

Optimization Algorithm Behavior
Modeling:
A Study on the Travelling Salesman
Problem

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Outline

- 1 Research Background
- 2 Methodology
 - Modeling
 - Curve Fitting Method
 - Results
 - Predict of Unseen Instances Performance
 - Predicton of Future Progress

Outline

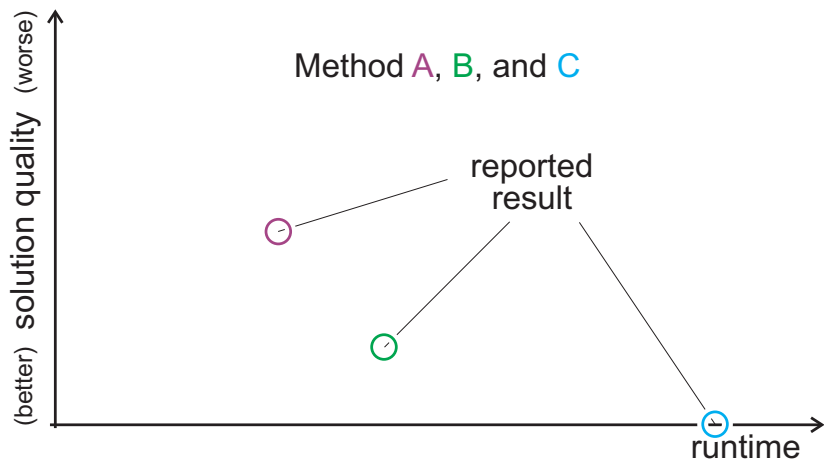
1 Reseach Background

2 Methodology

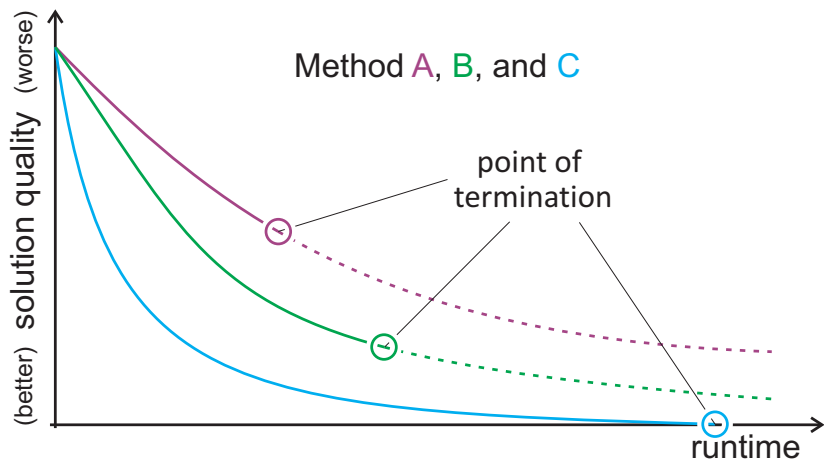
Anytime Algorithm

- 1 The solution quality of anytime algorithm improves step by step.
- 2 Anytime algorithm can provide approximate solution for problems at anytime during their run.
- 3 Many optimization algorithms belong to anytime algorithm like EA, local search algorithms.

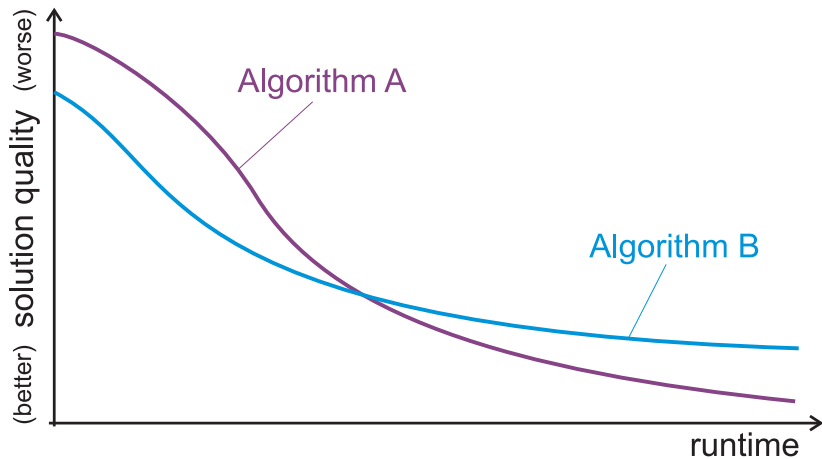
Existing Methods



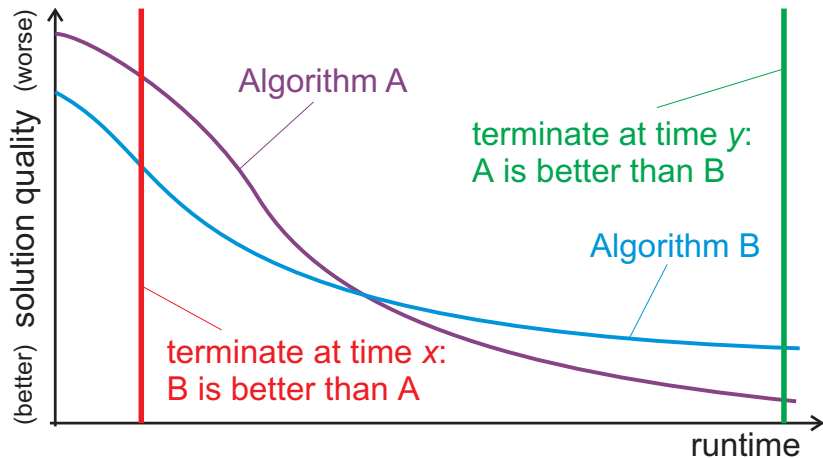
Existing Methods



Existing Methods



Existing Methods



What We Want to do

- 1** We investigate the time-quality relationship of optimization processes.
- 2** Analyze algorithms and problems based on the information gained.
- 3** Provide high-level information for the algorithms based on the whole runtime behavior of algorithms.

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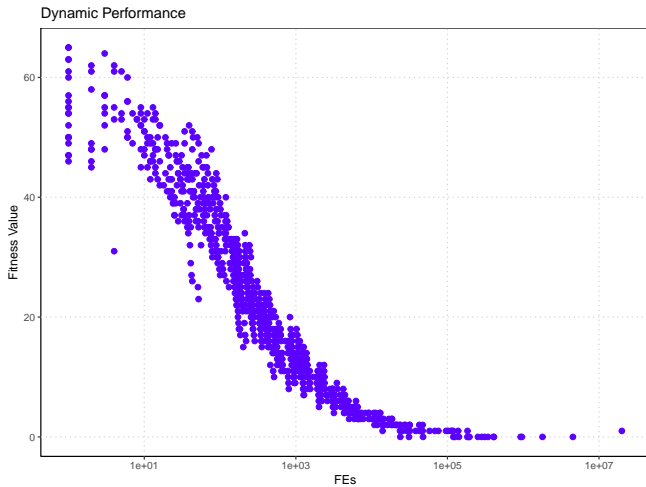
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Proposed Models

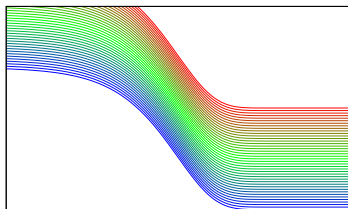


Model Propose

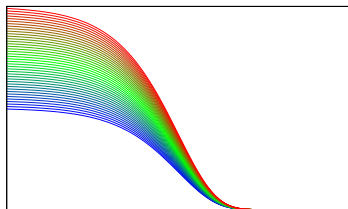
Table: The formulas for the four S-shaped models

Model	Shortcut	Formula
Decay	DCM	$A + B \exp(Cx^D)$
Logistic	LGM	$A + B/(1 + \exp(C \log(x) + D))$
Gompertz	GPM	$A + B \exp(C \ln(x + D))$
Exp-linear	EPM	$A + B \exp(C \exp(Dx))$

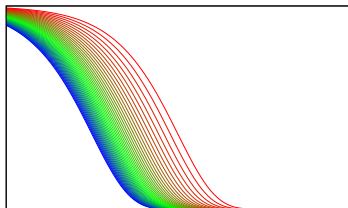
LGMP Behavior



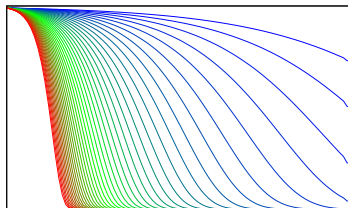
$$A \in [0, 50]$$



$$B \in [50, 100]$$



$$C \in [-0.1, -0.01]$$



$$D \in [0.3, 3]$$

Table: The meaning of the model parameters of the positive-B-shape of the Logistic model (LGMP).

Para.	Model Parameters	Algorithm Performance
A	Vertical offset of curves	The best performance of algorithms can get
B	Vertical range of curves	Algorithms' initial solution quality
C/D	Steepness of curves	Learning rates of algorithms

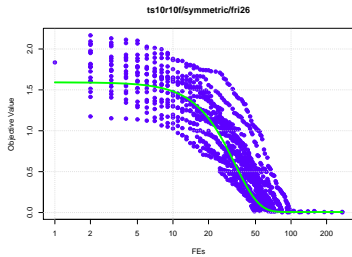
Curve Fitting Method

Cost function of fitting:

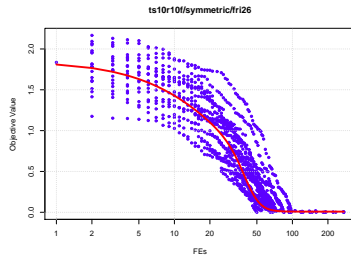
$$\min_{A,B,C,D} \frac{1}{n} \sum_{i=1}^n \frac{(f(x_i; A, B, C, D) - y_i)^2}{y_i} \quad (1)$$

- 1 Optimization: Levenberg-Marquardt algorithm
- 2 LM with Intelligent initialization strategy
 - Solve non-linear equations.
 - Limit the range value according to datasets.
 - Randomly generated based on Gaussian distribution.
 - Multiple Restarts.

Table



Parameter-Based Model



ANN

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Existing Method Shortcomings

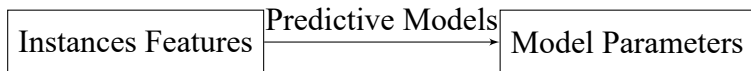
- 1 Algorithm selection is very important to solve problems:
 - Computational Expensive
 - Only Predict Arbitrary Runtime
- 2 Contribution:
 - Predict the full runtime of behaviors of new instances
 - Select algorithms based on computational budgets

Ideas

- 1 If we can get the predicted parameters A, B, C, D, that is the whole runtime behavior of algorithms

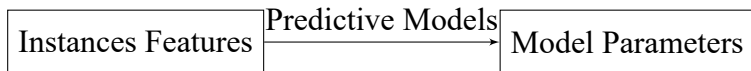
Ideas

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Ideas

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Whole runtime behavior

Process of Parameter Prediction

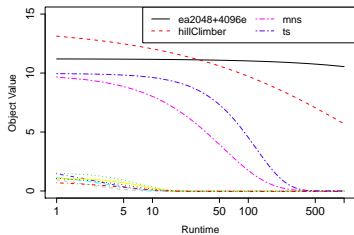
1 Feature Selection:

- Person Correlation (Top N)
- Spearman Correlation (Top N)
- MIC Correlation (Top N)
- Model-based Select (Top N)

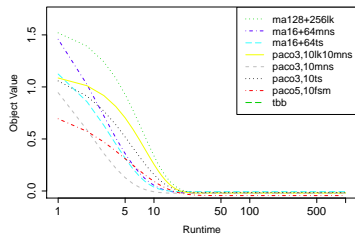
2 Ranking: Frequency of features in all correlation method (Top N).

3 Prediction Model: Neural Networks with different hidden nodes and layers.

Results



All Predict Instances



Subset of All predict Instances

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WMC Method

- 1 Representing algorithm behavior as function can compute, for any point in time, which solution quality the algorithm likely obtained
- 2 Used for prediction future progress of algorithms in the running optimization process.
- 3 $(train_t, test_t) = (50, 100)$ stands for predicting the complete algorithm behavior $(50, 100]$ during the process of first 50 time interval data collected.

WMC Method

$$q_p = \sum_{i=1}^m w_i q_i \quad (2)$$

where q_p is the prediction value of quality q in $test_t$ FE interval, and m is the number of models, which be 8 in our case. w_i is the weights of each model, which inverse ratio with the Φ_i , that is:

$$w_i = \frac{1}{m-1} \sum_{i=1}^m \left(1 - \frac{\Phi_i}{\sum_{i=1}^m \Phi_i}\right) \quad (3)$$

Results

Table: The prediction results between ANN and WMC in hill climber algorithms

Instance	WMC			ANN		
	10100	50100	1001000	10100	50100	1001000
burma14	0.00257	0.00268	0.00268	0.00119	0.00210	0.00161
ulysses16	0.00024	0.00079	0.00192	0.00119	0.00060	0.00178
gr17	0.00036	0.00069	0.00082	0.00035	0.00109	0.00067
gr21	0.00182	0.00427	0.00427	0.00686	0.00488	0.00437
ulysses22	0.00419	0.00098	0.00154	0.00167	0.00096	0.00153
gr24	0.00671	0.00146	0.00183	0.00287	0.00095	0.00125
fri26	0.00710	0.00127	0.00188	0.00736	0.00100	0.00048
bayg29	0.03006	0.00186	0.00193	0.01200	0.00123	0.00144
bays29	0.02468	0.00147	0.00001	0.00850	0.00140	0.00005
dantzig42	0.04709	0.00127	0.00287	0.03903	0.00050	0.00051
swiss42	0.03742	0.00127	0.00154	0.04487	0.00057	0.00123
att48	0.13983	0.00115	0.00113	0.08355	0.00052	0.00059
gr48	0.10958	0.00219	0.00190	0.10147	0.00083	0.00140
hk48	0.07905	0.00186	0.00588	0.11502	0.00068	0.00147
eil51	0.07627	0.00085	0.00234	0.10236	0.00032	0.00139

Thanks and Questions!