Evolving Exact Integer Algorithms with Genetic Programming

Thomas Weise, Mingxu Wan, Ke Tang, and Xin Yao

Nature Inspired Computation and Applications Laboratory
University of Science and Technology of China (USTC)
Hefei 230027, Anhui, China
tweise@ustc.edu.cn · http://www.it-weise.de/

IEEE Congress on Evolutionary Computation (IEEE CEC'14) @
2014 IEEE World Congress on Computational Intelligence (WCCI'14)
2014-07-10 ThE1-3 13:50 at Beijing, China,
Beijing International Convention Center, Room 203C

Contents

This is the presentation for conference paper [?]. You can find the citation information and reference at the end of these slides, including a download link for the paper.

Introduction

- Evolve algorithms with Genetic Programming.

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula
- Exact integer computations

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula
- Exact integer computations: Result is either right or wrong

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula
- Exact integer computations: Result is either right or wrong
- Hard for GP

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula
- Exact integer computations: Result is either right or wrong
- Hard for GP
  - rugged fitness landscape

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula
- Exact integer computations: Result is either right or wrong
- Hard for GP
  - rugged fitness landscape
  - epistasis, deceptiveness

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula
- Exact integer computations: Result is either right or wrong
- Hard for GP
  - rugged fitness landscape
  - epistasis, deceptiveness
- Questions

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula
- Exact integer computations: Result is either right or wrong
- Hard for GP
  - rugged fitness landscape
  - epistasis, deceptiveness
- Questions:
  - Can this actually be done with GP?

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula
- Exact integer computations: Result is either right or wrong
- Hard for GP
  - rugged fitness landscape
  - epistasis, deceptiveness
- Questions:
  - Can this actually be done with GP for non-trivial problems?

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula
- Exact integer computations: Result is either right or wrong
- Hard for GP
  - rugged fitness landscape
  - epistasis, deceptiveness
- Questions:
  - Can this actually be done with GP for non-trivial problems?
  - How can we improve the chance to success?

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula
- Exact integer computations: Result is either right or wrong
- Hard for GP
  - rugged fitness landscape
  - epistasis, deceptiveness
- Questions:
  - Can this actually be done with GP for non-trivial problems?
  - How can we improve the chance to success?
- Contributions

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions **+** control flow **+** memory $\neq$ Formula
- Exact integer computations: Result is either right or wrong
- Hard for GP
  - rugged fitness landscape
  - epistasis, deceptiveness
- Questions:
  - Can this actually be done with GP for non-trivial problems?
  - How can we improve the chance to success?
- Contributions:
  - Benchmark cases

Introduction

- Evolve algorithms with Genetic Programming.
- Algorithms $\equiv$ instructions + control flow + memory $\neq$ Formula
- Exact integer computations: Result is either right or wrong
- Hard for GP
  - rugged fitness landscape
  - epistasis, deceptiveness
- Questions:
  - Can this actually be done with GP for non-trivial problems?
  - How can we improve the chance to success?
- Contributions:
  - Benchmark cases
  - 2 technologies, 1 works, 1 doesn't

Benchmark Problems

Benchmark Problems

- A benchmark problem in GP is defined by

Benchmark Problems

- A benchmark problem in GP is defined by:
  1. the instruction set,

Benchmark Problems

- A benchmark problem in GP is defined by:
  1. the instruction set,

Benchmark Problems

- A benchmark problem in GP is defined by:
  1. the instruction set,
  2. the objective function f,

Benchmark Problems

- A benchmark problem in GP is defined by:
  1. the instruction set,
  2. the objective function f,
  3. the algorithm φ that we want to evolve,

Benchmark Problems

- A benchmark problem in GP is defined by:
  1. the instruction set,
  2. the objective function f,
  3. the algorithm φ that we want to evolve (and tc corresponding training cases t_i),

Benchmark Problems

Instruction Set:

Benchmark Problems

Instruction Set: **+**

Benchmark Problems

Instruction Set: **+**, **-**

Benchmark Problems

Instruction Set: **+, -, ***

Benchmark Problems

Instruction Set: **+, -, *, /**

Benchmark Problems

Instruction Set: **+, -, *, /, %**

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{E} \in \{0, 1\}$

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{E} \in \{0, 1\}$, m_1 m_2 m_3 m_4 m_5

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{E} \in \{0, 1\}$, m_1 m_2 m_3 m_4 m_5, **=**

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{C} \in \{0, 1\}$, m_1 m_2 m_3 m_4 m_5, **=**, **;**

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{C} \in \{0, 1\}$, m_1 m_2 m_3 m_4 m_5, **=**, **;**, **loop**

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{C} \in \{0, 1\}$, m_1 m_2 m_3 m_4 m_5, **=**, **;**, **loop**

Objective Function: $f(x)$ = Error Rate

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{E} \in \{0,1\}$, m_1 m_2 m_3 m_4 m_5, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{}$

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{E} \in \{0,1\}$, m_1 m_2 m_3 m_4 m_5, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc \text{ of training cases}}$

Benchmark Problems

Instruction Set: **+**, **-**, *****, ~~**/**~~, **%**, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 ~~m_3~~ ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. po2 [1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$

Benchmark Problems

Instruction Set: **+**, **-**, *****, ~~**/**~~, **%**, $\mathfrak{E} \in \{0,1\}$, m_1 m_2 ~~m_3~~ ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. po2 [1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. su2 [1]: sum of first t_i natural numbers

Benchmark Problems

Instruction Set: **+**, **-**, *****, ~~**/**~~, **%**, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 ~~m_3~~ ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}12 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$

Benchmark Problems

Instruction Set: **+**, **-**, *****, ~~**/**~~, **%**, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells

Benchmark Problems

Instruction Set: **+**, **-**, *****, ~~**/**~~, **%**, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}31 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{E} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}40 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{E} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0
10. `ssq`: $\varphi_{10}(t_i) = \sum_{i=1}^{t_i} i^2$

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0
10. `ssq`: $\varphi_{10}(t_i) = \sum_{i=1}^{t_i} i^2$
11. `sra`: $\varphi_{11}(t_i) = \sqrt{t_i}$

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{E} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0
10. `ssq`: $\varphi_{10}(t_i) = \sum_{i=1}^{t_i} i^2$
11. `sra`: $\varphi_{11}(t_i) = \sqrt{t_i}$
12. `srb`: $\varphi_{12}(t_i) = \lfloor \sqrt{t_i} \rfloor$

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0
10. `ssq`: $\varphi_{10}(t_i) = \sum_{i=1}^{t_i} i^2$
11. `sra`: $\varphi_{11}(t_i) = \sqrt{t_i}$
12. `srb`: $\varphi_{12}(t_i) = \lfloor \sqrt{t_i} \rfloor$
13. `ild`: $\mathrm{ld}^\star(y) = \begin{cases} 0 & \text{if } t_i \leq 1 \\ 1 + \mathrm{ld}^\star(\mathrm{ld}(y)) & \text{otherwise} \end{cases}$

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 m_3 $\cancel{\mathsf{m}_4}$ $\cancel{\mathsf{m}_5}$, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0
10. `ssq`: $\varphi_{10}(t_i) = \sum_{i=1}^{t_i} i^2$
11. `sra`: $\varphi_{11}(t_i) = \sqrt{t_i}$
12. `srb`: $\varphi_{12}(t_i) = \lfloor \sqrt{t_i} \rfloor$
13. `ild`: $\mathrm{ld}^\star(y) = \begin{cases} 0 & \text{if } t_i \leq 1 \\ 1 + \mathrm{ld}^\star(\mathrm{ld}(y)) & \text{otherwise} \end{cases}$
14. `lsb`: least significant bit in the two's complement

Benchmark Problems

Instruction Set: **+**, **-**, ~~*~~, ~~/~~, ~~%~~, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0
10. `ssq`: $\varphi_{10}(t_i) = \sum_{i=1}^{t_i} i^2$
11. `sra`: $\varphi_{11}(t_i) = \sqrt{t_i}$
12. `srb`: $\varphi_{12}(t_i) = \lfloor \sqrt{t_i} \rfloor$
13. `ild`: $\mathrm{ld}^\star(y) = \begin{cases} 0 & \text{if } t_i \leq 1 \\ 1 + \mathrm{ld}^\star(\mathrm{ld}(y)) & \text{otherwise} \end{cases}$
14. `lsb`: least significant bit in the two's complement
15. `mul`: $\varphi_{15}(t_i) = t_{i,1} * t_{i,2}$

Benchmark Problems

Instruction Set: **+**, **-**, ~~*~~, ~~/~~, ~~%~~, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0
10. `ssq`: $\varphi_{10}(t_i) = \sum_{i=1}^{t_i} i^2$
11. `sra`: $\varphi_{11}(t_i) = \sqrt{t_i}$
12. `srb`: $\varphi_{12}(t_i) = \lfloor \sqrt{t_i} \rfloor$
13. `ild`: $\mathrm{ld}^\star(y) = \begin{cases} 0 & \text{if } t_i \leq 1 \\ 1 + \mathrm{ld}^\star(\mathrm{ld}(y)) & \text{otherwise} \end{cases}$
14. `lsb`: least significant bit in the two's complement
15. `mul`: $\varphi_{15}(t_i) = t_{i,1} * t_{i,2}$
16. `qad`: $\varphi_{16}(t_i) = (t_i - 1)(t_i + 2)$

Benchmark Problems

Instruction Set: **+**, **-**, ~~*~~, ~~/~~, ~~%~~, $\mathfrak{C} \in \{0,1\}$, m_1 m_2 m_3 ~~m_4~~ ~~m_5~~, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0
10. `ssq`: $\varphi_{10}(t_i) = \sum_{i=1}^{t_i} i^2$
11. `sra`: $\varphi_{11}(t_i) = \sqrt{t_i}$
12. `srb`: $\varphi_{12}(t_i) = \lfloor \sqrt{t_i} \rfloor$
13. `ild`: $\mathrm{ld}^\star(y) = \begin{cases} 0 & \text{if } t_i \leq 1 \\ 1 + \mathrm{ld}^\star(\mathrm{ld}(y)) & \text{otherwise} \end{cases}$
14. `lsb`: least significant bit in the two's complement
15. `mul`: $\varphi_{15}(t_i) = t_{i,1} * t_{i,2}$
16. `qad`: $\varphi_{16}(t_i) = (t_i - 1)(t_i + 2)$
17. `mod`: $\varphi_{17}(t_i) = t_{i,1} \bmod t_{i,2}$

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{C} \in \{0,1\}$, $\mathtt{m}_1$ $\mathtt{m}_2$ $\mathtt{m}_3$ $\mathtt{m}_4$ $\mathtt{m}_5$, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0
10. `ssq`: $\varphi_{10}(t_i) = \sum_{i=1}^{t_i} i^2$
11. `sra`: $\varphi_{11}(t_i) = \sqrt{t_i}$
12. `srb`: $\varphi_{12}(t_i) = \lfloor \sqrt{t_i} \rfloor$
13. `ild`: $\mathrm{ld}^\star(y) = \begin{cases} 0 & \text{if } t_i \leq 1 \\ 1 + \mathrm{ld}^\star(\mathrm{ld}(y)) & \text{otherwise} \end{cases}$
14. `lsb`: least significant bit in the two's complement
15. `mul`: $\varphi_{15}(t_i) = t_{i,1} * t_{i,2}$
16. `qad`: $\varphi_{16}(t_i) = (t_i - 1)(t_i + 2)$
17. `mod`: $\varphi_{17}(t_i) = t_{i,1} \bmod t_{i,2}$
18. `mi5`: minimum of five

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{C} \in \{0,1\}$, $\mathtt{m}_1$ $\mathtt{m}_2$ $\mathtt{m}_3$ $\mathtt{m}_4$ $\mathtt{m}_5$, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0
10. `ssq`: $\varphi_{10}(t_i) = \sum_{i=1}^{t_i} i^2$
11. `sra`: $\varphi_{11}(t_i) = \sqrt{t_i}$
12. `srb`: $\varphi_{12}(t_i) = \lfloor \sqrt{t_i} \rfloor$
13. `ild`: $\mathrm{ld}^\star(y) = \begin{cases} 0 & \text{if } t_i \leq 1 \\ 1 + \mathrm{ld}^\star(\mathrm{ld}(y)) & \text{otherwise} \end{cases}$
14. `lsb`: least significant bit in the two's complement
15. `mul`: $\varphi_{15}(t_i) = t_{i,1} * t_{i,2}$
16. `qad`: $\varphi_{16}(t_i) = (t_i - 1)(t_i + 2)$
17. `mod`: $\varphi_{17}(t_i) = t_{i,1} \bmod t_{i,2}$
18. `mi5`: minimum of five
19. `ma5`: maximum of five

Benchmark Problems

Instruction Set: **+**, **-**, *****, **/**, **%**, $\mathfrak{C} \in \{0,1\}$, $\mathtt{m}_1$ $\mathtt{m}_2$ $\mathtt{m}_3$ $\mathtt{m}_4$ $\mathtt{m}_5$, **=**, **;**, **loop**

Objective Function: $f(x) = \dfrac{\text{\# training cases } t_i \text{ with } \varphi(t_i) \neq x(t_i)}{\text{total number } tc{=}100 \text{ of training cases}}$

1. `po2`[1]: $\varphi_1(t_i) = t_i^3 + t_i^2 + 2 * t_i$
2. `su2`[1]: sum of first t_i natural numbers
3. `fac`[1, 2]: $\varphi_3(t_i) = t_i!$
4. `po3`, `su3`: `po2`, `su2` with 3 memory cells
6. `gcd`[1, 3]: compute greatest common divisor
7. `exp`: $\varphi_7(t_i) = 2^{t_i}$
8. ℓ`20`: return 0 if $t_i < 20$, 1 otherwise
9. `prm`: returns 1 if t_i is prime, 0
10. `ssq`: $\varphi_{10}(t_i) = \sum_{i=1}^{t_i} i^2$
11. `sra`: $\varphi_{11}(t_i) = \sqrt{t_i}$
12. `srb`: $\varphi_{12}(t_i) = \lfloor \sqrt{t_i} \rfloor$
13. `ild`: $\mathrm{ld}^\star(y) = \begin{cases} 0 & \text{if } t_i \leq 1 \\ 1 + \mathrm{ld}^\star(\mathrm{ld}(y)) & \text{otherwise} \end{cases}$
14. `lsb`: least significant bit in the two's complement
15. `mul`: $\varphi_{15}(t_i) = t_{i,1} * t_{i,2}$
16. `qad`: $\varphi_{16}(t_i) = (t_i - 1)(t_i + 2)$
17. `mod`: $\varphi_{17}(t_i) = t_{i,1} \bmod t_{i,2}$
18. `mi5`: minimum of five
19. `ma5`: maximum of five
20. `sm5`: sum of five

Investigated Ideas

Loop Instructions

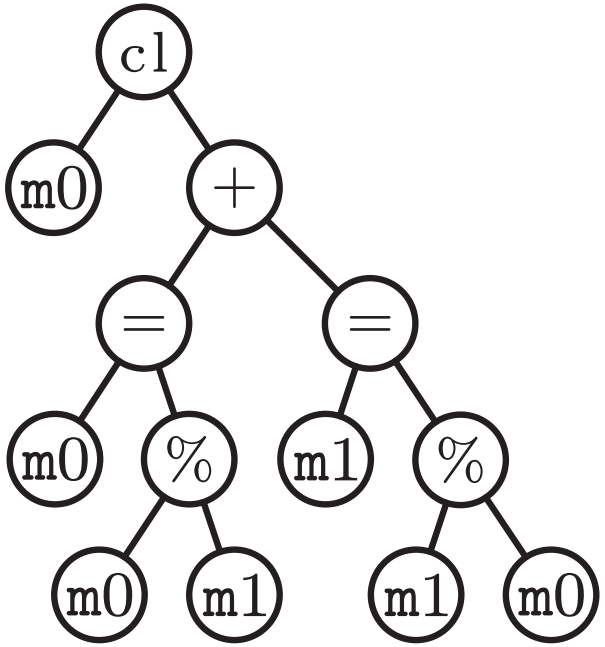
- We test six different loop instructions

Loop Instructions

- We test six different loop instructions
  1. `CL`: execute loop body n times

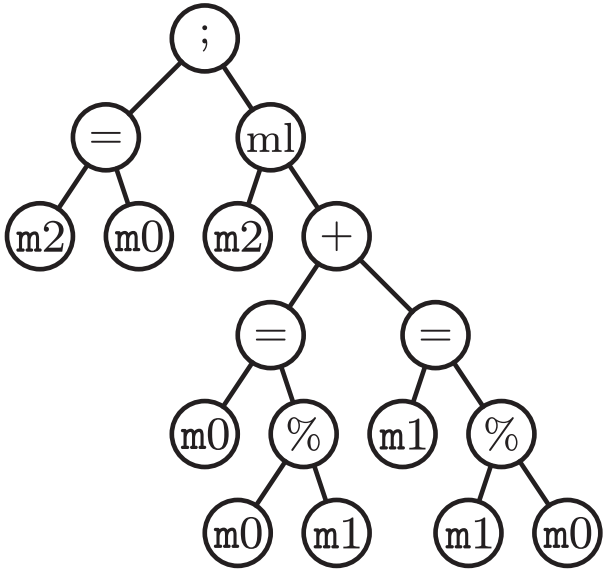
Loop Instructions

- We test six different loop instructions
  1. `CL`: execute loop body n times



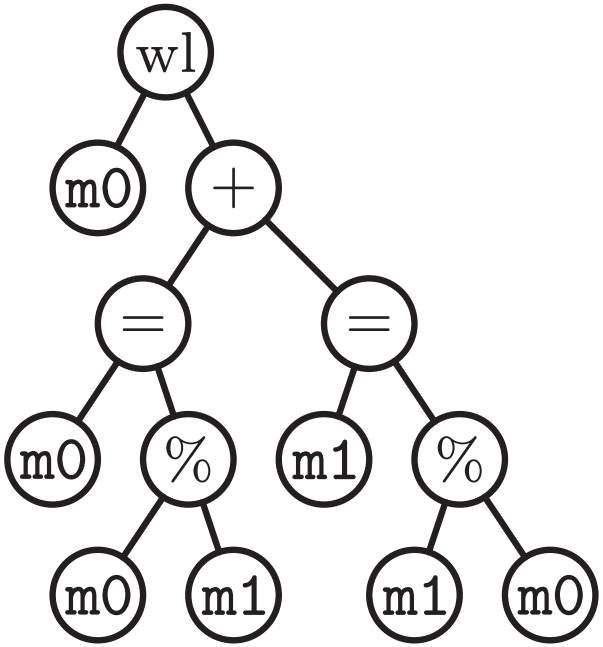
Loop Instructions

- We test six different loop instructions
  1. CL: execute loop body n times
  2. ML: decrease memory cell in each iteration, stop when 0 is reached



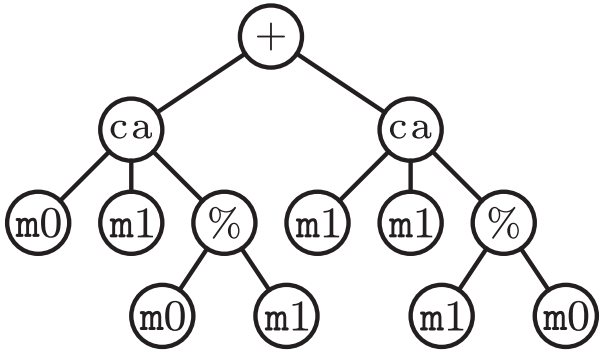
Loop Instructions

- We test six different loop instructions
  1. `CL`: execute loop body n times
  2. `ML`: decrease memory cell in each iteration, stop when 0 is reached
  3. `WL`: execute until expression becomes 0



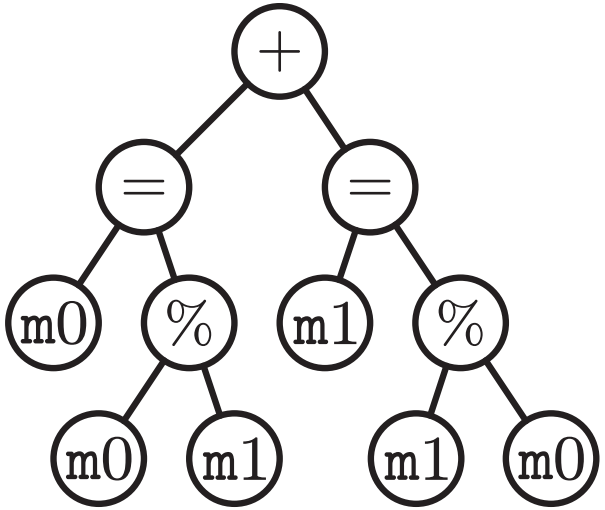
Loop Instructions

- We test six different loop instructions
  1. CL: execute loop body n times
  2. ML: decrease memory cell in each iteration, stop when 0 is reached
  3. WL: execute until expression becomes 0
  4. CA: conditional "=" assignment, execute program as long as variables change



Loop Instructions

- We test six different loop instructions
  1. `CL`: execute loop body n times
  2. `ML`: decrease memory cell in each iteration, stop when 0 is reached
  3. `WL`: execute until expression becomes 0
  4. `CA`: conditional "**=**" assignment, execute program as long as variables change
  5. `IL`: execute loop body (and program) as long as variables change



Loop Instructions

- We test six different loop instructions
  1. `CL`: execute loop body n times
  2. `ML`: decrease memory cell in each iteration, stop when 0 is reached
  3. `WL`: execute until expression becomes 0
  4. `CA`: conditional "**=**" assignment, execute program as long as variables change
  5. `IL`: execute loop body (and program) as long as variables change
  6. `IC`: `IL` $\oplus$ `CA`

Fighting Epistasis: Transactional Memory

- Program behavior depends on

Fighting Epistasis: Transactional Memory

- Program behavior depends on instructions

Fighting Epistasis: Transactional Memory

- Program behavior depends on **a)** instructions and **b)** their order

Fighting Epistasis: Transactional Memory

- Program behavior depends on **a)** instructions and **b)** their order $\Longrightarrow$ epistasis!

Fighting Epistasis: Transactional Memory

- Program behavior depends on **a)** instructions and **b)** their order $\Longrightarrow$ epistasis!
- Transactional Memory (`TM`) [4]

Fighting Epistasis: Transactional Memory

- Program behavior depends on **a)** instructions and **b)** their order $\Longrightarrow$ epistasis!
- Transactional Memory (`TM`) [4]
  - hierarchical `TM`: each control scope (**;**, **loop**) has *two* memory records

$$m_0 = m_1 + 1$$
$$m_1 = m_0 + 2$$

Fighting Epistasis: Transactional Memory

- Program behavior depends on **a)** instructions and **b)** their order $\Longrightarrow$ epistasis!
- Transactional Memory (TM) [4]
  - hierarchical TM: each control scope (**;**, **loop**) has *two* memory records
  - write memory: changes by write instruction **=**

$$m_0 = m_1 + 1$$
$$m_1 = m_0 + 2$$

Fighting Epistasis: Transactional Memory

- Program behavior depends on **a)** instructions and **b)** their order $\Longrightarrow$ epistasis!
- Transactional Memory (TM) [4]
  - hierarchical TM: each control scope (**;**, **loop**) has *two* memory records
  - write memory: changes by write instruction **=**
  - read memory: values returned by variable access m_i

$$m_0 = m_1 + 1$$
$$m_1 = m_0 + 2$$

Fighting Epistasis: Transactional Memory

- Program behavior depends on **a)** instructions and **b)** their order $\Longrightarrow$ epistasis!
- Transactional Memory (`TM`) [4]
  - hierarchical `TM`: each control scope (**;**, **loop**) has *two* memory records
  - write memory: changes by write instruction **=**
  - read memory: values returned by variable access m_i
  - write memory is comitted at end of control scope

m_0 = m_1 + 1
m_1 = m_0 + 2
(commit)

Fighting Epistasis: Transactional Memory

- Program behavior depends on **a)** instructions and **b)** their order $\Longrightarrow$ epistasis!
- Transactional Memory (`TM`) [4]
  - hierarchical `TM`: each control scope (**;**, **loop**) has *two* memory records
  - write memory: changes by write instruction **=**
  - read memory: values returned by variable access m_i
  - write memory is comitted to memory of parent scope at end of control scope
  - Order of instructions becomes less important

m_0 = m_1 + 1
m_1 = m_0 + 2
(commit)

Fighting Epistasis: Transactional Memory

- Program behavior depends on **a)** instructions and **b)** their order $\Longrightarrow$ epistasis!
- Transactional Memory (`TM`) [4]
  - hierarchical `TM`: each control scope (**;**, **loop**) has *two* memory records
  - write memory: changes by write instruction **=**
  - read memory: values returned by variable access m_i
  - write memory is comitted to memory of parent scope at end of control scope
  - Order of instructions becomes less important
- Can this reduce epistasis?

```
m0 = m1 + 1
m1 = m0 + 2
(commit)
```

≡

```
m1 = m0 + 2
m0 = m1 + 1
(commit)
```

Fighting Epistasis: Transactional Memory

- Program behavior depends on **a)** instructions and **b)** their order $\Longrightarrow$ epistasis!
- Transactional Memory (`TM`) [4]
  - hierarchical `TM`: each control scope (**;**, **loop**) has *two* memory records
  - write memory: changes by write instruction **=**
  - read memory: values returned by variable access m_i
  - write memory is comitted to memory of parent scope at end of control scope
  - Order of instructions becomes less important
- Can this reduce epistasis?

```
m0 = m1 + 1
m1 = m0 + 2
(commit)
```

≡

```
m1 = m0 + 2
m0 = m1 + 1
(commit)
```

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
  - Not necessarily

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
  - Not necessarily: deceptiveness, many local optima

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
  - Not necessarily: deceptiveness, many local optima
  - But selection is *very* likely to prefer x_1!

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
  - Not necessarily: deceptiveness, many local optima
  - But selection is *very* likely to prefer x_1!
  - GP will sooner or later converge.

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones
  - f defines behavioral semantics (to some degree)

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones
  - f defines behavioral semantics (to some degree)
  - tc training cases $\Longrightarrow tc + 1$ possible f values

Fighting Deceptiveness: FFA

- Genetic Programming minimizes f
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones
  - f defines behavioral semantics (to some degree)
  - tc training cases $\Longrightarrow tc + 1$ possible *behaviors*

Fighting Deceptiveness: FFA

- FFA-Genetic Programming minimizes H
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones
  - f defines behavioral semantics (to some degree)
  - tc training cases $\Longrightarrow tc + 1$ possible *behaviors*
  - Fitness $H[f(x)]$: How often has a given f value been encountered during the GP process?

Fighting Deceptiveness: FFA

- FFA-Genetic Programming minimizes H
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones
  - f defines behavioral semantics (to some degree)
  - tc training cases $\Longrightarrow tc + 1$ possible *behaviors*
  - Fitness $H[f(x)]$: How often has a given f value been encountered during the GP process?
  - high H-value: behavior seen often

Fighting Deceptiveness: FFA

- FFA-Genetic Programming minimizes H
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones
  - f defines behavioral semantics (to some degree)
  - tc training cases $\Longrightarrow tc + 1$ possible *behaviors*
  - Fitness $H[f(x)]$: How often has a given f value been encountered during the GP process?
  - high H-value: behavior seen often $\Longrightarrow$ not new

Fighting Deceptiveness: FFA

- FFA-Genetic Programming minimizes H
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones
  - f defines behavioral semantics (to some degree)
  - tc training cases $\Longrightarrow tc + 1$ possible *behaviors*
  - Fitness $H[f(x)]$: How often has a given f value been encountered during the GP process?
  - high H-value: behavior seen often $\Longrightarrow$ not new $\Longrightarrow$ simple to discover

Fighting Deceptiveness: FFA

- FFA-Genetic Programming minimizes H
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones
  - f defines behavioral semantics (to some degree)
  - tc training cases $\Longrightarrow tc + 1$ possible *behaviors*
  - Fitness $H[f(x)]$: How often has a given f value been encountered during the GP process?
  - high H-value: behavior seen often $\Longrightarrow$ not new $\Longrightarrow$ simple to discover $\Longrightarrow$ uninteresting

Fighting Deceptiveness: FFA

- FFA-Genetic Programming minimizes H
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones
  - f defines behavioral semantics (to some degree)
  - tc training cases $\Longrightarrow tc+1$ possible *behaviors*
  - Fitness $H\left[f(x)\right]$: How often has a given f value been encountered during the GP process?
  - high H-value: behavior seen often $\Longrightarrow$ not new $\Longrightarrow$ simple to discover $\Longrightarrow$ uninteresting
  - low H-value: behavior new

Fighting Deceptiveness: FFA

- FFA-Genetic Programming minimizes H
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones
  - f defines behavioral semantics (to some degree)
  - tc training cases $\Longrightarrow tc + 1$ possible *behaviors*
  - Fitness $H[f(x)]$: How often has a given f value been encountered during the GP process?
  - high H-value: behavior seen often $\Longrightarrow$ not new $\Longrightarrow$ simple to discover $\Longrightarrow$ uninteresting
  - low H-value: behavior new $\Longrightarrow$ program does something which may be worth exploring

Fighting Deceptiveness: FFA

- FFA-Genetic Programming minimizes H
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
  - Favour *new* behaviors instead of "good" ones
  - f defines behavioral semantics (to some degree)
  - tc training cases $\Longrightarrow tc+1$ possible *behaviors*
  - Fitness $H[f(x)]$: How often has a given f value been encountered during the GP process?
  - high H-value: behavior seen often $\Longrightarrow$ not new $\Longrightarrow$ simple to discover $\Longrightarrow$ uninteresting
  - low H-value: behavior new $\Longrightarrow$ program does something which may be worth exploring
  - H is a dynamic fitness function: behaviors get "worse" over time

Fighting Deceptiveness: FFA

- FFA-Genetic Programming minimizes H
- ...but: *Is a program x_1 that can solve 60% of the training cases closer to a solution than a program x_2 that can solve 10%?*
- Prevent convergence: Frequency Fitness Assignment (FFA) [5]
- Can FFA improve exploration and fight deceptiveness?

Experiments

Experiments

- On each of the new benchmarks, we tested

Experiments

- On each of the new benchmarks, we tested 6 setups
  - 6 loop structures

Experiments

- On each of the new benchmarks, we tested 6 $\times$ 2 setups
  - 6 loop structures
  - 2 memory models

Experiments

- On each of the new benchmarks, we tested $6 \times 2 \times 2 = 24$ setups
  - 6 loop structures
  - 2 memory models
  - 2 fitness functions

Experiments

- On each of the new benchmarks, we tested 24 setups
  - 6 loop structures
  - 2 memory models
  - 2 fitness functions for
  - 100 runs each

Experiments

- On each of the new benchmarks, we tested 24 setups
  - 6 loop structures
  - 2 memory models
  - 2 fitness functions for
  - 100 runs each
- population size 1000

Experiments

- On each of the new benchmarks, we tested 24 setups
  - 6 loop structures
  - 2 memory models
  - 2 fitness functions for
  - 100 runs each
- population size 1000

Experiments

- On each of the new benchmarks, we tested 24 setups
  - 6 loop structures
  - 2 memory models
  - 2 fitness functions for
  - 100 runs each
- population size 1000, 100 generations

Experiments

- On each of the new benchmarks, we tested 24 setups
  - 6 loop structures
  - 2 memory models
  - 2 fitness functions for
  - 100 runs each
- population size 1000, 100 generations, tournament selection with 7 contestants

Experiments

- On each of the new benchmarks, we tested 24 setups
  - 6 loop structures
  - 2 memory models
  - 2 fitness functions for
  - 100 runs each
- population size 1000, 100 generations, tournament selection with 7 contestants, 10% mutation

Experiments

- On each of the new benchmarks, we tested 24 setups
  - 6 loop structures
  - 2 memory models
  - 2 fitness functions for
  - 100 runs each
- population size 1000, 100 generations, tournament selection with 7 contestants, 10% mutation, 90% crossover

Results

- 48 000 runs

Results

- 48 000 runs by only 5422 (11%) were successful.

Results

- 48 000 runs by only 5422 (11%) were successful.
- What factors contribute to success (or failure)?

Results

- 48 000 runs by only 5422 (11%) were successful.
- What factors contribute to success (or failure)?
- Let's look at the success rates...

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.12	0.09	0.05	0.05	**0.13**	0.22	0.00	0.65	0.00	0.01	0.06	0.00	0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.14	0.03	0.04	0.01	0.01	0.37	**0.20**	0.93	0.00	0.00	0.01	0.00	0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.18	**0.66**	0.08	0.32	0.27	0.36	0.11	0.70	0.00	**0.21**	0.03	0.00	0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.11	0.00	0.08	0.00	0.00	0.42	0.00	0.51	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.07	0.06
SM-DIR-CA	0.11	0.00	0.08	0.00	0.00	0.55	0.00	0.93	0.01	0.00	0.14	0.01	**0.13**	0.03	0.00	0.00	0.00	0.40	0.50	0.00	0.14
SM-DIR-IL	0.09	0.01	0.11	0.01	0.00	0.10	0.00	0.81	0.00	0.00	0.04	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.06
SM-DIR-IC	0.04	0.03	0.00	0.00	0.00	0.68	0.00	0.88	0.01	0.00	0.13	0.02	0.05	0.02	0.01	0.00	0.00	0.43	0.55	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	0.00	0.01	**0.07**	0.00	0.00	0.72	**0.41**	**0.31**	0.19	0.17	0.05	0.20
SM-FFA-ML	0.14	0.57	0.15	**0.46**	0.45	0.47	0.12	0.92	0.02	0.05	0.05	0.00	0.00	0.00	**0.89**	0.30	0.15	0.00	0.00	0.06	**0.24**
SM-FFA-WL	**0.32**	0.00	**0.17**	0.02	0.04	0.46	0.00	0.90	0.01	0.00	0.01	0.01	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
SM-FFA-CA	0.17	0.00	0.05	0.00	0.02	0.80	0.00	0.97	0.02	0.00	**0.22**	0.06	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
SM-FFA-IL	0.21	0.03	0.11	0.00	0.00	0.42	0.01	0.93	0.01	0.00	0.15	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10
SM-FFA-IC	0.12	0.02	0.01	0.03	0.00	0.76	0.00	0.90	**0.05**	0.00	0.16	0.03	0.01	0.01	0.01	0.00	0.00	**0.72**	**0.74**	0.04	0.18
TM-DIR-CL	0.12	0.07	0.07	0.03	0.01	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.05	0.04	0.00	0.00	0.13	0.08
TM-DIR-ML	0.17	0.27	0.04	0.14	0.43	0.01	0.00	0.30	0.00	0.01	0.18	0.00	0.03	0.00	0.03	0.00	0.01	0.00	0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00	0.11	0.00	0.24	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05	0.83	0.00	0.66	0.00	0.00	0.11	0.01	0.06	0.01	0.04	0.01	0.00	0.24	0.30	0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00	0.04	0.00	0.75	0.00	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09	0.05	0.00	0.45	0.00	0.00	0.04	0.00	0.03	0.02	0.00	0.00	0.00	0.05	0.15	0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01	0.00	0.02	**1.00**	0.00	0.00	0.00	0.00	0.01	0.00	0.34	0.32	0.07	0.00	0.00	0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	0.13	**0.58**	0.00	0.00	0.72	0.00	0.03	0.05	0.00	0.01	0.00	0.07	0.09	0.04	0.00	0.00	0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	0.00	0.00	0.17	0.00	0.74	0.01	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.08	0.16	**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	0.06	0.21	**0.87**	0.00	0.76	0.01	0.02	0.12	0.01	0.00	**0.04**	0.03	0.00	0.00	0.45	0.63	0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.12	0.09	0.05	0.05	**0.13**	0.22	0.00	0.65	0.00	0.01	0.06	0.00	0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.14	0.03	0.04	0.01	0.01	0.37	**0.20**	0.93	0.00	0.00	0.01	0.00	0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.18	**0.66**	0.08	0.32	0.27	0.36	0.11	0.70	0.00	**0.21**	0.03	0.00	0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.11	0.00	0.08	0.00	0.00	0.42	0.00	0.51	0.00	0.00	0.01	0.00	0.00	0.01	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.07	0.06
SM-DIR-CA	0.11	0.00	0.08	0.00	0.00	0.55	0.00	0.93	0.01	0.00	0.14	0.01	**0.13**	0.03	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.14
SM-DIR-IL	0.09	0.01	0.11	0.01	0.00	0.10	0.00	0.81	0.00	0.00	0.04	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.06
SM-DIR-IC	0.04	0.03	0.00	0.00	0.00	0.68	0.00	0.88	0.01	0.00	0.13	0.02	0.05	0.02	0.01	0.00	0.00	0.43	0.55	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	0.00	0.01	**0.07**	0.00	0.00	0.72	**0.41**	**0.31**	0.19	0.17	0.05	0.20
SM-FFA-ML	0.14	0.57	0.15	**0.46**	0.45	0.47	0.12	0.92	0.02	0.05	0.05	0.00	0.00	0.00	**0.89**	0.30	0.15	0.00	0.00	0.06	**0.24**
SM-FFA-WL	**0.32**	0.00	**0.17**	0.02	0.04	0.46	0.00	0.90	0.01	0.00	0.01	0.01	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
SM-FFA-CA	0.17	0.00	0.05	0.00	0.02	0.80	0.00	0.97	0.02	0.00	**0.22**	0.06	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
SM-FFA-IL	0.21	0.03	0.11	0.00	0.00	0.42	0.01	0.93	0.01	0.00	0.15	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10
SM-FFA-IC	0.12	0.02	0.01	0.03	0.00	0.76	0.00	0.90	**0.05**	0.00	0.16	0.03	0.01	0.01	0.01	0.00	0.00	**0.72**	**0.74**	0.04	0.18
TM-DIR-CL	0.12	0.07	0.07	0.03	0.01	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.05	0.04	0.00	0.00	0.13	0.08
TM-DIR-ML	0.17	0.27	0.04	0.14	0.43	0.01	0.00	0.30	0.00	0.01	0.18	0.00	0.03	0.00	0.03	0.00	0.01	0.00	0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00	0.11	0.00	0.24	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05	0.83	0.00	0.66	0.00	0.00	0.11	0.01	0.06	0.01	0.04	0.01	0.00	0.24	0.30	0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00	0.04	0.00	0.75	0.00	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09	0.05	0.00	0.45	0.00	0.00	0.04	0.00	0.03	0.02	0.00	0.00	0.00	0.05	0.15	0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01	0.00	0.02	**1.00**	0.00	0.00	0.00	0.00	0.01	0.00	0.34	0.32	0.07	0.00	0.00	0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	0.13	**0.58**	0.00	0.00	0.72	0.00	0.03	0.05	0.00	0.01	0.00	0.07	0.09	0.04	0.00	0.00	0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	0.00	0.00	0.17	0.00	0.74	0.01	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.08	0.16	**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	0.06	0.21	**0.87**	0.00	0.76	0.01	0.02	0.12	0.01	0.00	**0.04**	0.03	0.00	0.00	0.45	0.63	0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

benchmarks

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.12	0.09	0.05	0.05	**0.13**	0.22	0.00	0.65	0.00	0.01	0.06	0.00	0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.14	0.03	0.04	0.01	0.01	0.37	**0.20**	0.93	0.00	0.00	0.01	0.00	0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.18	**0.66**	0.08	0.32	0.27	0.36	0.11	0.70	0.00	**0.21**	0.03	0.00	0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.11	0.00	0.08	0.00	0.00	0.42	0.00	0.51	0.00	0.00	0.01	0.00	0.00	0.01	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.07	0.06
SM-DIR-CA	0.11	0.00	0.08	0.00	0.00	0.55	0.00	0.93	0.01	0.00	0.14	0.01	**0.13**	0.03	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.14
SM-DIR-IL	0.09	0.01	0.11	0.01	0.00	0.10	0.00	0.81	0.00	0.00	0.04	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.06
SM-DIR-IC	0.04	0.03	0.00	0.00	0.00	0.68	0.00	0.88	0.01	0.00	0.13	0.02	0.05	0.02	0.01	0.00	0.00	0.43	0.55	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	0.00	0.01	**0.07**	0.00	0.00	0.72	**0.41**	**0.31**	0.19	0.17	0.05	0.20
SM-FFA-ML	0.14	0.57	0.15	**0.46**	0.45	0.47	0.12	0.92	0.02	0.05	0.05	0.00	0.00	0.00	**0.89**	0.30	0.15	0.00	0.00	0.06	**0.24**
SM-FFA-WL	**0.32**	0.00	**0.17**	0.02	0.04	0.46	0.00	0.90	0.01	0.00	0.01	0.01	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
SM-FFA-CA	0.17	0.00	0.05	0.00	0.02	0.80	0.00	0.97	0.02	0.00	**0.22**	0.06	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
SM-FFA-IL	0.21	0.03	0.11	0.00	0.00	0.42	0.01	0.93	0.01	0.00	0.15	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10
SM-FFA-IC	0.12	0.02	0.01	0.03	0.00	0.76	0.00	0.90	**0.05**	0.00	0.16	0.03	0.01	0.01	0.01	0.00	0.00	**0.72**	**0.74**	0.04	0.18
TM-DIR-CL	0.12	0.07	0.07	0.03	0.01	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.05	0.04	0.00	0.00	0.13	0.08
TM-DIR-ML	0.17	0.27	0.04	0.14	0.43	0.01	0.00	0.30	0.00	0.01	0.18	0.00	0.03	0.00	0.03	0.00	0.01	0.00	0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00	0.11	0.00	0.24	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05	0.83	0.00	0.66	0.00	0.00	0.11	0.01	0.06	0.01	0.04	0.01	0.00	0.24	0.30	0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00	0.04	0.00	0.75	0.00	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09	0.05	0.00	0.45	0.00	0.00	0.04	0.00	0.03	0.02	0.00	0.00	0.00	0.05	0.15	0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01	0.00	0.02	**1.00**	0.00	0.00	0.00	0.00	0.01	0.00	0.34	0.32	0.07	0.00	0.00	0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	[illegible]	[illegible]	[illegible]	[illegible]	0.72	0.00	0.03	0.05	0.00	0.01	0.00	0.07	0.09	0.04	0.00	0.00	0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	[illegible]	[illegible]	[illegible]	[illegible]	0.74	0.01	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.08	0.16	**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	[illegible]	[illegible]	[illegible]	[illegible]	0.76	0.01	0.02	0.12	0.01	0.00	**0.04**	0.03	0.00	0.00	0.45	0.63	0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

benchmarks

setups

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.1												0.00	0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.1												0.00	0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.1												0.00	0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.1												0.00	0.00	0.01					0	0.07	0.06
SM-DIR-CA	0.1												0.01	**0.13**	0.03					0	0.00	0.14
SM-DIR-IL	0.0												0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.06
SM-DIR-IC	0.0												0.02	0.05	0.02	0.01	0.00	0.00	0.43	0.55	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	0.00	0.01	**0.07**	0.00	0.00	0.72	**0.41**	**0.31**	0.19	0.17	0.05	0.20
SM-FFA-ML	0.14	0.57	0.15	**0.46**	0.45	0.47	0.12	0.92	0.02	0.05	0.05	0.00	0.00	0.00	**0.89**	0.30	0.15	0.00	0.00	0.06	**0.24**
SM-FFA-WL	**0.32**	0.00	**0.17**	0.02	0.04	0.46	0.00	0.90	0.01	0.00	0.01	0.01	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
SM-FFA-CA	0.17	0.00	0.05	0.00	0.02	0.80	0.00	0.97	0.02	0.00	**0.22**	0.06	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
SM-FFA-IL	0.21	0.03	0.11	0.00	0.00	0.42	0.01	0.93	0.01	0.00	0.15	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10
SM-FFA-IC	0.12	0.02	0.01	0.03	0.00	0.76	0.00	0.90	**0.05**	0.00	0.16	0.03	0.01	0.01	0.01	0.00	0.00	**0.72**	**0.74**	0.04	0.18
TM-DIR-CL	0.12	0.07	0.07	0.03	0.01	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.05	0.04	0.00	0.00	0.13	0.08
TM-DIR-ML	0.17	0.27	0.04	0.14	0.43	0.01	0.00	0.30	0.00	0.01	0.18	0.00	0.03	0.00	0.03	0.00	0.01	0.00	0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00	0.11	0.00	0.24	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05	0.83	0.00	0.66	0.00	0.00	0.11	0.01	0.06	0.01	0.04	0.01	0.00	0.24	0.30	0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00	0.04	0.00	0.75	0.00	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09	0.05	0.00	0.45	0.00	0.00	0.04	0.00	0.03	0.02	0.00	0.00	0.00	0.05	0.15	0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01	0.00	0.02	**1.00**	0.00	0.00	0.00	0.00	0.01	0.00	0.34	0.32	0.07	0.00	0.00	0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	0.			0.00	0.72	0.00	0.03	0.05	0.00	0.01	0.00	0.07	0.09	0.04	0.00	0.00	0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	0			0.00	0.74	0.01	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.08	0.16	**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	0.			0.00	0.76	0.01	0.02	0.12	0.01	0.00	**0.04**	0.03	0.00	0.00	0.45	0.63	0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

aggregated results:

- with FFA (and f directly DIR)
- with TM (and simple memory SM)

benchmarks

setups

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.1												0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.1												0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.1												0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.1												0.00	0.01						0.07	0.06
SM-DIR-CA	0.1												0.01	**0.13**	0.03					0.00	0.14
SM-DIR-IL	0.0												0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.06
SM-DIR-IC	0.0												0.02	0.05	0.02	0.01	0.00	0.00	0.43	0.55	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	0.00	0.01	**0.07**	0.00	0.00	0.72	**0.41**	**0.31**	0.19	0.17	0.05	0.20
SM-FFA-ML	0.14	0.57	0.15	**0.46**	0.45	0.47	0.12	0.92	0.02	0.05	0.05	0.00	0.00	0.00	**0.89**	0.30	0.15	0.00	0.00	0.06	**0.24**
SM-FFA-WL	**0.32**	0.00	**0.17**	0.02	0.04	0.46	0.00	0.90	0.01	0.00	0.01	0.01	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
SM-FFA-CA	0.17	0.00	0.05	0.00	0.02	0.80	0.00	0.97	0.02	0.00	**0.22**	0.06	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
SM-FFA-IL	0.21	0.03	0.11	0.00	0.00	0.42	0.01	0.93	0.01	0.00	0.15	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10
SM-FFA-IC	0.12	0.02	0.01	0.03	0.00	0.76	0.00	0.90	**0.05**	0.00	0.16	0.03	0.01	0.01	0.01	0.00	0.00	**0.72**	**0.74**	0.04	0.18
TM-DIR-CL	0.12	0.07	0.07	0.03	0.01	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.05	0.04	0.00	0.00	0.13	0.08
TM-DIR-ML	0.17	0.27	0.04	0.14	0.43	0.01	0.00	0.30	0.00	0.01	0.18	0.00	0.03	0.00	0.03	0.00	0.01	0.00	0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00	0.11	0.00	0.24	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05	0.83	0.00	0.66	0											0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00	0.04	0.00	0.75	0											0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09	0.05	0.00	0.45	0											0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01	0.00	0.02	**1.00**	0											0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	0.			0.00	0.72	0											0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	0			0.00	0.74	0											**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	0.			0.00	0.76	0											0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

aggregated results:

- with FFA (and f directly DIR)
- with TM (and simple memory SM)

benchmarks

setups

aggregated results:

- over benchmark instances
- over setups

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.12	0.09	0.05	0.05	**0.13**	0.22	0.00	0.65	0.00	0.01	0.06	0.00	0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.14	0.03	0.04	0.01	0.01	0.37	**0.20**	0.93	0.00	0.00	0.01	0.00	0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.18	**0.66**	0.08	0.32	0.27	0.36	0.11	0.70	0.00	**0.21**	0.03	0.00	0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.11	0.00	0.08	0.00	0.00	0.42	0.00	0.51	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.07	0.06
SM-DIR-CA	0.11	0.00	0.08	0.00	0.00	0.55	0.00	0.93	0.01	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.14
SM-DIR-IL	0.09	0.01	0.11	0.01	0.00	0.10	0.00	0.81	0.00	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.06
SM-DIR-IC	0.04	0.03	0.00	0.00	0.00	0.68	0.00	0.88	0.01	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.05	0.20
SM-FFA-ML	0.14	0.57	0.15	**0.46**	0.45	0.47	0.12	0.92	0.02	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.06	**0.24**
SM-FFA-WL	**0.32**	0.00	**0.17**	0.02	0.04	0.46	0.00	0.90	0.01	0.00	0.01	0.01	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
SM-FFA-CA	0.17	0.00	0.05	0.00	0.02	0.80	0.00	0.97	0.02	0.00	**0.22**	0.06	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
SM-FFA-IL	0.21	0.03	0.11	0.00	0.00	0.42	0.01	0.93	0.01	0.00	0.15	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10
SM-FFA-IC	0.12	0.02	0.01	0.03	0.00	0.76	0.00	0.90	**0.05**	0.00	0.16	0.03	0.01	0.01	0.01	0.00	0.00	**0.72**	**0.74**	0.04	0.18
TM-DIR-CL	0.12	0.07	0.07	0.03	0.01	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.05	0.04	0.00	0.00	0.13	0.08
TM-DIR-ML	0.17	0.27	0.04	0.14	0.43	0.01	0.00	0.30	0.00	0.01	0.18	0.00	0.03	0.00	0.03	0.00	0.01	0.00	0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00	0.11	0.00	0.24	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05	0.83	0.00	0.66	0.00	0.00	0.11	0.01	0.06	0.01	0.04	0.01	0.00	0.24	0.30	0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00	0.04	0.00	0.75	0.00	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09	0.05	0.00	0.45	0.00	0.00	0.04	0.00	0.03	0.02	0.00	0.00	0.00	0.05	0.15	0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01	0.00	0.02	**1.00**	0.00	0.00	0.00	0.00	0.01	0.00	0.34	0.32	0.07	0.00	0.00	0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	0.13	**0.58**	0.00	0.00	0.72	0.00	0.03	0.05	0.00	0.01	0.00	0.07	0.09	0.04	0.00	0.00	0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	0.00	0.00	0.17	0.00	0.74	0.01	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.08	0.16	**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	0.06	0.21	**0.87**	0.00	0.76	0.01	0.02	0.12	0.01	0.00	**0.04**	0.03	0.00	0.00	0.45	0.63	0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

FFA increases success rate by 40%

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.12	0.09	0.05	0.05	**0.13**	0.22	0.00	0.65	0.00	0.01	0.06	0.00	0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.14	0.03	0.04	0.01	0.01	0.37	**0.20**	0.93	0.00	0.00	0.01	0.00	0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.18	**0.66**	0.08	0.32	0.27	0.36	0.11	0.70	0.00	**0.21**	0.03	0.00	0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.11	0.00	0.08	0.00	0.00	0.42	0.00	0.51	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.07	0.06
SM-DIR-CA	0.11	0.00	0.08	0.00	0.00	0.55	0.00	0.93	0.01	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.14
SM-DIR-IL	0.09	0.01	0.11	0.01	0.00	0.10	0.00	0.81	0.00	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.06
SM-DIR-IC	0.04	0.03	0.00	0.00	0.00	0.68	0.00	0.88	0.01	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.05	0.20
SM-FFA-ML	0.14	0.57	0.15	**0.46**	0.45	0.47	0.12	0.92	0.02	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.06	**0.24**
SM-FFA-WL	**0.32**	0.00	**0.17**	0.02	0.04	0.46	0.00	0.90	0.01	0.00	0.01	0.01	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
SM-FFA-CA	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	**0.22**	0.06	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
SM-FFA-IL	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10
SM-FFA-IC	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.03	0.01	0.01	0.01	0.00	0.00	**0.72**	**0.74**	0.04	0.18
TM-DIR-CL	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.00	0.00	0.26	0.05	0.04	0.00	0.00	0.13	0.08
TM-DIR-ML	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.03	0.00	0.03	0.00	0.01	0.00	0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00	0.11	0.00	0.24	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05	0.83	0.00	0.66	0.00	0.00	0.11	0.01	0.06	0.01	0.04	0.01	0.00	0.24	0.30	0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00	0.04	0.00	0.75	0.00	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09	0.05	0.00	0.45	0.00	0.00	0.04	0.00	0.03	0.02	0.00	0.00	0.00	0.05	0.15	0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01	0.00	0.02	**1.00**	0.00	0.00	0.00	0.00	0.01	0.00	0.34	0.32	0.07	0.00	0.00	0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	0.13	**0.58**	0.00	0.00	0.72	0.00	0.03	0.05	0.00	0.01	0.00	0.07	0.09	0.04	0.00	0.00	0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	0.00	0.00	0.17	0.00	0.74	0.01	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.08	0.16	**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	0.06	0.21	**0.87**	0.00	0.76	0.01	0.02	0.12	0.01	0.00	**0.04**	0.03	0.00	0.00	0.45	0.63	0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

FFA increases success rate by 40%

TM *de*creases success rate on 17 out of 20 benchmarks

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.12	0.09	0.05	0.05	**0.13**	0.22	0.00	0.65	0.00	0.01	0.06	0.00	0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.14	0.03	0.04	0.01	0.01	0.37	**0.20**	0.93	0.00	0.00	0.01	0.00	0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.18	**0.66**	0.08	0.32	0.27	0.36	0.11	0.70	0.00	**0.21**	0.03	0.00	0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.11	0.00	0.08	0.00	0.00	0.42	0.00	0.51	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.07	0.06
SM-DIR-CA	0.11	0.00	0.08	0.00	0.00	0.55	0.00	0.93	0.01	0.00	0.14	0.01	**0.13**	0.03	0.00	0.00	0.00	0.40	0.50	0.00	0.14
SM-DIR-IL	0.09	0.01	0.11	0.01	0.00	0.10	0.00	0.81	0.00	0.00	0.04	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.06
SM-DIR-IC	0.04	0.03	0.00	0.00	0.00	0.68	0.00	0.88	0.01	0.00	0.13	0.02	0.05	0.02	0.01	0.00	0.00	0.43	0.55	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	0.00	0.01	**0.07**	0.00	0.00	0.72	**0.41**	**0.31**	0.19	0.17	0.05	0.20
SM-FFA-ML	0.14	0.57	0.15	**0.46**	0.45	0.47	0.12	0.92	0.02	0.05	0.05	0.00	0.00	0.00	**0.89**	0.30	0.15	0.00	0.00	0.06	**0.24**
SM-FFA-WL	**0.32**	0.00	**0.17**	0.02	0.04	0.46	0.00	0.90	0.01	0.00	0.01	0.01	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
SM-FFA-CA	0.17	0.00	0.05	0.00	0.02	0.80	0.00	0.97	0.02	0.00	**0.22**	0.06	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
SM-FFA-IL	0.21	0.03	0.11	0.00	0.00	0.42	0.01	0.93	0.01	0.00	0.15	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10
SM-FFA-IC	0.12	0.02	0.01	0.03	0.00	0.76	0.00	0.90	**0.05**	0.00	0.16	0.03	0.01	0.01	0.01	0.00	0.00	**0.72**	**0.74**	0.04	0.18
TM-DIR-CL	0.12	0.07	0.07	0.03	0.01	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.05	0.04	0.00	0.00	0.13	0.08
TM-DIR-ML	0.17	0.27	0.04	0.14	0.43														0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00														0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05														0.30	0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00														0.01	0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09	0.05	0.00	0.45	0.00	0.00	0.04	0.00	0.03	0.02	0.00	0.00	0.00	0.05	0.15	0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01	0.00	0.02	**1.00**	0.00	0.00	0.00	0.00	0.01	0.00	0.34	0.32	0.07	0.00	0.00	0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	0.13	**0.58**	0.00	0.00	0.72	0.00	0.03	0.05	0.00	0.01	0.00	0.07	0.09	0.04	0.00	0.00	0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	0.00	0.00	0.17	0.00	0.74	0.01	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.08	0.16	**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	0.06	0.21	**0.87**	0.00	0.76	0.01	0.02	0.12	0.01	0.00	**0.04**	0.03	0.00	0.00	0.45	0.63	0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

- Best Setup: SM-FFA-ML

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.12	0.09	0.05	0.05	**0.13**	0.22	0.00	0.65	0.00	0.01	0.06	0.00	0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.14	0.03	0.04	0.01	0.01	0.37	**0.20**	0.93	0.00	0.00	0.01	0.00	0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.18	**0.66**	0.08	0.32	0.27	0.36	0.11	0.70	0.00	**0.21**	0.03	0.00	0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.11	0.00	0.08	0.00	0.00	0.42	0.00	0.51	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.07	0.06
SM-DIR-CA	0.11	0.00	0.08	0.00	0.00	0.55	0.00	0.93	0.01	0.00	0.14	0.01	**0.13**	0.03	0.00	0.00	0.00	0.40	0.50	0.00	0.14
SM-DIR-IL	0.09	0.01	0.11	0.01	0.00	0.10	0.00	0.81	0.00	0.00	0.04	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.06
SM-DIR-IC	0.04	0.03	0.00	0.00	0.00	0.68	0.00	0.88	0.01	0.00	0.13	0.02	0.05	0.02	0.01	0.00	0.00	0.43	0.55	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	0.00	0.01	**0.07**	0.00	0.00	0.72	**0.41**	**0.31**	0.19	0.17	0.05	0.20
SM-FFA-ML	0.14	0.57	0.15	**0.46**	0.45	0.47	0.12	0.92	0.02	0.05	0.05	0.00	0.00	0.00	**0.89**	0.30	0.15	0.00	0.00	0.06	**0.24**
SM-FFA-WL	**0.32**	0.00	**0.17**	0.02	0.04	0.46	0.00	0.90	0.01	0.00	0.01	0.01	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
SM-FFA-CA	0.17	0.00	0.05	0.00	0.02	0.80	0.00	0.97	0.02	0.00	**0.22**	0.06	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
SM-FFA-IL	0.21	0.03	0.11	0.00	0.00	0.42	0.01	0.93	0.01	0.00	0.15	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10
SM-FFA-IC	0.12	0.02	0.01	0.03	0.00														**0.74**	0.04	0.18
TM-DIR-CL	0.12	0.07	0.07	0.03	0.01														0.00	0.13	0.08
TM-DIR-ML	0.17	0.27	0.04	0.14	0.43														0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00														0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05														0.30	0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00														0.01	0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09														0.15	0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01														0.00	0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	0.13	**0.58**														0.00	0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	0.00	0.00	0.17	0.00	0.74	0.01	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.08	0.16	**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	0.06	0.21	**0.87**	0.00	0.76	0.01	0.02	0.12	0.01	0.00	**0.04**	0.03	0.00	0.00	0.45	0.63	0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

- Best Setup: SM-FFA-ML
- Best loop structures: ML and CA (16% success rate)

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.12	0.09	0.05	0.05	**0.13**	0.22	0.00	0.65	0.00	0.01	0.06	0.00	0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.14	0.03	0.04	0.01	0.01	0.37	**0.20**	0.93	0.00	0.00	0.01	0.00	0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.18	**0.66**	0.08	0.32	0.27	0.36	0.11	0.70	0.00	**0.21**	0.03	0.00	0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.11	0.00	0.08	0.00	0.00	0.42	0.00	0.51	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.07	0.06
SM-DIR-CA	0.11	0.00	0.08	0.00	0.00	0.55	0.00	0.93	0.01	0.00	0.14	0.01	**0.13**	0.03	0.00	0.00	0.00	0.40	0.50	0.00	0.14
SM-DIR-IL	0.09	0.01	0.11	0.01	0.00	0.10	0.00	0.81	0.00	0.00	0.04	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.06
SM-DIR-IC	0.04	0.03	0.00	0.00	0.00	0.68	0.00	0.88	0.01	0.00	0.13	0.02	0.05	0.02	0.01	0.00	0.00	0.43	0.55	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	0.00	0.01	**0.07**	0.00	0.00	0.72	**0.41**	**0.31**	0.19	0.17	0.05	0.20
SM-FFA-ML	0.14	0.57	0.15	**0.46**	0.45	0.47	0.12	0.92	0.02	0.05	0.05	0.00	0.00	0.00	**0.89**	0.30	0.15	0.00	0.00	0.06	**0.24**
SM-FFA-WL	**0.32**	0.00	**0.17**	0.02	0.04	0.46	0.00	0.90	0.01	0.00	0.01	0.01	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
SM-FFA-CA	0.17	0.00	0.05	0.00	0.02	0.80	0.00	0.97	0.02	0.00	**0.22**	0.06	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
SM-FFA-IL	0.21	0.03	0.11	0.00	0.00														0.00	0.05	0.10
SM-FFA-IC	0.12	0.02	0.01	0.03	0.00														**0.74**	0.04	0.18
TM-DIR-CL	0.12	0.07	0.07	0.03	0.01														0.00	0.13	0.08
TM-DIR-ML	0.17	0.27	0.04	0.14	0.43														0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00														0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05														0.30	0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00														0.01	0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09														0.15	0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01														0.00	0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	0.13	**0.58**														0.00	0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	0.00	0.00														0.16	**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	0.06	0.21	**0.87**	0.00	0.76	0.01	0.02	0.12	0.01	0.00	**0.04**	0.03	0.00	0.00	0.45	0.63	0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

- Best Setup: SM-FFA-ML
- Best loop structures: ML and CA (16% success rate)
- Worst loop structures: IL and WL

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.12	0.09	0.05	0.05	**0.13**	0.22	0.00	0.65	0.00	0.01	0.06	0.00	0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.14	0.03	0.04	0.01	0.01	0.37	**0.20**	0.93	0.00	0.00	0.01	0.00	0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.18	**0.66**	0.08	0.32	0.27	0.36	0.11	0.70	0.00	**0.21**	0.03	0.00	0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.11	0.00	0.08	0.00	0.00	0.42	0.00	0.51	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.07	0.06
SM-DIR-CA	0.11	0.00	0.08	0.00	0.00	0.55	0.00	0.93	0.01	0.00	0.14	0.01	**0.13**	0.03	0.00	0.00	0.00	0.40	0.50	0.00	0.14
SM-DIR-IL	0.09	0.01	0.11	0.01	0.00	0.10	0.00	0.81	0.00	0.00	0.04	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.06
SM-DIR-IC	0.04	0.03	0.00	0.00	0.00	0.68	0.00	0.88	0.01	0.00	0.13	0.02	0.05	0.02	0.01	0.00	0.00	0.43	0.55	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	0.00	0.01	**0.07**	0.00	0.00	0.72	**0.41**	**0.31**	0.19	0.17	0.05	0.20
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.00	**0.89**	0.30	0.15	0.00	0.00	0.06	**0.24**
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
SM-FFA-CA	0.17	0.00	0.05	0.00	0.02	0.80	0.00	0.97	0.02	0.00	**0.22**	0.06	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
SM-FFA-IL	0.21	0.03	0.11	0.00	0.00	0.42	0.01	0.93	0.01	0.00	0.15	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10
SM-FFA-IC	0.12	0.02	0.01	0.03	0.00	0.76	0.00	0.90	**0.05**	0.00	0.16	0.03	0.01	0.01	0.01	0.00	0.00	**0.72**	**0.74**	0.04	0.18
TM-DIR-CL	0.12	0.07	0.07	0.03	0.01	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.05	0.04	0.00	0.00	0.13	0.08
TM-DIR-ML	0.17	0.27	0.04	0.14	0.43	0.01	0.00	0.30	0.00	0.01	0.18	0.00	0.03	0.00	0.03	0.00	0.01	0.00	0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00	0.11	0.00	0.24	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05	0.83	0.00	0.66	0.00	0.00	0.11	0.01	0.06	0.01	0.04	0.01	0.00	0.24	0.30	0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00	0.04	0.00	0.75	0.00	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09	0.05	0.00	0.45	0.00	0.00	0.04	0.00	0.03	0.02	0.00	0.00	0.00	0.05	0.15	0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01	0.00	0.02	**1.00**	0.00	0.00	0.00	0.00	0.01	0.00	0.34	0.32	0.07	0.00	0.00	0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	0.13	**0.58**	0.00	0.00	0.72	0.00	0.03	0.05	0.00	0.01	0.00	0.07	0.09	0.04	0.00	0.00	0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	0.00	0.00	0.17	0.00	0.74	0.01	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.08	0.16	**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	0.06	0.21	**0.87**	0.00	0.76	0.01	0.02	0.12	0.01	0.00	**0.04**	0.03	0.00	0.00	0.45	0.63	0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

Easiest problems: ℓ20, gcd, ma5

Setup	po2	su2	po3	su3	fac	gcd	exp	ℓ20	prm	ssq	sra	srb	ild	lsb	mul	qad	mod	mi5	ma5	sm5	All
DIR	0.10	0.10	0.05	0.05	0.07	0.29	0.03	0.66	0.00	**0.02**	0.06	0.00	**0.03**	**0.01**	0.12	0.01	0.03	0.10	0.13	0.06	0.10
FFA	**0.18**	**0.11**	**0.09**	**0.07**	**0.13**	**0.41**	**0.03**	**0.86**	**0.01**	0.01	**0.08**	**0.02**	0.02	**0.01**	**0.17**	**0.09**	**0.05**	**0.20**	**0.24**	**0.07**	**0.14**
SM	**0.16**	**0.12**	**0.09**	**0.07**	0.07	**0.48**	**0.05**	**0.87**	**0.01**	**0.02**	**0.08**	**0.02**	**0.03**	0.01	**0.23**	**0.07**	**0.07**	**0.20**	**0.23**	0.04	**0.15**
TM	0.12	0.09	0.05	0.05	**0.13**	0.22	0.00	0.65	0.00	0.01	0.06	0.00	0.02	**0.01**	0.06	0.04	0.01	0.09	0.14	**0.09**	0.09
SM-DIR-CL	0.14	0.03	0.04	0.01	0.01	0.37	**0.20**	0.93	0.00	0.00	0.01	0.00	0.00	0.00	0.44	0.07	0.22	0.04	0.07	0.11	0.13
SM-DIR-ML	0.18	**0.66**	0.08	0.32	0.27	0.36	0.11	0.70	0.00	**0.21**	0.03	0.00	0.01	0.00	0.63	0.01	0.13	0.00	0.00	0.04	0.19
SM-DIR-WL	0.11	0.00	0.08	0.00	0.00	0.42	0.00	0.51	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.07	0.06
SM-DIR-CA	0.11	0.00	0.08	0.00	0.00	0.55	0.00	0.93	0.01	0.00	0.14	0.01	**0.13**	0.03	0.00	0.00	0.00	0.40	0.50	0.00	0.14
SM-DIR-IL	0.09	0.01	0.11	0.01	0.00	0.10	0.00	0.81	0.00	0.00	0.04	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.06
SM-DIR-IC	0.04	0.03	0.00	0.00	0.00	0.68	0.00	0.88	0.01	0.00	0.13	0.02	0.05	0.02	0.01	0.00	0.00	0.43	0.55	0.02	0.14
SM-FFA-CL	**0.32**	0.03	0.16	0.00	0.03	0.41	0.19	**1.00**	0.00	0.00	0.01	**0.07**	0.00	0.00	0.72	**0.41**	**0.31**	0.19	0.17	0.05	0.20
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00	0.00	**0.89**	0.30	0.15	0.00	0.00	0.06	**0.24**
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.05	0.01	0.00	0.00	0.00	0.00	0.02	0.06	0.10
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.02	0.01	0.00	0.00	0.00	0.64	**0.74**	0.00	0.19
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10
SM-FFA-IC	0.12	0.02	0.01	0.03	0.00	0.76	0.00	0.90	**0.05**	0.00	0.16	0.03	0.01	0.01	0.01	0.00	0.00	**0.72**	**0.74**	0.04	0.18
TM-DIR-CL	0.12	0.07	0.07	0.03	0.01	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.05	0.04	0.00	0.00	0.13	0.08
TM-DIR-ML	0.17	0.27	0.04	0.14	0.43	0.01	0.00	0.30	0.00	0.01	0.18	0.00	0.03	0.00	0.03	0.00	0.01	0.00	0.00	0.14	0.09
TM-DIR-WL	0.15	0.00	0.07	0.00	0.00	0.11	0.00	0.24	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.03	0.08	0.04
TM-DIR-CA	0.04	0.05	0.04	0.08	0.05	0.83	0.00	0.66	0.00	0.00	0.11	0.01	0.06	0.01	0.04	0.01	0.00	0.24	0.30	0.10	0.13
TM-DIR-IL	0.03	0.01	0.02	0.00	0.00	0.04	0.00	0.75	0.00	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.05
TM-DIR-IC	0.04	0.03	0.00	0.01	0.09	0.05	0.00	0.45	0.00	0.00	0.04	0.00	0.03	0.02	0.00	0.00	0.00	0.05	0.15	0.06	0.05
TM-FFA-CL	0.20	0.04	0.12	0.06	0.01	0.00	0.02	**1.00**	0.00	0.00	0.00	0.00	0.01	0.00	0.34	0.32	0.07	0.00	0.00	0.09	0.11
TM-FFA-ML	0.24	0.28	0.09	0.13	**0.58**	0.00	0.00	0.72	0.00	0.03	0.05	0.00	0.01	0.00	0.07	0.09	0.04	0.00	0.00	0.12	0.12
TM-FFA-WL	0.17	0.00	0.15	0.00	0.00	0.17	0.00	0.74	0.01	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.08	0.16	**0.16**	0.08
TM-FFA-CA	0.15	0.14	0.01	0.06	0.21	**0.87**	0.00	0.76	0.01	0.02	0.12	0.01	0.00	**0.04**	0.03	0.00	0.00	0.45	0.63	0.12	0.18
TM-FFA-IL	0.03	0.06	0.03	0.02	0.02	0.42	0.00	0.76	0.00	0.00	0.09	0.00	0.03	**0.04**	0.00	0.00	0.00	0.00	0.01	0.04	0.08
TM-FFA-IC	0.05	0.14	0.00	0.04	0.15	0.09	0.01	0.66	0.01	0.00	0.05	0.01	0.00	0.01	0.00	0.00	0.00	0.29	0.44	0.01	0.10
All	0.14	0.10	0.07	0.06	0.10	0.35	0.03	0.76	0.01	0.01	0.07	0.01	0.02	0.01	0.14	0.05	0.04	0.15	0.19	0.06	0.12

Easiest problems: ℓ20, gcd, ma5

Hardest problems: prm, lsb, srb, ssq

Summary

Summary

- Evolution of non-trivial exact integer algorithms is possible

Summary

- Evolution of non-trivial exact integer algorithms is possible
- ...but a hard problem

Summary

- Evolution of non-trivial exact integer algorithms is possible
- ...but a hard problem
- Search for novel behaviors [6, 7] (via FFA [5]) = good

Summary

- Evolution of non-trivial exact integer algorithms is possible
- ...but a hard problem
- Search for novel behaviors [6, 7] (via FFA [5]) = good
- Transactional memory idea = not good

Summary

- Evolution of non-trivial exact integer algorithms is possible
- ...but a hard problem
- Search for novel behaviors [6, 7] (via FFA [5]) = good
- Transactional memory idea = not good
- Benchmark set: will hopefully be interesting for a long time

Evolved Programs

CL-SM program for srb

- Computes square root of a number
- Contains the same code twice . . . remove one copy and it stops working for the single input 5

Listing 1: Manual Translation to Java

```
static int srb(int m0) {
  int m2 = 0, t = 0;

  for (int i = m0; i > 0; i--) {
    t  = m0 / (1 + m2);
    m2 = (1 + m2 - ((t != 0) , (m2 / t) , m2));
    t  = m0 / (1 + m2);
    m2 = (1 + m2 - ((t != 0) , (m2 / t) , m2));
  }
  return m2;
}
```

CA-TM program for lsb

Listing 2: Manual Translation to Java

```
static int gpLSB(int m0) {
  int m1 = 0, m2 = 0, tm0 = m0, tm2 = m2;

  for (;;) {  // small m0 -> many iterations
    if (m0 != 0) {
      tm2 = (m1 - m0);
      m1  = 1;
      if (m2 != 0) {
        tm2 = (m2 - m0);
        if (tm2 != 0) tm0 = (m0 % tm2);
      }
    }
    if ((tm0 == m0) && (tm2 == m2))
        return m2;
    m0 = tm0;
    m2 = tm2;
  }
}
```

- Calculates value of least significant bit
- Faster for larger inputs (probably utilizes integer overflow. . .)

CL-TM program for qad

- Calculates $(m_0 - 1)(m_0 - 2)$ without multiplication

Listing 3: Manual Translation to Java

```
static int qad(int m0) {
  int m1 = 0, t = (m0 - 1), tm1 = 0;

  t = m0 - 1;
  for (int i = t; i > 0; i--) {
    tm1 = (m1 + m0);
    t   = m0 + m0 + m1 - 1;
    m1  = tm1;
  }

  return (m0 - (1 - t));
}
```

CA–TM program for gcd

Listing 4: Manual Translation to Java

```
static int gcd(int m0, int m1) {
  int m2 = 0, tm0 = 0, tm1 = 0, tm2 = 0;

  for (;;) {
    if (m1 != 0)
      tm1 = m0;

    tm0 = (m0 != 0) , (m1 % m0) , m1;

    if ((m0 != 0) && (m1 != 0) && ((m1 % m0) == 0))
      tm2 = m0;

    if ((m0 == tm0) && (m1 == tm1) && (m2 == tm2))
      return m2;

    m0 = tm0;
    m1 = tm1;
    m2 = tm2;
  }
}
```

- Calculates value of greatest common divisor

IC-SM program for mi5

Listing 5: Manual Translation to Java

```
static int mi5(int m0, int m1, int m2, int m3, int m4) {
  int t;

  for(;;) {
    t = m4;
    if(m4 > m0) m4 = m0;
    if(m4 > m2) m4 = m2;

    if((m4 == t) && (m2 == m1) &&
       (m1 == m3)) return m4;

    m2 = m1;
    m1 = m3;
  }
}
```

- Calculates minimum of five variables in a “serial” fashion

WL-SM program for 120

- Checks if a number is less than 20, without having any constant larger than 1
- (the constants in the listing are simplified results of calculations)

Listing 6: Manual Translation to Java

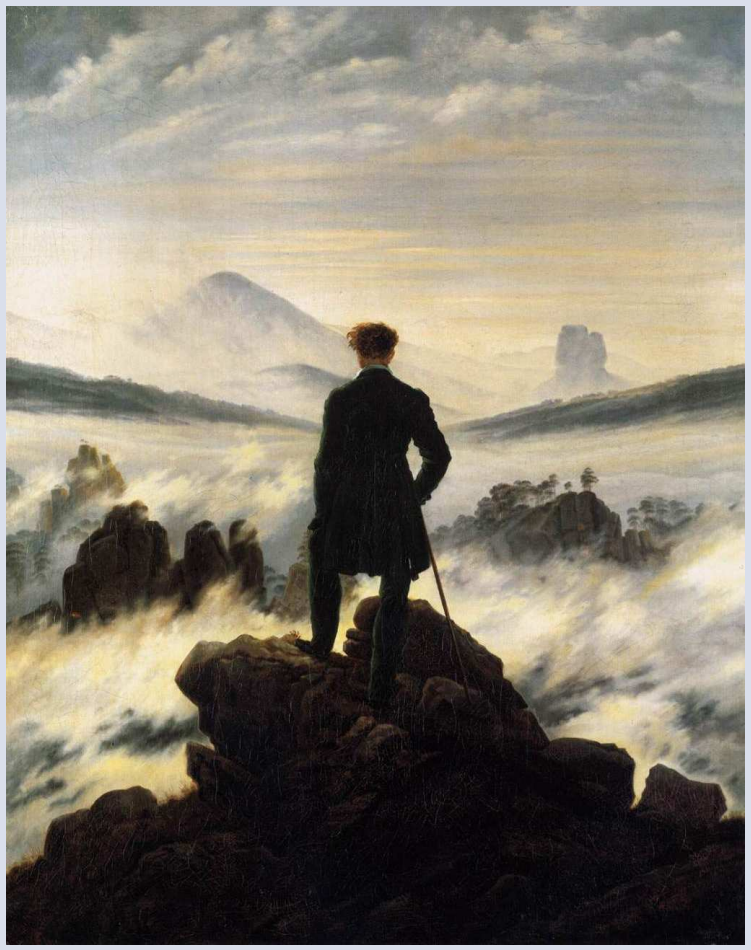
```
static int l20(int m0) {
  int t = ((m0 - 2) / 3) / 3; // integer division!
  if (t < 2) return 1;
  else       return 0;
}
```

谢谢！

Thank you very much for your kind attention.

Any questions?

email: tweise@ustc.edu.cn
web: http://www.it-weise.de/



References

References I

1. Mingxu Wan, Thomas Weise, and Ke Tang. Novel loop structures and the evolution of mathematical algorithms. In Sara Silva, James A. Foster, Miguel Nicolau, Penousal Machado, and Mario Giacobini, editors, *Proceedings of the 14th European Conference on Genetic Programming (EuroGP'11)*, volume 6621/2011 of *Lecture Notes in Computer Science (LNCS)*, pages 49–60, Torino, Italy, April 27–29, 2011. Berlin, Germany: Springer-Verlag GmbH. 10.1007/978-3-642-20407-4_5. URL http://www.it-weise.de/documents/files/WWT2011NLSATEOMA.pdf.
2. Guang Chen and Mengjie Zhang. Evolving while-loop structures in genetic programming for factorial and ant problems. In Shichao Zhang and Ray Jarvis, editors, *Advances in Artificial Intelligence. Proceedings of the 18th Australian Joint Conference on Artificial Intelligence (AI'05)*, volume 3809/2005 of *Lecture Notes in Computer Science (LNCS)*, pages 1079–1085, Sydney, NSW, Australia: University of Technology, Sydney (UTS), December 5–9, 2005. Berlin, Germany: Springer-Verlag GmbH. 10.1007/11589990_144.
3. Timothy Lai. Discovery of understandable math formulas using genetic programming. In *Genetic Algorithms and Genetic Programming at Stanford*, pages 118–127. Stanford, CA, USA: Stanford University Bookstore, Stanford University, Fall 2003. URL http://www.genetic-programming.org/sp2003/Lai.pdf.
4. Thomas Weise and Ke Tang. Evolving distributed algorithms with genetic programming. *IEEE Transactions on Evolutionary Computation (IEEE-EC)*, 16(2):242–265, April 2012. 10.1109/TEVC.2011.2112666. URL http://www.it-weise.de/documents/files/WT2011EVDAWGP.pdf.
5. Thomas Weise, Mingxu Wan, Ke Tang, Pu Wang, Alexandre Devert, and Xin Yao. Frequency fitness assignment. *IEEE Transactions on Evolutionary Computation (IEEE-EC)*, 18(2):226–243, April 2014. 10.1109/TEVC.2013.2251885. URL http://www.it-weise.de/documents/files/WWTWDY2014FFA.pdf.
6. Joel Lehman and Kenneth Owen Stanley. Abandoning objectives: Evolution through the search for novelty alone. *Evolutionary Computation*, 19(2):189–223, Summer 2011. 10.1162/EVCO_a_00025. URL http://eplex.cs.ucf.edu/papers/lehman_ecj10.pdf.
7. Marcus Hutter and Shane Legg. Fitness uniform optimization. *IEEE Transactions on Evolutionary Computation (IEEE-EC)*, 10(5): 568–589, October 2006. 10.1109/TEVC.2005.863127. URL http://arxiv.org/abs/cs/0610126v1.

Citation Information

```
@inproceedings{WWTY2014EEIAWGP,
  author    = {Thomas Weise and Mingxu Wan and Ke Tang and Xin Yao},
  title     = {{Evolving Exact Integer Algorithms with Genetic
                Programming}},
  booktitle = {Proceedings of the IEEE Congress on Evolutionary
               Computation (CEC'14), Proceedings of the 2014 World
               Congress on Computational Intelligence (WCCI'14)},
  publisher = {Los Alamitos, CA, USA: IEEE Computer Society Press},
  address   = {Beijing, China: Beijing International Convention
               Center (BICC)},
  pages     = {1816--1823},
  year      = {2014},
  month     = jul # {~6--11, },
  doi       = {10.1109/CEC.2014.6900292},
  isbn      = {978-1-4799-1488-3},
},
```