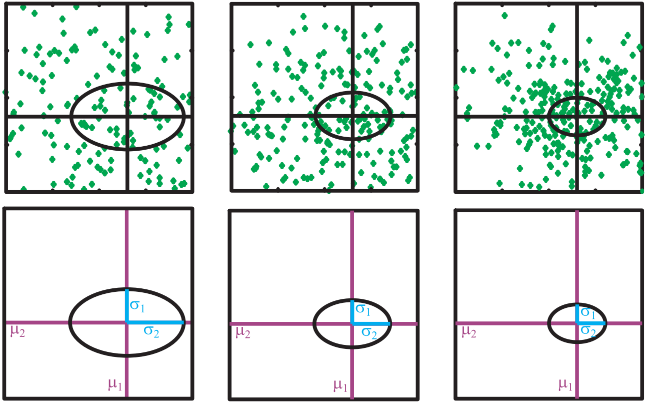

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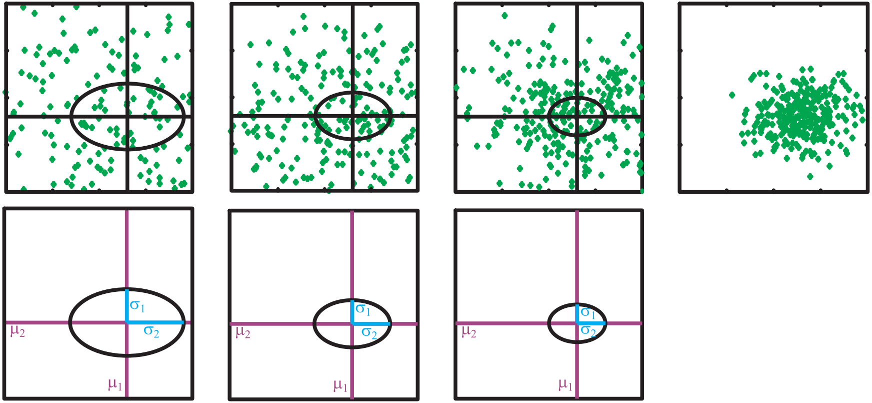

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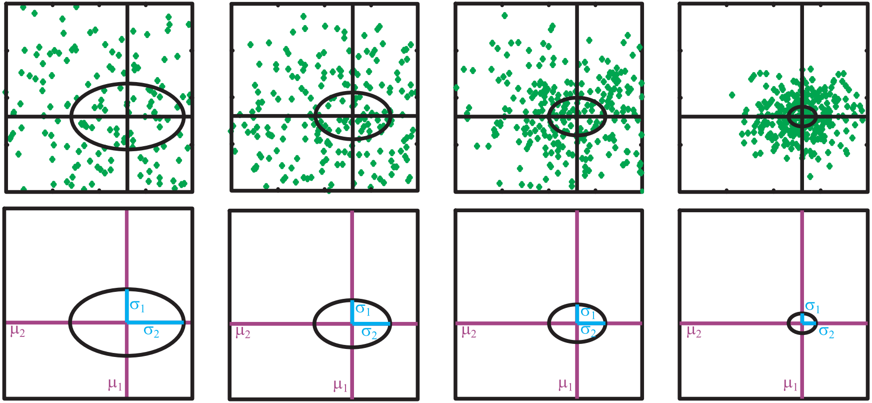

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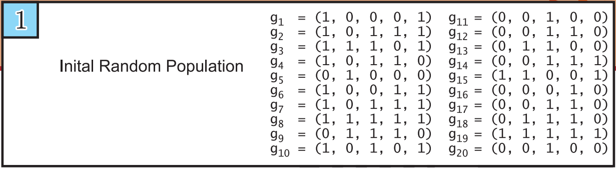
- $M[i] =$ estimated proability that the bit at locus i of a globally optimal genotype $g^{\star}$ should be 0
- Initialization: $M[i] = 0.5 \forall i \in 1 \ldots n$

1. Create a statistical model of possible solutions (UMDA: $(0.5, \cdots, 0.5)$)



Inital Random Population

$g_1 = (1, 0, 0, 0, 1)$ $g_{11} = (0, 0, 1, 0, 0)$
$g_2 = (1, 0, 1, 1, 1)$ $g_{12} = (0, 0, 1, 1, 0)$
$g_3 = (1, 1, 1, 0, 1)$ $g_{13} = (0, 1, 1, 0, 0)$
$g_4 = (1, 0, 1, 1, 0)$ $g_{14} = (0, 0, 1, 1, 1)$
$g_5 = (0, 1, 0, 0, 0)$ $g_{15} = (1, 1, 0, 0, 1)$
$g_6 = (1, 0, 0, 1, 1)$ $g_{16} = (0, 0, 0, 1, 0)$
$g_7 = (1, 0, 1, 1, 1)$ $g_{17} = (0, 0, 1, 1, 0)$
$g_8 = (1, 1, 1, 1, 1)$ $g_{18} = (0, 1, 1, 1, 0)$
$g_9 = (0, 1, 1, 1, 0)$ $g_{19} = (1, 1, 1, 1, 1)$
$g_{10} = (1, 0, 1, 0, 1)$ $g_{20} = (0, 0, 1, 0, 0)$

1. Create a statistical model of possible solutions (UMDA: $(0.5, \cdots, 0.5)$)
2. ps times do:
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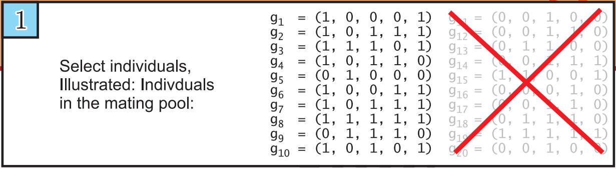

1

Select individuals, Illustrated: Indivduals in the mating pool:

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$g_3 = (1, 1, 1, 0, 1)$
$g_4 = (1, 0, 1, 1, 0)$
$g_5 = (0, 1, 0, 0, 0)$
$g_6 = (1, 0, 0, 1, 1)$
$g_7 = (1, 0, 1, 1, 1)$
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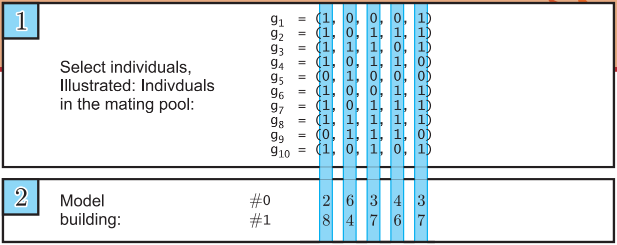

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2 Model building:

#0	2	6	3	4	3
#1	8	4	7	6	7

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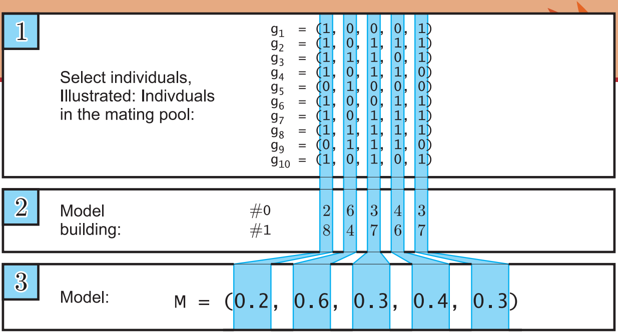

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`M = (0.2, 0.6, 0.3, 0.4, 0.3)`

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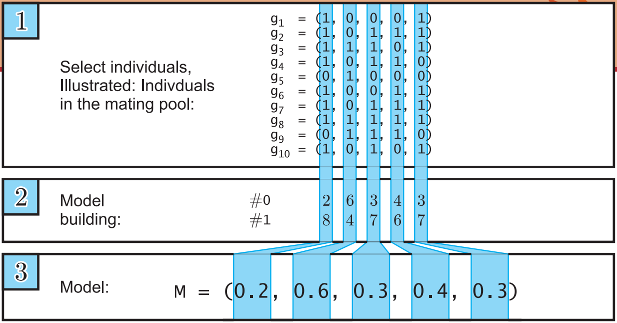

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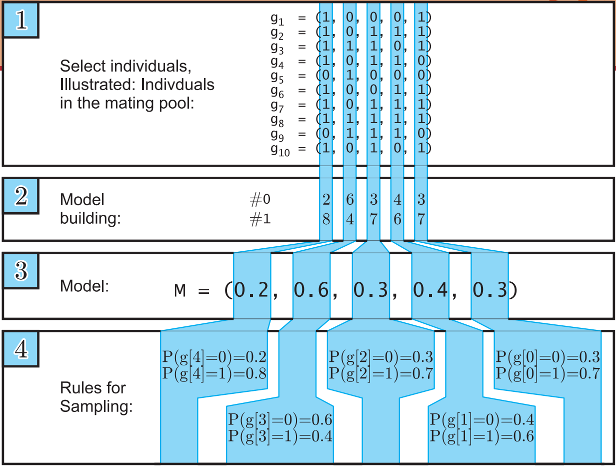

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P(g[4]=0)=0.2
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P(g[3]=0)=0.6
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P(g[2]=0)=0.3
P(g[2]=1)=0.7

P(g[1]=0)=0.4
P(g[1]=1)=0.6

P(g[0]=0)=0.3
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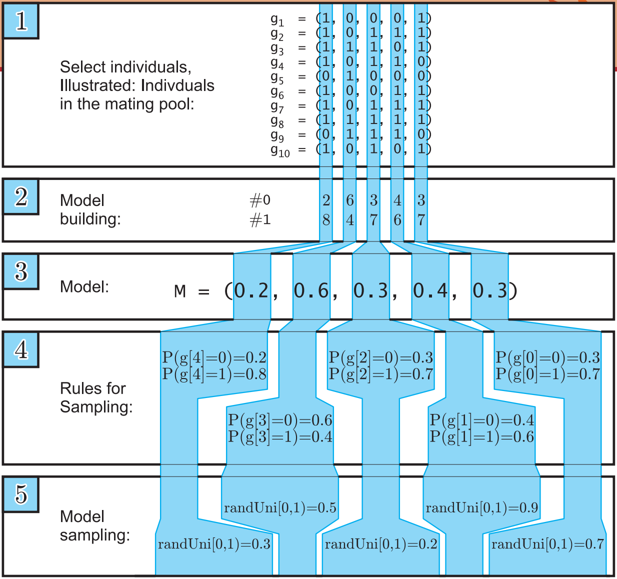

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randUni[0,1)=0.3
randUni[0,1)=0.5
randUni[0,1)=0.2
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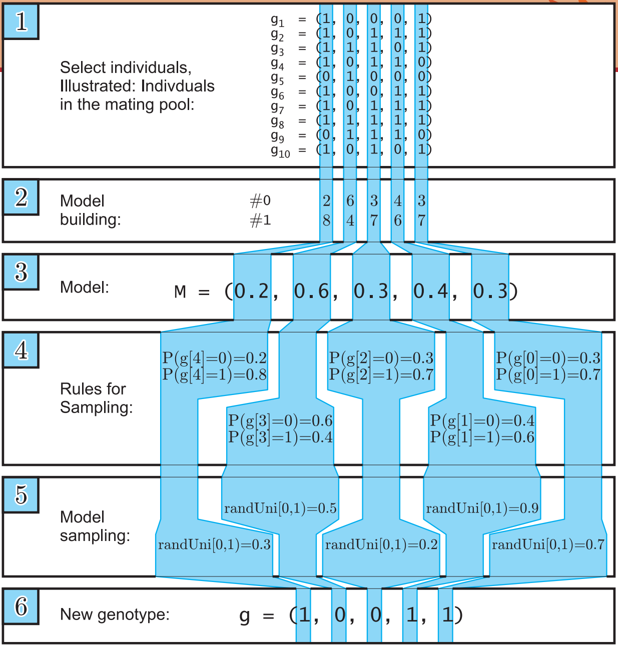

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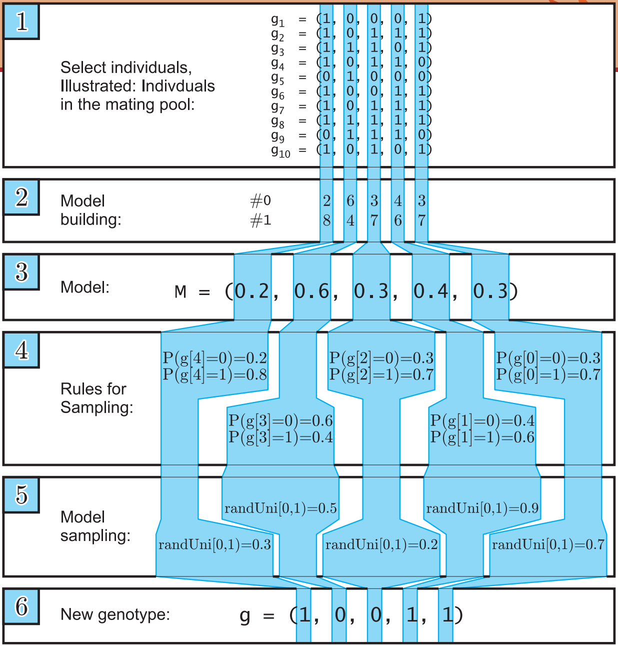

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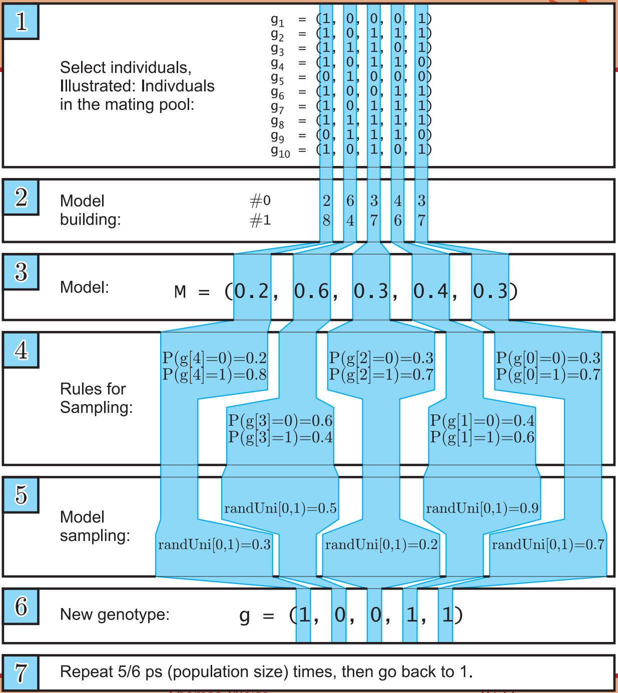

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2 Model building:

#0	2	6	3	4	3
#1	8	4	7	6	7

3 Model: `M = (0.2, 0.6, 0.3, 0.4, 0.3)`

4 Rules for Sampling:

P(g[4]=0)=0.2
P(g[4]=1)=0.8

P(g[2]=0)=0.3
P(g[2]=1)=0.7

P(g[0]=0)=0.3
P(g[0]=1)=0.7

P(g[3]=0)=0.6
P(g[3]=1)=0.4

P(g[1]=0)=0.4
P(g[1]=1)=0.6

5 Model sampling:

randUni[0,1)=0.5
randUni[0,1)=0.9
randUni[0,1)=0.3
randUni[0,1)=0.2
randUni[0,1)=0.7

6 New genotype: `g = (1, 0, 0, 1, 1)`

7 Repeat 5/6 ps (population size) times, then go back to 1.

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- Learning rate λ determines influence of the old model M' and temporary model M_T on new model: $M = (1-\lambda)M' + \lambda M_T$

cGA

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Compact: Ideal for implementation in hardware or on small/weak devices

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$M \longleftarrow \text{buildModelCompGA}(g_1, g_2, M', ps)$

```
In: $M$' – the old model;   Out: $M$ – the new model
begin
    $M \longleftarrow M'$
    if $g_2$ is better than $g_1$ then
      exchange $g_1$ and $g_2$     // $g_1$ is now always better than $g_2$

    for $i \longleftarrow 1$ up to $n$ do
        if $g_1[i] \neq g_2[i]$ then
            if $g_1[i] = 0$ then $M[i] \longleftarrow M'[i] + \frac{1}{ps}$

            else $M[i] \longleftarrow M'[i] - \frac{1}{ps}$
```

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- Convergence: Achieved when $M[i] \in \{0, 1\} \forall i \in 1 \ldots n$

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  7. $M.\vec{\sigma} = \sigma_{\text{red}} * M'.\vec{\sigma}$ (where $\sigma_{\text{red}} \in [0, 1)$ reduces the standard deviation step by step)

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2. Univariate EDAs
3. Multivariate EDAs
4. Tree-based EDA
5. Summary

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- Multi-variate EDAs are more complicated but can deal with epistasis while univariate ones will produce bad results in these cases

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- Probabilities updated in a rather complex way towards the best tree found

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- Easy to implement for both bit strings and real vectors
- Univariate EDAs: simple
- Multivariate EDAs: more complicated, but can work better if epistasis is present
- Anything which can be evolved with an EA can be evolved with an EDA (and vice versa)

谢谢

Thank you

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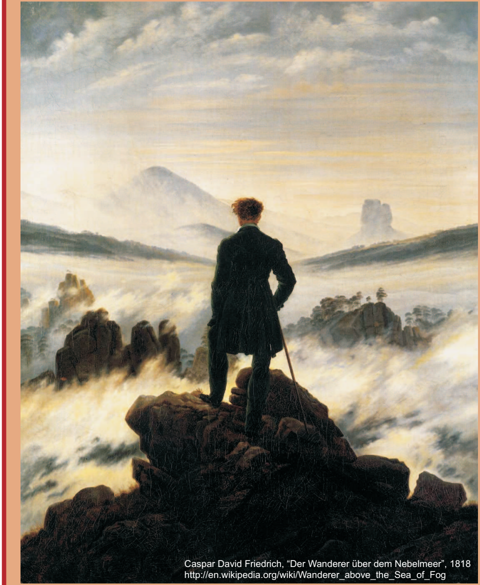

Caspar David Friedrich, “Der Wanderer über dem Nebelmeer”, 1818
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