Metaheuristic Optimization

6. Random Walk

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1. Random Walks

website

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- Based on this idea, the hill climber generates modified copies of the current solution and accepts them if they are better than the old solution.
- What would happen if we would always accept them?

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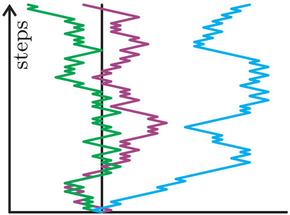

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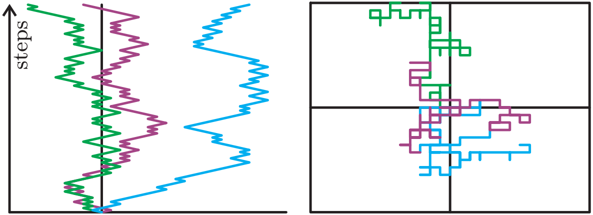

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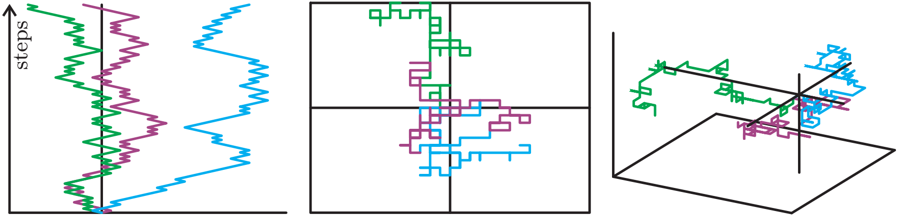

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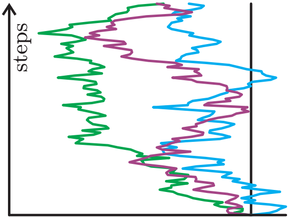

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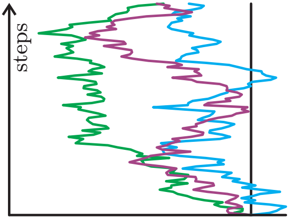

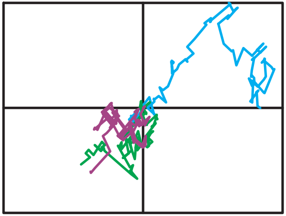

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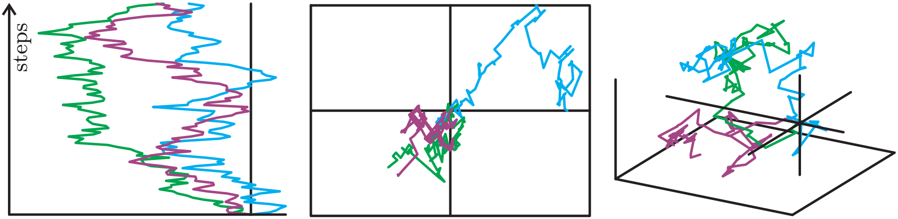

$p_{best} \longleftarrow \text{randomWalk}(f)$

Input: f: the objective function subject to minization
Input: [implicit] shouldTerminate: the termination criterion
Data: p_{new}: the new solution to be tested
Output: p_{best}: the best individual ever discovered

begin
- $p_{best}.g \longleftarrow \text{create}()$
- $p_{best}.x \longleftarrow \text{gpm}(p_{best}.g)$
- $p_{best}.y \longleftarrow f(p_{best}.x)$
- $p_{new} \longleftarrow p_{best}$
- **while** $\neg \mathit{shouldTerminate}$ **do**
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4. go back to 2, until termination criterion is met

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  2. combinatorial optimization (e.g., for TSP over permutations).

Implementing the Random Walk

Listing: The Random Walk Algorithm

```
public class RandomWalk<G, X> extends OptimizationAlgorithm<G, X> {
  public Individual<G, X> solve(final IObjectiveFunction<X> f) {
    Individual<G, X> pstar, pnew;

    pstar = new Individual<>();
    pnew = new Individual<>();

    pstar.g = this.nullary.create(this.random);
    pstar.x = this.gpm.gpm(pstar.g);
    pstar.v = f.compute(pstar.x);
    pnew.assign(pstar);

    while (!(this.termination.shouldTerminate())) {
      pnew.g = this.unary.mutate(pnew.g, this.random);
      pnew.x = this.gpm.gpm(pnew.g);
      pnew.v = f.compute(pnew.x);

      if (pnew.v <= pstar.v) {
        pstar.assign(pnew);
      }
    }

    return pstar;
  }
}
```

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- When the information we get, e.g., from objective function evaluation, is used efficiently
- Comparison with algorithms that do not use this information!

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- Hence, their performance is much worse, similar to random sampling.
- This means that the idea of expection some sort of "gradient" in the search space towards better solutions is reasonable.

谢谢
Thank you

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Caspar David Friedrich, “Der Wanderer über dem Nebelmeer”, 1818
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